ENFORCED PRIVACY: FROM PRACTICE TO THEORY
Summary

Privacy protection is an important requirement in both everyday life and the Internet. As the Internet is an open network, adversaries can observe and manipulate data flowing over it. To ensure privacy in communications over open networks, cryptographic protocols have been widely used, and thus, proposing such protocols has become a popular research area. However, design of cryptographic protocols is difficult and error-prone. Thus, verification of such protocols is a necessary step before implementation. Formal analysis has shown its strength in proving or disproving privacy properties of cryptographic protocols, contrary to informal verification of cryptographic protocols which is not suitable for finding subtle privacy flaws.

To formally verify whether a protocol satisfies a property, there are usually three steps: 1) formally model the protocol, 2) formalise the property and 3) decide whether the formalised property is satisfied on the formal model. Depending on the differences on the formalisms used to model the protocol and the property, there are various formal approaches. Once a formal approach is chosen, that is, the first and third steps are determined, one only needs to focus on the second step.

In this thesis, we use a formalism called the applied pi calculus. The applied pi calculus provides an intuitive way to model cryptographic protocols. In addition, the applied pi calculus is equipped with proof techniques for privacy properties modelled as equivalences of processes. Furthermore, the verification of a protocol modelled in the applied pi calculus is supported by an automatic verification tool ProVerif.

Many privacy properties have been proposed, most of which are with respect to an adversary controlling the network. Recently, a stronger privacy property was identified in the e-voting domain. This privacy property assumes that the adversary can perform extra actions, namely bribing or coercing voters, to obtain additional information. To distinguish such strong privacy properties, we from here refer to these as enforced privacy, capturing the idea that the system enforces privacy, even if users try to reveal themselves due to bribery or coercion. Properties such as receipt-freeness and coercion-resistance, have been formalised in e-voting, to formally verify whether a protocol satisfies enforced privacy. The leading work of formalising enforced privacy in the applied pi calculus is the DKR framework, proposed by Delaune et al.

Following studies of enforced privacy in e-voting, protocols ensuring enforced privacy have also been proposed in other domains, e-auctions and e-health. However, such protocols have not been formally analysed. Thus, we first formally define enforced privacy in the e-auction and e-health domains. To validate the formal definitions, we formally verify enforced privacy in a case study in each domain.
First, we study the enforced privacy requirements in the e-auction and e-health domains. In both, enforced privacy is important. In the e-auction domain, there exists a type of auction, sealed-bid e-auction, where bidders seal their bids. In such auctions, it is required that a bidder should not be able to prove his bid to a coercer, so that malicious bidders cannot win with an unreasonable low price by controlling other bidder’s bidding prices. In the e-health domain, it is required that doctors should not be able to prove prescriptions of their patients to pharmaceutical companies, to prevent pharmaceutical companies bribing or coercing doctors to favour their medicine.

Second, we formally define enforced privacy properties in e-auctions and e-health. To do so, we first study whether we could apply the formalisations in voting to e-auctions and e-health. We found that we cannot directly adopt these existing formalisations. The first reason is that classical privacy, on which enforced privacy is built on, differs from domain to domain. In e-voting, the voting result is published. When the result is unanimous, the adversary knows every voter’s vote. Consequently, DKR defines classical privacy property in voting as: The adversary cannot detect any difference when two voters swap their votes. In effect, the definition uses a counter-balancing vote. In sealed-bid e-auction, a non-winning bidder’s bid is not revealed. Thus, we do not need a counter-balancing bid. Classical privacy in e-auction can be defined, in a classical way, as: The adversary cannot detect any difference when the bidder changes his bid. However, we do need a higher bid, so that the target bid does not win. The second reason that we cannot directly adopt the existing formalisations in e-voting to e-auctions and e-health, is that the amount of information the adversary bribes or coerces differs. In e-voting, each voter normally only votes once. Similarly, in sealed-bid e-auctions, each bidder bids only once. However, in e-health, a doctor uses the same protocol multiple times to prescribe medicine for patients. Classical privacy in e-health requires that the adversary cannot detect any difference when two doctors swap their prescriptions. This requirements focus on a specific session of a doctor. The corresponding enforced privacy focuses on the specific session as well. That is, not all the doctor’s information is revealed to the adversary. Therefore, we need to model doctors only partially communicating with the adversary, contrasting with in e-voting and e-auctions, voters/bidders fully communicating with the adversary.

Third, besides enforced privacy, we identify a new privacy property, independency of privacy in e-health. Unlike e-voting and e-auction systems, where roles can be naturally divided into two types: voters/bidders and authorities, e-health systems involve much more roles: doctors, patients, pharmacists, medical administrations, insurance companies. Some of these roles have access to sensitive data, for example, pharmacists have access to prescriptions. However, these roles may not be trustworthy, for example, the pharmacists may be bribed to reveal the link between a doctor and his prescriptions. To prevent this, it requires the pharmacist be independent of a doctor’s privacy. To capture this requirement, we formally define the property independency of prescribing-privacy. This property has also been verified in a case study.

Based on the experience of domain-specific formalisation of enforced privacy, we propose a formal framework, in which enforced privacy properties can be defined in a domain-independent manner. Inspired by the demand that only part of the
user process communicates with the adversary, we extend formalisation of bribery and coercion in e-voting, to allow us to specify the shared information with the adversary. This captures a variety of collaborations between users and the adversary. Each enforced privacy property and independency privacy property is parametrised with a collaboration specification. Specific enforced privacy and independency of privacy properties can be expressed by instantiating the parameter.

Inspired by the requirement in e-health that a pharmacist should not be able to help break a doctor’s privacy, we notice that third parties may influence a target user’s privacy. Here, pharmacists, cooperate with the adversary to break a target user’s privacy. In such cases, we say that third parties have negative influence on the target user’s privacy. On the other hand, third parties may help strengthen the target user’s privacy, by cooperating with the target user. We say that third parties have positive influence. For instance, a voter can swap receipts with a coerced voter, if the voter votes as the coercer wants, so that the coerced voter can vote freely. We say that the third parties and the target user form a coalition. A coalition can be specified in a similar way as the collaboration. The difference is that in collaboration, we do not need to specify the behaviour of the adversary because the adversary is modelled as the environment; in coalition, both the third parties and the target user’s behaviour need to be specified. We can formally define a coalition using a coalition specification. Correspondingly, we formalise privacy properties parametrised with coalitions. As an additional benefit, we can use coalitions to model various domain-specific classical privacy properties. For instance, vote-privacy, where a counter-balancing voter is required, can be considered as a coalition privacy with a coalition of the target voter and a counter-balancing voter. Finally, we prove the relations between the privacy properties defined in the formal framework.
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Introduction

In the physical world, privacy is assured by means such as locks and curtains. Communication over the Internet introduces new privacy risks. For example, a malicious user may spy on another user’s e-mails. As the usage of Internet-based services continues to grow, privacy on the Internet is becoming more and more important. This has led to a proliferation of privacy notions, including notions focused on communication privacy and those focused on data-bases, etc. Recently, a strong privacy notion has been proposed for e-voting. This new notion includes the assumption that adversaries can bribe or coerce users. This thesis studies the privacy effect of such an assumption in other domains and proposes a domain-independent notion, using a formal approach.

In the introduction, we discuss the privacy issues and privacy requirements in Internet-based services in Section 1.1. The approach that we use to address the privacy issues is introduced in Section 1.2. Next, detailed research challenges are presented in Section 1.3. Finally, we describe the organisation of the thesis in Section 1.4.

1.1 Privacy and enforced privacy

Privacy has been considered as an important requirement in daily life. For instance, curtains are used to keep people’s private life beyond the general public’s observation. In some cases, privacy is even required by law. For example, the U.S. HIPAA Privacy Rule (The Health Insurance Portability and Accountability Act of 1996) states that a patient’s medical records are required to be confidential in health care. Even in cases where privacy is not a legal requirement, privacy may remain a desirable and marketable property. For instance, people may want to publish an article anonymously to avoid problems, if the article is against a group of people’s benefits. Consequently, in the analogous Internet-based activities, privacy protection is required as well. In addition, in order to attract users, providers for Internet-based services where privacy is optional may offer privacy as a distinguishing feature. For instance, privacy protection is an aimed feature in the Internet search engine DuckDuckGo.

However, there have been more and more high-profile privacy incidents in Internet-based services, such as, private messages sent between users and Bloomberg’s financial terminals having leaked [6], a user’s activities being viewed by people who should not be able to do so in facebook [4]. These incidents cause more and more potential privacy breaking worries, for example, when a user posts a photo on facebook, additional data, such as the time, date, and place are recorded, the user can...
be traced using the collected data by facebook [2]. In order to register a service, a
user often needs to provide personal information to the service provider. Normally
the requested information is used for the purpose of authenticating users or pro-
viding better services. However, this provides the service providers an opportunity
to know more than the user expected, from the collected data. For example,

- Google can track search terms via cookies and IP addresses, thus can build
  a profile of you based on your gmail account and your search queries, for
  example, who you are, where you go and what you do.

  “We are moving to a Google that knows more about you.”
  – Google CEO Eric Schmidt, 10 February 2005 [3]

- Mobile phone companies can log users locations and associations between
  users’ credit card companies and other financial organisations [7].

- Internet service providers can log all transactions, monitor email, web ac-
  cesses, etc. [5]

Even if the service providers are all trustworthy on not revealing privacy, sending
information over the Internet may also leak information due to eavesdroppers on
the Internet. If the transmitted information is not well protected, an observer on
the Internet can obtain private information of users. For example, by observing the
location attached to a service query “what is the nearest coffee shop”, the observer
knows the location of the user.

All these privacy concerns lead to research on how to develop systems satisfying a
desired privacy property. Various techniques have been applied to ensure privacy.
For instance,

- Laws and regulations exist to ensure privacy on an administrative level. For
  example the European Union’s data protection directive (95/46/EC) regulates
  the processing of personal data.

- Access control exists to ensure privacy in the sense that only authorised
  roles can access protected data. Such an example is role-based access con-
  trol [RCHS03].

- Cryptographic protocols have been employed to ensure privacy on commu-
  nications over open networks (such as, the Internet). For example protocols
  using zero-knowledge proofs, allow an individual to prove a statement with-
  out revealing his secrets.

We observe that privacy controls focus on different interests. A breach of privacy
on any domain affects privacy in general. Hence, ensuring privacy requires that
privacy is assured on each individual domain.

For each domain, there are specific approaches towards ensuring privacy. And
in general, an approach used to ensure privacy in one domain does not translate
to another. For example, zero-knowledge proofs cannot be applied on laws. In
addition, for each domain, there are specific privacy requirements. For instance,
in cryptographic protocols:
1.1 Privacy and enforced privacy

- Adversaries, observing and manipulating messages over open networks, need to be considered. The use of computers and networks, in addition to facilitate honest users, provides adversaries stronger computing ability and the ability to control the networks.

- Privacy of encryption keys, which may not even be mentioned in some access controls (such as laws), may be critical in cryptographic protocols.

- Moreover, data can be easily spread over networks. And once the data is revealed, it is hardly possible to destroy it, as it may has been stored multiple times by many users. Thus, privacy breach, such as revealing private data, in the digital world may be more harmful than in the real world.

Therefore, domain specific expertise is required to design or evaluate specific privacy controls. In this thesis, we focus on privacy on the level of cryptographic protocols.

To capture privacy requirements in cryptographic protocols, privacy properties are proposed. In order to define a privacy property precisely, the adversary abilities need to be defined first. The best known adversary model is the Dolev-Yao adversary, who controls the whole network (observing, blocking, modifying, injecting messages on the network). Depending on protected items in various domains, many privacy properties have been defined, e.g., anonymity, untraceability, unlinkability, etc. A common feature for such privacy properties is that they assume users want to keep their information private. In other words, a system satisfies privacy under the assumption that users honestly follow the system execution.

However, this assumption has been shown not suitable in some cases. For instance, in e-voting, a voter may want to sell his vote to a vote-buyer. As vote-selling harms the e-voting system, e-voting systems require that a voter should not be able to prove his vote to others. In addition, a coercer may be able to force (using a gun or a threat) a voter to vote a certain candidate or vote in a certain way. Hence, e-voting systems also require coercion-resistance. Similar requirements have also been identified in other domains. For instance, some e-auction systems require that the adversary should not be able to coerce a non-winning bidder to show his bid. In e-health, a doctor should not be able to prove his prescriptions to a pharmaceutical company even with the help of pharmacists.

To capture these requirements, privacy properties have been proposed, for instance, receipt-freeness in e-voting and e-auctions, coercion-resistance in e-voting.

- Receipt-freeness in e-voting: a voter should not have a receipt to prove his vote to any vote-buyer;

- Receipt-freeness in sealed-bid e-auctions: a non-winning bidder should not be able to prove his bid to other bidders;

- Coercion-resistance in e-voting: a coercer should not be able to coerce a voter to vote in a certain way.

These properties all follow the same idea: If a system is designed in such a way that a user can lie to the adversary about the target information, a vote or a bid,
and the adversary cannot tell whether the user has lied, then the system satisfies this type of privacy.

We call this type of privacy, where users are assumed to reveal information to the adversary (by bribery/coercion or other methods), enforced privacy, meaning that the system enforces privacy upon its users instead of users desiring privacy. Enforced privacy properties have recently been identified. Compared to the classical privacy properties where users do not cooperate with the adversary, research on enforced privacy properties is less mature. This thesis focuses on enforced privacy.

1.2 Formal approach

As stated in the previous section, information sharing between users, which are distributed over open networks, follows communication protocols. Communication among users is open to interference by adversaries controlling the network. To ensure security and privacy in communications over open networks, cryptography is widely used in protocols. Hence, proposing cryptographic protocols has become a popular research area.

However, equipping protocols with cryptography is not a guarantee for security or privacy. In fact, the design of cryptographic protocols is notoriously difficult and error-prone. For instance, the Needham-Schroeder protocol [NS78] was found to allow the adversary to reuse an old and compromised session key [DS81]. An RFID protocol [HM04] with an untraceability claim was found flawed [Av05].

At the beginning, a claimed property of a protocol is empirically verified. Researchers study a protocol in detail and decide whether the protocol satisfies a claimed property. As flaws in cryptographic protocols are often subtle and counterintuitive, it is easy to make undetected mistakes in such informal verification. Thus, informal analysis is too prone to error to reliably verify cryptographic protocols, especially when the protocols are getting more and more complex. Protocols tend to involve increasing utilisation of cryptographic primitives and increasing concurrent distributed programs which can be executed by a large population of agents.

As informal reasoning was getting less reliable and efficient, people turned to formal approaches, a mathematically based technique, to help detect flaws. The use of formal approaches was initiated by Dolev and Yao to analyse secrecy properties of protocols. The main ideas are that

1. the adversary controls the whole network; and
2. encryptions are perfect, meaning that the adversary cannot undo an encryption and find the plain text.

Since the adversary controls the whole network, and communicated messages all pass through the network, the adversary can be modelled as a buffer between users. The messages from an honest user are considered to be sent to the adversary and the messages received by an honest user are considered to be sent by the adversary. In addition, the adversary can block, redirect and alter messages, and generate fresh messages. Security properties and privacy properties of a protocol are modelled as properties of such a model of the protocol.
1.3 Research questions

Formal verification has been successfully used in proving or disproving the satisfaction of a claimed property. In several cases, formal verification found security or privacy flaws in protocols which were thought to be secure or privacy-preserving, e.g., see [Low96, CKS04, DKR09]. Therefore, before using a cryptographic protocol, formal verification is an important step.

To formally verify whether a protocol satisfies a property, there are usually three steps:

1. formally model the protocol,
2. formalise the property, and
3. decide whether the formalised property is satisfied on the formal model.

Depending on the differences on the formalisms used to model the protocol and the property, and verification algorithms used to decide whether a property is satisfied on a formal model, there are various formal approaches. In general, in the literature, the study of specification languages for modelling a protocol and its properties, and verification algorithms is rather mature. In contrast to this, the specification of privacy properties is still in its infancy. Formalisation of privacy properties is not as standard as other properties, like security. New privacy notions arise from time to time. In addition, privacy requirements differ from domain to domain. Thus, we use an existing formalism, the applied pi calculus, and focus on formalising privacy properties. The applied pi calculus provides an intuitive way to model cryptographic protocols. In addition, the applied pi calculus is equipped with proof techniques for privacy properties modelled as equivalences of processes. Furthermore, the verification of a protocol modelled in the applied pi calculus is supported by an automatic verification tool ProVerif. Moreover, a leading work in formalising enforced privacy properties in e-voting, receipt-freeness and coercion-resistance, uses the applied pi calculus. Therefore, in this thesis, we focus on studying and formalising enforced privacy using the applied pi calculus, learning from the formalisations of enforced privacy in e-voting.

1.3 Research questions

Current research on enforced privacy focuses on the e-voting domain. However, bribery and coercion, in general, are domain independent. Thus, the adversary can bribe or coerce users in any domain. Therefore, a natural question to ask is: Are privacy requirements against bribery and coercion relevant in other domains? If the answer is yes, then how do the requirements in other domains differ from those in e-voting?

We answer these questions by studying enforced privacy requirements in e-voting, e-auctions and e-health. We observe that enforced privacy is indeed required in other domains. In e-auctions, it requires that a non-winning bidder should not be able to prove his bid to a coercer; in e-health, it requires that a doctor should not be able to prove his prescriptions to a pharmaceutical company.

Enforced privacy requirements differ in domains. Despite the difference in the protected items, privacy requirements may also differ in the way a user cooperates
Chapter 1  Introduction

with the adversary. For example, in e-voting, the adversary may instruct a voter to vote for a certain candidate, to vote the same as another voter (copy another’s vote), to abstain etc. In general, theses instructions may also be given by the adversary in e-auctions. However, in some sealed-bid auctions where each bidder is required to submit a bid, the adversary, in order to achieve the goal of controlling the price, would not give the abstaining instruction to bidders. In e-health, abstention is not a realistic instruction as well, since it conflicts the adversary’s goal. In addition, privacy notions may differ for the various kinds of users. For example, in e-voting, enforced privacy needs to be satisfied for all voters; in some sealed bid e-auctions, enforced privacy only needs to be satisfied for non-winning bidders. Finally, privacy requirements may differ in the influence of third parties. In e-voting and e-auctions, in general, two roles are involved and one of them is assumed to be honest. In e-health, many roles are involved, typically including: doctors, patients, pharmacists, insurance companies, etc. Some of these roles are in general not trustworthy, e.g., pharmacists. Thus, it is required that pharmacists should not be able to help prove a doctor’s prescriptions.

Due to these differences in domains, formalisations of privacy properties in e-voting, in general, cannot be directly adopted in other domains; for example, new privacy properties taking pharmacists into consideration need to be formalised in e-health.

Since enforced privacy is required and the formalisation of privacy properties is a necessary and important step in deciding whether a protocol satisfies such a privacy requirement, a research question arises:

**Research question 1:** How should enforced privacy be formalised in the e-auction and e-health domain?

Enforced privacy notions, e.g., receipt-freeness and coercion-resistance, have been formalised in e-voting. The first piece of work is the formalisation of receipt-freeness using the applied pi calculus. Later, following the direction of the applied pi calculus, a general formal framework for enforced privacy in e-voting has been developed – the DKR formal framework. Other formal frameworks, using other formal approaches or focusing on variations of enforced privacy properties, have also been developed in e-voting.

In e-auctions, enforced privacy properties have been discussed and protocols preserving them have been proposed. However, these properties are not formalised, protocols are not formally verified. Similarly, enforced privacy requirements can be found in the e-health domain, but no formalisation has been developed. Thus, a research task is to formally define enforced privacy in such domains. To do so, we first want to see whether the existing formalisations in e-voting can be reused. Among the few formal frameworks, we choose to follow the DKR formal framework, due to the convenience of the applied pi calculus.

We observe that although the formal definitions of enforced privacy in e-voting cannot be adopted due to the variety in classical privacy properties on which enforced privacy is built, the formalisation of bribery and coercion can be reused in general. Following the DKR style of defining the cooperation between bribed or coerced users and the adversary, we formalise enforced privacy in e-auction and e-health protocols in the applied pi calculus. This formalisation has been applied to two case studies – a sealed-bid e-auction protocol proposed by Abe and Suzuki
1.3 Research questions

(AS02) and an e-health protocol proposed by de Decker et al. (DLV08).

It is not convenient to formalise enforced privacy in every domain where it is
required. And the formalisations in the three domains follow the same idea and
only differ in minor details. These lead us to consider the generalisation of formal
definitions for enforced privacy.

Research question 2: How can we develop a generic formal framework in which
we are able to define various enforced privacy notions?

We learn from the DKR framework in e-voting and are inspired by a formal frame-
work for defining generic classical privacy in the applied pi calculus. With the
help of the knowledge and experience of formalising enforced privacy in e-auctions
and e-health, we propose a formal framework. This framework presents a stan-
dard form of protocols which enables us to formalise privacy in a generic manner.
We notice that the formalisation of bribery and coercion in e-voting, e-auctions
and e-health are instances of various types of cooperation between users and the
adversary and cooperation is not limited to bribery and coercion. To be able to
formalise more types of cooperation, a generic way of modelling cooperating users
is proposed. By allowing a precise specification of which information has been
shared with the adversary, the formalisation of bribery, coercion and many other
types of cooperation can be instantiated.

In addition, in this formal framework, we formally define a classical privacy notion
– data privacy. This formal definition is domain-independent and covers many
privacy requirements. It serves as the foundational privacy notion on top of which
enforced privacy notions can be built. Using the formalisation of coercion, we are
able to formalise various enforced privacy properties.

However, since data privacy does not cover all classical privacy formalisations,
e.g., vote-privacy cannot be instantiated as data privacy, enforced privacy notions
based on data privacy inherently have the same limitation, e.g., receipt-freeness
defined in DKR framework cannot be instantiated. We observe that those classi-
cal privacy formalisations which cannot be instantiated as data privacy, normally
require certain behaviour of users who are not the target user, e.g., vote-privacy
requires a voter besides a target voter to counter balance the target voter’s vote,
bid-privacy requires a bidder who bids higher than the target bidder. We consider
that in these privacy formalisations, the target user needs third parties’ help. On
the other hand, third parties helping to break a target user’s privacy has been
formalised in e-health. A natural research direction is to formalise privacy and
enforced privacy which takes third parties’ influence into account.

Research question 3: How do we formally define privacy notions taking third
parties’ influence into account in our framework?

We notice that the influence of a third party can be positive and negative. The
positive influence includes both active help and passive help from third parties. In
some cases, although third parties do not actively help, they do assist to create an
environment in which privacy may be satisfied. The pharmacists in e-health can
be considered as instances of such negative third parties.

The negative influence of third parties is modelled as the third parties cooperating
with the adversary. Using the formalisation of cooperation between users and the
adversary mentioned in the previous question, we are able to formalise the privacy notions taking third parties’ negative influence into account. The positive influence of third parties is modelled as a coalition of a target user and the third parties. A coalition is formalised as a transformation of the original target user and third party processes in the formal framework. Using the formalisation of coalition, we are able to formalise the privacy notions taking third parties’ positive influence into account. Finally, since many privacy notions have been formalised, we provide a hierarchy of these privacy notions and prove the relations between them.

1.4 Thesis overview

The first research question is answered in Chapter 4 and Chapter 5. And the last two research questions are answered in Chapter 6. The remaining part of the thesis is organised as follows.

Chapter 2: Related work In this chapter, we discuss the adversary model, privacy notions and enforced privacy notions in the literature.

Chapter 3: Preliminaries In this chapter, we focus on the applied pi calculus. We motivate the use of the applied pi calculus, briefly introduce the language, and present how enforced privacy in e-voting is formalised in the applied pi calculus.

Chapter 4: Enforced privacy in e-auctions This chapter focuses on enforced privacy in the e-auction domain. We first briefly introduce e-auctions. Next, privacy and enforced privacy requirements in e-auction systems are discussed. Then, we formalise a classical privacy notion – bid-privacy for non-winning bidders and an enforced privacy notion – receipt-freeness for non-winning bidders in the applied pi calculus. Finally, a case study – the AS02 protocol is introduced, modelled and analysed.

The main contribution of this chapter is the formalisation of enforced privacy in e-auctions and the formal analysis of the AS02 e-auction protocol. This chapter is based on work with Hugo Jonker and Jun Pang, published in [FAST10].

Chapter 5: Enforced privacy in e-health This chapter focuses on enforced privacy in the e-health domain. We briefly discuss privacy issues in e-health. Differing from e-auctions and e-voting, where enforced privacy is well identified, there is only few work on enforced privacy in e-health. Thus, enforced privacy in this domain remains a challenge. We contribute to this challenge in formally defining privacy notions which capture the enforced privacy requirements. Enforced privacy for doctor’s prescription behaviour is formalised. In addition, we formally define the privacy notions which capture the requirement to enforce a third party (pharmacist) to respect a target user’s privacy or enforced privacy (doctor prescribing privacy). Finally, a case study is performed to validate the formalisations of privacy notions. This case study includes a description of the DLV08 e-health protocol, the formal modelling of the protocol and the formal analysis. Few privacy flaws have been identified due to the ambiguous assumptions of the protocol and suggestions to
1.4 Thesis overview

address these flaws are proposed. The main contribution of this chapter is as follows: 1) We formalised enforced privacy in e-health; 2) identified and formalised independency of (enforced) privacy in e-health; and 3) formally analysed the DLV08 e-health protocol. This chapter is based on two published papers [FHIES11, ESORICS12].

Chapter [6]: Enforced privacy in the presence of others In this chapter, we generalise the domain specific enforced privacy. In addition, we take third parties’ influence on a target user’s privacy into account. Two types of influences are considered: positive (helping to maintain privacy) and negative (helping to break privacy). In order to formally define the above privacy in a domain-independent manner, we propose a formal framework. In the framework, a standard form of protocols is defined using the applied pi calculus. Based on the standard form, a classical privacy notion – data privacy, is formally defined which is used as the foundation of enforced privacy notions. In addition, a formalisation of a variety of ways for cooperation between users and the adversary is proposed. Using this formalisation, enforced privacy is defined based on data privacy. The behaviour of third parties is formalised in the formal framework as well: the negative behaviour is modelled as the cooperation between the third parties and the adversary; the positive behaviour is modelled as a coalition between the third parties and the target user. Using this formalisation, we formalise privacy notions with third party influences in the framework. Finally, a hierarchy of enforced privacy properties is built to show the relations between these notions. The relations are proved in the appendix.

The main contribution of this chapter is as follows: 1) We formalised collaboration between users and the adversary and the collaboration between users, in a formal framework. 2) We identified coalition privacy and formalised them in the formal framework, as well as other (enforced) privacy properties. 3) Finally, we proved the relations between the formalised privacy properties. This chapter is based on a paper with Hugo Jonker and Jun Pang published in [ESORICS13].

Chapter [7]: Conclusions and future remarks Finally, the thesis is concluded in Chapter [7].
Related work

This chapter is organised as follows. Section 2.1 introduces privacy properties of cryptographic protocols previously proposed in the literature, and Section 2.2 discusses existing formalisations of these privacy properties.

2.1 Privacy properties of cryptographic protocols

In general, privacy hides the correspondence between users and the users’ items (such as identities and votes) from the adversary. To define a privacy property, first of all, we need to define the adversary ability. As 1) our main focus is privacy of cryptographic protocols and 2) in such protocols, the use of the network is essential, the adversary from which privacy is protected is from the network. Many such adversary models exist, for instance, passive adversaries which only observe the network, and active adversaries which also actively communicate with the participants. Among those adversary models, the best-known is the Dolev-Yao adversary.

Dolev-Yao adversary \([DY83]\). The adversary has the following abilities:

- Controlling the network: The adversary is able to eavesdrop, block and inject messages on the network.
- Computational ability: The adversary can extract data from messages and compose new messages from known data.
- Initiating data and sessions: The adversary can generate fresh data as needed and can initiate a conversation with any user.
- Initial knowledge: The adversary has a set of initial knowledge containing public information, such as public keys.

This adversary is considered as the strongest adversary controlling the whole network \([Cer01]\). When a system satisfies privacy with respect to the Dolev-Yao adversary, it also satisfies privacy with respect to weaker adversaries. Thus, in this thesis, we only consider privacy properties with respect to the Dolev-Yao adversary, that is, the variations of privacy properties due to weaker adversary models are neglected.

We show privacy properties of cryptographic protocols in the rest of this section. These properties are classified into two categories by whether the assumption that
the adversary bribes or coerces users is made. Privacy properties without this assumption, named *classical* privacy properties, are introduced in Section 2.1.1. Privacy properties with the assumption, named *enforced* privacy properties, are presented in Section 2.1.2.

### 2.1.1 Classical privacy properties

Classical privacy has been studied for long time [Cha85]. Classical privacy properties have been proposed for systems in various domains, such as, e-mail, digital cash, e-voting, e-auction, RFID (radio-frequency identification), location based services, etc. These properties can be distinguished by the protected items. Protected items naturally vary in domains. Even in the same domain, protected items can be different. For example, in e-mail systems, three privacy properties are distinguished:

- **Sender anonymity**: the adversary cannot identify the sender of a given message [SD02].
- **Recipient anonymity**: the adversary cannot identify the recipient of a given message [SD02].
- **Relation anonymity**: the adversary cannot link a sender to a receiver [PK00].

Variations of anonymity can be found in other systems, for instance, payment anonymity, where the adversary cannot identify the initiator of a payment.

Another typical privacy property is untraceability – the adversary cannot tell whether two choices (sending message \(A\) versus \(B\), or voting for candidate \(C\) versus \(D\)) are made by the same user. This property was first proposed to capture the requirement that a bank should not be able to trace payment records to the same account [Cha88]. Later, this privacy property is showed to be highly desired in location-based services [DDS11] and RFID systems [HMZH08, vMR08]. It has been shown that anonymity and untraceability are not comparable with respect to which one is stronger ([ACRR10]).

In addition to the above mentioned privacy properties, a large number of other privacy properties have also been proposed [BMW03, CL01, HM08]. With the number of properties increasing, different names were used to capture the same requirement, and the same name was used to capture different requirements by different authors (for examples, see [BP11]). Pfitzmann et al. proposed terminology of privacy properties which distinguishes *anonymity, unlinkability, unobservability* and *pseudonymity* [PK00].

- **Anonymity** is defined as not being identifiable within a set of subjects;
- **Unlinkability** is the state that two or more items are no more and no less related after the adversary’s observation than they are related concerning the adversary’s a-priori knowledge;

\footnote{Note that the authors of [ACRR10] use a different terminology for untraceability, namely unlinkability.}
2.1 Privacy properties of cryptographic protocols

- Unobservability is defined as items being indistinguishable from any other items;
- Pseudonymity is defined as the state of using a pseudonym as identity.

![Diagram of relations between anonymity and unobservability](Image)

Figure 2.1: Relations between anonymity and unobservability [PK00].

The relations of these privacy properties are as follows: unlinkability is a sufficient condition for anonymity, but not a necessary condition; unobservability implies anonymity. In the context where sending and receiving of messages are the items of interest, anonymity can be defined as unlinkability of an item and identifier of a subject, and the relations between anonymity and unobservability are as shown in Figure 2.1.

Note that other generalisations of privacy properties exist. For instance, Chothia et al., distinguish player anonymity and choice anonymity as follows [COPD06].

- Player anonymity is defined as the adversary being unable to distinguish who makes a particular choice (data or action).
- Choice anonymity is defined as the adversary being unable to distinguish which choice a player made.

Differences between the various privacy properties are often subtle, since they are used to capture similar requirements. In addition, due to the differences in context, privacy properties are often not easy to compare. For example, some authors state that anonymity and unlinkability are actually the same, while others distinguish them [BP11]. Bohli and Pashalidis proposed a framework in which privacy properties are defined in a consistent and comparable manner [BP11]. This framework considers systems having finite runs. Each run is uniquely associated with a user. They modelled the correspondence between the runs and the set of its users as a function $f$. Privacy aims to ensure properties of $f$. Privacy properties considered in the framework describe potentially different degrees to which $f$ remains hidden from the adversary. The privacy properties are distinguished by the information potentially revealed to the adversary:

- the set of user identifiers ($U_f$);
- the number of runs corresponding to each participant ($Q_f$);
- the partition of runs that is induced by $f$ ($P_f$);
The following privacy properties are considered: strong anonymity (SA), strong unlinkability with participation hiding (SUP), strong unlinkability with usage hiding (SUU), weak unlinkability with participation hiding (WUP), weak unlinkability with usage hiding (WUU), weak unlinkability (WU), pseudonymity (PS), anonymity (AN), and weak anonymity (WA). The properties are defined in Table 2.1. The relations between the privacy properties are shown in the hierarchy in Figure 2.2.

<table>
<thead>
<tr>
<th>privacy property</th>
<th>A system providing the property hides $f$ except for</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA</td>
<td>$</td>
</tr>
<tr>
<td>SUP</td>
<td>$U_f$</td>
</tr>
<tr>
<td>SUU</td>
<td>$C_f$</td>
</tr>
<tr>
<td>WUP</td>
<td>$U_f$ and $C_f$</td>
</tr>
<tr>
<td>WUU</td>
<td>$Q_f$</td>
</tr>
<tr>
<td>WU</td>
<td>$P_f$</td>
</tr>
<tr>
<td>PS</td>
<td>$P_f$ and $U_f$</td>
</tr>
<tr>
<td>AN</td>
<td>$P_f$ and $Q_f$</td>
</tr>
</tbody>
</table>

Table 2.1: Definitions of privacy properties [BP11].

2.1.2 Enforced privacy properties

A common feature of the above privacy properties is that users are assumed to keep their information private. Recently, it has been identified that in some cases users may reveal their private information due to bribery, blackmail or extortion, and such revealing information is different from the revealing information by the compromised users. Normally, compromised users are considered as part of the adversary, as the adversary fully controls the compromised users. However, the bribed or coerced users may lie to the adversary if it is possible. Thus, bribed or coerced users are not fully trusted by the adversary.
Privacy properties that account for revealing information from bribed or coerced users (enforced privacy properties) are first proposed in e-voting. To prevent vote-buying, a property named receipt-freeness is proposed [BT94]. An e-voting protocol is receipt-free if a voter cannot prove his vote to a vote-buyer. Later, a stronger property coercion-resistance is proposed to prevent coercion in e-voting [Oka97]. An e-voting protocol is coercion-resistant if a voter cannot be forced to vote in a certain way. Coercion-resistance is stronger than receipt-freeness because a vote-buyer is not assumed to interrupt the voting process, while a coercer can communicate with the voter during the voting process. Inspired by the enforced privacy properties in e-voting, similar properties are proposed in e-auctions [SM00, AS02]. In particular, in sealed-bid e-auctions, receipt-freeness for non-winning bidders is required to prevent the adversary from trying to control the winning price of the auction. Similar requirements have been found in the e-health domain, for example, a doctor should not be able to prove his prescriptions to a pharmaceutical company [IDLVV08].

2.2 Formalisations of privacy properties

In order to verify a claimed privacy property of a protocol, precise definitions of the property are required. A privacy property can be defined in different manners. For instance, we distinguish binary privacy from quantitative privacy.

- Binary privacy: A protocol either satisfies a privacy property or not.
- Quantitative privacy: It defines to which extent a protocol satisfies a claimed privacy property. For example, sender anonymity can be quantified by the number of participants from which the adversary cannot identify the sender [Cha88].

Quantitative enforced privacy properties have been defined for e-voting in a formal framework proposed by Jonker, Pang and Mauw – the JMP framework [JPM09]. In this framework, the enforced privacy property, coercion-resistance, is quantified using the size of a candidate set. If a voter is coerced to vote a candidate in the set, the voter can vote for another candidate in the set and lie to the adversary, without the adversary detecting whether the voter lied. Many ways to quantify privacy can be found [Cha88, RR98, BPS00]. How to quantify (enforced) privacy is beyond our concern. This thesis focuses on the first category, binary privacy.

Definitions of a privacy property also vary depending on the verification techniques used to prove the satisfaction of the definition. We distinguish directly proving a privacy property from proving a privacy property in a symbolic model. Directly proving (e.g., using game-based provable security) is normally achieved by showing that the adversary cannot solve the underlying hard problem in order to break the property, for example, integer factoring, discrete logarithm, 3-SAT, etc.

- Game-based provable security: A privacy property is defined as a game of the adversary and a hypothetical challenger. The privacy property is satisfied if the adversary cannot win the game.

\[^{2}\text{This relation is based on the formal definitions of the two properties proposed by Delaune et al. [DKR09]. In informal definitions, the distinction between the two properties are not so clear.}\]
Chapter 2  Related work

isfied if no polynomially bounded adversary has a non-negligible advantage against the challenger in the game. Enforced privacy properties in e-voting have been defined in this way: receipt-freeness for a specific voting protocol (Prêt à Voter) \cite{KR11} and a generic coercion-resistance for the e-voting domain \cite{KTV10}.

• Symbolic model: Typically, the Dolev-Yao assumption is adopted: Cryptographic primitives are assumed to be perfect, e.g., the adversary cannot undo an encryption; and messages are considered to be abstract, e.g., data are expressed as symbols instead of bit-strings.

As verification in symbolic models is easier to carry out and is often supported by tools. This thesis lies in the symbolic category.

In this category, formalisations of privacy properties vary depending on the used formal models. For instance,

• using epistemic model \cite{SS99,HO05}: Protocols are modelled as knowledge of users and the adversary. Epistemic logic is used to reason about knowledge. Privacy properties are formalised as epistemic formulas. Enforced privacy property in e-voting have been formalised based on epistemic logic, for instance, receipt-freeness by Jonker and Pieters \cite{JP06} and a framework for coercion-resistance proposed by Küsters and Truderung – the KT framework \cite{KT09}.

• using process algebra: The behaviour of a system can be intuitively modelled as a process. Privacy properties are typically modelled as relations of processes.

Compared to epistemic logic, process algebra is better at modelling the behaviour of protocols. Process algebras are designed for specifying concurrent systems, and thus are very suitable to model e-services in which users are often highly distributed. In addition, process algebras are often equipped with proof techniques for process equivalences and some of them are supported by automatic verification tools. In this thesis, privacy properties are formalised using process algebra.

Many process algebras are used to model cryptographic protocols and formalise privacy properties, for example, CSP (communicating sequential processes) \cite{Hoa78,Sch96,SS96,OC02}, spi calculus \cite{AG97} and the applied pi calculus \cite{AF01,KR05,DKR09}. Enforced privacy properties were first formalised using the applied pi calculus for a specific e-voting protocol \cite{KR05}. Later, a framework for e-voting was proposed using the applied pi calculus – the DKR framework \cite{DKR09}. In addition, enforced privacy properties for weighted voting were proposed using the applied pi calculus as well – the DLL framework proposed by Dreier, Lafourcade and Lakhnech \cite{DLL12}. The formalisation using the applied pi calculus is rather mature. Furthermore, compared to other process algebras, the applied pi calculus provides an intuitive way to model cryptographic protocols. And the verification of many properties is supported by the verification tool – ProVerif. Thus, in this thesis, we use the applied pi calculus to model protocols and formalise enforced privacy properties. In order to formalise enforced privacy properties, the cooperation between users and the adversary needs to be formalised first. So far in the
literature, the cooperation is only formalised in the context of e-voting as bribery or coercion using the applied pi calculus. This formalisation is proposed in a framework by Delaune, Kremer and Ryan – the DKR framework and is applied in many formal definitions of enforced privacy properties \cite{KR05,BHM08,DKR09,DLL12}. Thus, we follow the methodology of the DKR framework.

The DKR framework. In this framework, e-voting protocols are modelled as processes in which voter processes and authority processes run in parallel, using the applied pi calculus. To model the enforced privacy property – receipt-freeness, the bribery of a voter is formalised as the voter forwarding all private information to the adversary over a fresh channel. The process for a bribed voter is constructed from the original voter process – whenever a private data is generated or read-in in the original voter process, the data is sent to the adversary in the bribed voter process. Receipt-freeness is defined as follows: an e-voting protocol is receipt-free if there exists a process in which a voter can lie to the adversary and the adversary cannot distinguish the process from the constructed process in which the voter genuinely forwards information to the adversary. The coercion of a voter is stronger: The adversary is able to communicate with the coerced voter during the voting procedure. This is modelled by giving the adversary the ability to prepare information for the coerced voter. The coerced voter process is constructed from the original process in a similar way as constructing the bribed voter process. The main difference is that in the coerced voter process, the voter reads in information from the adversary over a particular channel if there exists information sending in the original voter process, and sends the information prepared by the adversary instead. Since the adversary prepares information for a coerced voter, the vote may be decided by the adversary instead of the voter. Thus, coercion-resistance is defined as: for any prepared information by the adversary (possibly empty) such that the intended vote is \(a\) (no matter it is decided by the adversary or the voter), there is a process in which the voter can vote for \(c\), and the adversary cannot distinguish the process from the process in which the voter genuinely shares information with the adversary and uses the information prepared by the adversary. Detailed formalisations are introduced in the next chapter (Chapter 3).
Preliminaries

As stated in Chapter 1, the design of cryptographic protocols is error-prone and formal verification has shown its strength in proving or disproving correctness of cryptographic protocols. To verify whether a protocol satisfies a claimed property, a necessary step is to specify the protocol. Therefore, in this chapter, we first introduce a formal language for modelling protocols – the applied pi calculus – in Section 3.1. Given a protocol modelled in the applied pi calculus, many properties can be automatically verified using the tool ProVerif. Thus, we introduce ProVerif in the next section (Section 3.2). As stated in Chapter 2, we focus on formalising enforced privacy following the DKR formal framework of modelling bribery and coercion. Thus, we show several definitions in the DKR framework in Section 3.3.

3.1 The applied pi calculus

The applied pi calculus is a language for modelling and analysing concurrent systems, in particular cryptographic protocols. The following briefly introduces its syntax, semantics and equivalence relations. It is mainly based on [AF01, RS10].

3.1.1 Syntax

The calculus assumes an infinite set of names, which are used to model communication channels or other atomic data, an infinite set of variables, which are used to model received messages, and a signature \( \Sigma \) consisting of a finite set of function symbols, which are used to model cryptographic primitives. Each function symbol has an arity. A function symbol with arity zero is a constant. Terms (which are used to model messages) are defined as names, variables, or function symbols applied to terms (see Figure 3.1).

\[
M, N, T ::= \text{terms} \\
a, b, c, m, n, \ldots \quad \text{names} \\
x, y, z \quad \text{variables} \\
f(M_1, \ldots, M_\ell) \quad \text{function application}
\]

Figure 3.1: Terms in the applied pi calculus.

**Example 3.1** (function symbols and terms). *Typical function symbols are enc with arity 2 for encryption, dec with arity 2 for decryption. The term for encrypting \( x \) with a key \( k \) is \( \text{enc}(x, k) \).*
The applied pi calculus assumes a sort system for terms. Terms can be of a base type (e.g., Key or a universal base type Data) or type $\text{Channel}(\omega)$ where $\omega$ is a type. A variable and a name can have any type. A function symbol can only be applied to and return, terms of base type. Terms are assumed to be well-sorted and substitutions preserve types.

Terms are often equipped with an equational theory $E$ – a set of equations on terms. The equational theory is normally used to capture features of cryptographic primitives. The equivalence relation induced by $E$ is denoted as $E$.

**Example 3.2 (equational theory).** The behaviour of symmetrical encryption and decryption can be captured by the following equation:

$$\text{dec}(\text{enc}(x, k), k) = E x,$$

where $x, k$ are variables.

Systems are described as processes: plain processes and extended processes (see Figure 3.2). In Figure 3.2 $M, N$ are terms, $n$ is a name, $x$ is a variable and $v$ is a metavariable, standing either for a name or a variable. The null process $0$ does nothing. The parallel composition $P \mid Q$ represents the sub-process $P$ and the sub-process $Q$ running in parallel. The replication $!P$ represents an infinite number of process $P$ running in parallel. The name restriction $\nu n. P$ bounds the name $n$ in the process $P$, which means the name $n$ is secret to the adversary. The conditional evaluation $M = E N$ represents equality over the equational theory rather than strict syntactic identity. The message input $\text{in}(v, x). P$ reads a message from channel $v$, and bounds the message to the variable $x$ in the following process $P$. The message output $\text{out}(v, M). P$ sends the message $M$ on the channel $v$, and then runs the process $P$. Extended processes add restriction on variables and active substitutions. The variable restriction $\nu x. A$ bounds the variable $x$ in the process $A$. The active substitution $\{M/x\}$ replaces variable $x$ with term $M$ in any process that it contacts with.
Example 3.3 (process and protocol). In a protocol, $A$ generates a nonce $m$, encrypts the message with a secret key $k$, then sends the encrypted message to $B$. The process modelling the behaviour of $A$ is:

$$Q_A := \nu.m \cdot \text{out}(ch, \text{enc}(m, k)).$$

where $ch$ is a public channel.

The process modelling the behaviour $B$ is:

$$Q_B := \text{in}(ch, x).$$

The process $Q$ models the complete protocol, composed of $A$ and $B$.

$$Q := \nu.k.(Q_A \parallel Q_B).$$

Names and variables have scopes. A name is *bound* if it is under restriction. A variable is *bound* by restrictions or inputs. Names and variables are *free* if they are not delimited by restrictions or by inputs. The sets of free names, free variables, bound names and bound variables of a process $A$ are denoted as $\text{fn}(A)$, $\text{fv}(A)$, $\text{bn}(A)$ and $\text{bv}(A)$, respectively. A term is *ground* when it does not contain variables. A process is *closed* if it does not contain free variables.

Example 3.4 (scopes of names and variables). In Example 3.3, $ch$ is a free name representing a public channel. Name $k$ is bound in process $Q$; name $m$ is bound in process $Q_A$. Variable $x$ is bound in process $Q_B$.

A frame is defined as an extended process built up from 0 and active substitutions by parallel composition and restrictions. The active substitutions in extended processes allow us to map an extended process $A$ to its frame $\text{frame}(A)$ by replacing every plain process in $A$ with 0.

Example 3.5 (frame). The frame of the process $\nu.m.(\text{out}(ch, x) \mid \{m/x\})$, denoted as $\text{frame}(\nu.m.(\text{out}(ch, x) \mid \{m/x\}))$, is $\nu.m.(\{m/x\})$.

The domain of a frame $B$, denoted as $\text{domain}(B)$, is the set of variables for which the frame defines a substitution.

Example 3.6 (domain). The domain of the frame in Example 3.5, denoted as $\text{domain}(\nu.m.(\{m/x\}))$ is $\{x\}$.

A context $C[.]$ is defined as a process with a hole, which may be filled with any process. An evaluation context is a context whose hole is not under a replication, a condition, an input or an output.

Example 3.7 (context). Process $\nu.k.(Q_A \mid _)$ is and evaluation context. When we fill the hole with process $Q_B$, we obtain the process in Example 3.3.

Finally, we abbreviate the process $\nu n_1 \ldots \nu n_n$ as $\nu \tilde{n}$, abbreviate the process $\nu n_1 \ldots \nu n_{i-1} \cdot \nu n_{i+1} \ldots \cdot \nu n_n$ as $\nu \tilde{n}/n_i$ (erasing process $\nu n_i$ from $\nu \tilde{n}$), and abbreviate $\{M_1/x_1\} \ldots \{M_n/x_n\}$ as $\{M_1/x_1, \ldots, M_n/x_n\}$. 
3.1.2 Operational semantics

The operational semantics of the applied pi calculus is defined by: 1) structural equivalence ($\equiv$), 2) internal reduction ($\rightarrow$), and 3) labelled reduction ($\overset{\alpha}{\rightarrow}$) of processes.

1) Informally, two processes are structurally equivalent if they model the same thing but differ in structure. Formally, structural equivalence of processes is the smallest equivalence relation on extended process that is closed by $\alpha$-conversion on names and variables, by application of evaluation contexts as shown in Figure 3.3.

<table>
<thead>
<tr>
<th>Rule</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>PAR - 0</td>
<td>$A</td>
</tr>
<tr>
<td>PAR - A</td>
<td>$A</td>
</tr>
<tr>
<td>PAR - C</td>
<td>$A</td>
</tr>
<tr>
<td>REPL</td>
<td>$!P \equiv P</td>
</tr>
<tr>
<td>SUBST</td>
<td>${M/x}</td>
</tr>
<tr>
<td>NEW - 0</td>
<td>$\nu u.0 \equiv 0$</td>
</tr>
<tr>
<td>NEW - C</td>
<td>$\nu u.\nu v.A \equiv \nu v.\nu u.A$</td>
</tr>
<tr>
<td>NEW - PAR</td>
<td>$A</td>
</tr>
<tr>
<td>ALIAS</td>
<td>$\nu x.{M/x} \equiv 0$</td>
</tr>
<tr>
<td>REWRITE</td>
<td>${M/x} \equiv {N/x}$ if $M =_E N$</td>
</tr>
</tbody>
</table>

Figure 3.3: Structural equivalence in the applied pi calculus.

2) Internal reduction is the smallest relation on extended processes closed under structural equivalence, application of evaluation of contexts as shown in Figure 3.4.

<table>
<thead>
<tr>
<th>Rule</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMM</td>
<td>$\text{out}(c, x).P</td>
</tr>
<tr>
<td>THEN</td>
<td>if $N =_E N$ then $P$ else $Q \rightarrow P$</td>
</tr>
<tr>
<td>ELSE</td>
<td>if $M =_E N$ then $P$ else $Q \rightarrow Q$ for ground terms $M, N$ where $M \neq_E N$</td>
</tr>
</tbody>
</table>

Figure 3.4: Internal reduction in the applied pi calculus.

3) The labelled reduction models the environment interacting with the processes. It defines a relation $A \overset{\alpha}{\rightarrow} A'$ as in Figure 3.5. The label $\alpha$ is either reading a term from the process’s environment, or sending a name or a variable of base type to the environment.

3.1.3 Equivalences

The applied pi calculus defines observational equivalence and labelled bisimilarity to model the indistinguishability of two processes by the adversary. It is proved that the two relations coincide [AF01, Liu11]. We mainly use the labelled bisimilarity for the convenience of proofs. Labelled bisimilarity is based on static equivalence: labelled bisimilarity compares the dynamic behaviour of processes, while static equivalence compares their static states (as represented by their frames).
### Definition 3.8 (static equivalence)

Two terms M and N are equal in the frame B, written as \((M =_E N)B\), iff there exists a set of restricted names \(\mathfrak{n}\) and a substitution \(\sigma\) such that \(B \equiv \nu\mathfrak{n}.\sigma, M\sigma =_E N\sigma\) and \(\mathfrak{n} \cap (\text{fn}(M) \cup \text{fn}(N)) = \emptyset\).

Closed frames B and B’ are statically equivalent, denoted as \(B \approx_s B’\), if:

1. \(\text{domain}(B) = \text{domain}(B’)\);
2. \(\forall \text{ terms } M, N: (M =_E N)B \iff (M =_E N)B’\).

Extended processes A, A’ are statically equivalent, denoted as \(A \approx_s A’\), if their frames are statically equivalent: \(\text{frame}(A) \approx_s \text{frame}(A’)\).

### Example 3.9 (equivalence of frames \([\text{AF01}]\))

The frame B and the frame B’, are equivalent. However, the two frames are not equivalent to frame B”, because the adversary can discriminate B” by testing \(y =_E f(x)\).

\[
\begin{align*}
B &:= \nu M.\{M/x\} | \nu N.\{N/y\} \\
B’ &:= \nu M.\{(f(M)/x) | \{g(M)/y\}\} \\
B” &:= \nu M.\{(M/x) | \{f(M)/y\}\}
\end{align*}
\]

where \(f\) and \(g\) are two function symbols without equations.

### Example 3.10 (static equivalence)

Process \(\{M/x\} | Q_1\) is statically equivalent to process \(\{M/x\} | Q_2\) where \(Q_1\) and \(Q_2\) are two closed plain processes, because the frame of the two processes are the statically equivalent, i.e., \(\{M/x\} \approx_s \{M/x\}\).

### Definition 3.11 (labelled bisimilarity)

Labelled bisimilarity \(\approx_\nu\) is the largest symmetric relation \(\mathcal{R}\) on closed extended processes, such that \(A \mathcal{R} B\) implies:

1. \(A \approx_s B\);
2. if \(A \xrightarrow{\alpha} A’\) then \(B \rightarrow^* B’\) and \(A’ \mathcal{R} B’\) for some \(B’\);
3. if \(A \xrightarrow{\alpha} A’\) and \(\text{fv}(\alpha) \subseteq \text{domain}(A)\) and \(\text{bn}(\alpha) \cap \text{fn}(B) = \emptyset\) then \(B \rightarrow^* \alpha \rightarrow^* B’\) and \(A’ \mathcal{R} B’\) for some \(B’\), where * denotes zero or more.
3.2 ProVerif

The verification of protocols modelled in the applied pi calculus is supported by an automatic verification tool ProVerif [Bla01, Bla02, Bla04]. The tool has been used to verify many secrecy, authentication and privacy properties, e.g., see [AB05, ABF07, BC08]. ProVerif takes a protocol and a property modelled in the applied pi calculus as input, returns a proof of correctness or flaws as output. A protocol modelled in the applied pi calculus is translated to Horn clauses [Hor51]. The adversary’s capabilities are modelled as Horn clauses as well. Using these clauses, verification of secrecy and authentication is equivalent to determining whether a particular state is reachable in the execution of the protocol.

Secrecy of a term is defined as the adversary cannot obtain the term by communicating with the protocol and/or applying cryptography on the output of the protocol [AB05]. The secrecy property is modelled as a predicate in ProVerif: the query of secrecy of term $M$ is “attack:$M$” [Bla01]. ProVerif determines whether the term $M$ can be inferred from the Horn clauses representing the adversary knowledge.

Authentication is captured by correspondence properties of events in processes: if one event happens the other event must have happened [ABF07, Bla09]. Events are tags which mark important stages reached by the protocol. Events have arguments, which allow us to express relationships between the arguments of events. A correspondence property is a formula of the form: $f(M) \rightsquigarrow g(N)$. That is, in any process if event $f(M)$ has been executed then the event $g(N)$ must have been previously executed and any relationship between $M$ and $N$ must be satisfied. To capture stronger authentication where an injective relationship between executions of participants is required, an injective correspondence property $f(M) \rightsquigarrow_{\text{inj}} g(N)$ is defined: in any process if event $f(M)$ is executed, there is a distinct earlier occurrence of the event $g(N)$ being executed and the relationship between $M$ and $N$ is satisfied.

A correspondence property $f(M) \rightsquigarrow g(N)$ can be translated into a secrecy property, thus can be verified in ProVerif [Bla02]. The translation is as follows: the begin event “$g(N).P$” is replaced with “if $N = N'$ then 0 else $P$” and the end event “$f(M).P''$” is replaced with “out(c, auth($M$)).P''”. For any closed term $N'$, if the modified process preserves the secrecy of auth($N'$), the process satisfies the correspondence property. The intuition is that when the process does not execute event $g(N)$, the event $f(M)$ cannot be executed, thus, the event $f(M)$ should be secret. This intuition is reflected in the translated process: given a term $N'$ as the parameter of the end event,

- if $N = N'$, the process stops, meaning that if the original process tries to execute $g(N)$, since the begin event is executed, the secrecy of the end event does not reflect the verification result of the correspondence property, thus the process stops;

- when $N' \neq N$, the original process does not execute $g(N)$, if the correspondence property is satisfied, the secrecy of $f(N')$ in the translated process
should hold, if the translated process does not satisfy secrecy of $f(N')$, then the trace in which the secrecy of $f(N')$ is flawed shows that the end event is executed without the begin event being executed before, thus the correspondence property is not satisfied.

The predicate to query non-injective correspondence property $f(M) \rightsquigarrow g(N)$ is $\text{evf}(M) \Rightarrow \text{evg}(N)$ and the predicate to query injective correspondence property $f(M) \rightsquigarrow_{\text{inj}} g(N)$ is $\text{evinjf}(M) \Rightarrow \text{evinjg}(N)$.

Secrecy and authentication are properties expressed as predicates on system behaviours. However, not all properties can be expressed as predicates on system behaviours. Many of such properties can be expressed as equivalences of processes, for example, strong secrecy which is defined as the adversary’s inability to distinguish when the secret changes. Therefore, in addition, ProVerif provides automatic verification of labelled bisimilarity of two processes which differ only in the choice of some terms [BAF08]. An operation “$\text{choice}[M, N]$” is introduced to model the different choices of a term in the two processes. Using this operation, the two processes can be written as one process – a bi-process.

Example 3.12. To verify the equivalence

$$\nu a.\nu b.\text{out}(ch, a).\text{out}(ch, e) \approx_i \nu a.\nu b.\text{out}(ch, b).\text{out}(ch, d)$$

where $ch$ is a public channel, $e$ and $d$ are two free names, we can query the following bi-process in ProVerif:

$$P := \nu a.\nu b.\text{out}(ch, \text{choice}[a, b]).\text{out}(ch, \text{choice}[e, d]).$$

Using the first parameter of all “$\text{choice}$” operations in a bi-process $P$, we obtain one side of the equivalence (denoted as $\text{fst}(P)$); using the second parameters, we obtain the other side (denoted as $\text{snd}(P)$).

Example 3.13. For the bi-process in Example 3.12, using the first parameter to replace each “$\text{choice}$” operation, we obtain $\nu a.\nu b.\text{out}(ch, a).\text{out}(ch, e)$, which is the left hand side of the equivalence in Example 3.12, using the second parameter to replace each “$\text{choice}$” operation, we obtain $\nu a.\nu b.\text{out}(ch, b).\text{out}(ch, d)$, which is the right hand side of the equivalence.

Given a bi-process $P$, ProVerif determines whether $\text{fst}(P)$ is labelled bisimilar to $\text{snd}(P)$. The fundamental idea is that ProVerif tries to prove that the executions of the bi-process are uniform: when $\text{fst}(P)$ can do a reduction to some $Q_1$, it implies that the bi-process can do a reduction to some bi-process $Q$, such that $\text{fst}(Q) \equiv Q_1$ and symmetrically for $\text{snd}(P)$ taking a reduction to $Q_2$.

ProVerif uses slightly different notations. We use both the notions in ProVerif and the applied pi calculus for the convenience of expression. The notations representing the same thing is presented in Table 3.1.
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### Notations in ProVerif

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<td>$M := E N$</td>
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<td></td>
<td>$P := \ldots$</td>
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Table 3.1: Different notations in ProVerif and applied pi.

### 3.3 The DKR framework

ProVerif has been successfully used for helping analyse enforced privacy properties which are formalised for specific e-voting protocols, using the applied pi calculus [KR05, BHM08]. The specific formalisations of enforced privacy have been generalised in the DKR framework [DKR09]. We introduce some definitions in the framework which are mentioned in this thesis.

In the framework, a voting protocol with $n_v$ of voters and $n_{ad}$ of authorities is modelled as $P_{vote} := \nu chandata.(P_K \mid P_{v_1} \mid \cdots \mid P_{v_{n_v}} \mid P_{ad_1} \mid \cdots \mid P_{ad_{n_{ad}}})$ where $P_{vi}$ is an instance of a voter, $P_{adj}$ is an instance of an authority, chandata is a set of private channel names and data, and $P_K$ generates and distributes keys.

One requirement in e-voting is to protect the link between a voter and his vote. This requirement is captured by the privacy property vote-privacy as in Definition 3.14. Since the final result of a voting is normally published, when voters vote unanimously, every voter’s vote is revealed. To avoid such situation, in the formalisation of vote-privacy, two voters are modelled and they need to vote differently.

**Definition 3.14** (vote-privacy [DKR09]). A voting protocol $P_{vote}$ respects vote-privacy if $C_v[P_{v A}\{a/vote\} \mid P_{v B}\{c/vote\}] \approx \ell C_v[P_{v A}\{c/vote\} \mid P_{v B}\{a/vote\}]$ for all possible $a$ and $c$.

In the definition, $C_v[] := \nu chandata.(P_K \mid P_{v_1} \mid \cdots \mid P_{v_{n_v-2}} \mid \_ \mid P_{ad_1} \mid \cdots \mid P_{ad_{n_{ad}}})$. $P_{v A}\{a/vote\}$ is a voter process in which the voted candidate vote is $a$, similarly, $P_{v B}\{c/vote\}$ is another voter process in which the voted candidate vote is $c$. On the right-hand side, the two voter processes swap voted candidates.

On top of vote-privacy, two enforced privacy properties are formalised: receipt-freeness and coercion-resistance. Receipt-freeness ensures that a voter cannot prove his vote to a vote-buyer. Coercion-resistance is defined to capture the requirement that a coercer should not be able to enforce a voter to vote in a certain way. In order to formalise the two enforced privacy properties, the behaviour of a bribed or coerced user needs to be formalised first.

The bribery of a voter is formalised as the voter forwarding all private information to the adversary over a fresh channel. Formally, the bribed behaviour of voter $P_v$ is given by $P^{chc}$ which is defined as follows:

**Definition 3.15** (process $P^{chc}$ [DKR09]). Let $P$ be a plain process and chc a fresh channel name. $P^{chc}$, the process that shares all of $P$’s secrets, is defined as:

- $0^{chc} \leq 0$, 

3.3 The DKR framework

- \((P \mid Q)^\text{chc} \equiv P^\text{chc} \mid Q^\text{chc}\),
- \((\nu n.P)^\text{chc} \equiv \nu n.\text{out}(\text{chc}, n).P^\text{chc}\) when \(n\) is a name of base type,
- \((\nu n.P)^\text{chc} \equiv \nu n.P^\text{chc}\) otherwise,
- \((\text{in}(v, x).P)^\text{chc} \equiv \text{in}(v, x).\text{out}(\text{chc}, x).P^\text{chc}\) when \(x\) is a variable of base type,
- \((\text{out}(v, M).P)^\text{chc} \equiv \text{out}(v, M).P^\text{chc}\),
- \((!P)^\text{chc} \equiv !P^\text{chc}\),
- \((\text{if } M = E N \text{ then } P \text{ else } Q)^\text{chc} \equiv \text{if } M = E N \text{ then } P^\text{chc} \text{ else } Q^\text{chc}\).

Delaune et al. also define the process transformation \(A^{\text{out}(\text{chc}, \cdot)}\), which can be considered as the process \(A\) hides \(\text{out}(\text{chc}, \cdot)\) (the outputs on the channel \(\text{chc}\)).

**Definition 3.16** (process \(A^{\text{out}(\text{chc}, \cdot)}\) [DKR09]). Let \(A\) be an extended process. We define the process \(A^{\text{out}(\text{chc}, \cdot)}\) as \(\nu \text{chc}.(A \parallel \text{in}(\text{chc}, x))\).

Using the above two definitions, receipt-freeness is defined as follows: There exists a process in which the bribed/coerced voter can lie to the adversary and the adversary cannot tell whether the voter lied.

**Definition 3.17** (receipt-freeness [DKR09]). A voting protocol \(P_{\text{vote}}\) is receipt-free if there exists a closed plain process \(P_f\) such that

- \(P_{f}^{\text{out}(\text{chc}, \cdot)} \approx_{\ell} P_v A\{a\/\text{vote}\},\)
- \(C_v[P_v A\{c\/\text{vote}\}^\text{chc} \mid P_v B\{a\/\text{vote}\}] \approx_{\ell} C_v[P_{f} \mid P_v B\{c\/\text{vote}\}]\)

Process \(P_f\) is the process in which the voter lies to the adversary. The first equivalence shows the real behaviour of process \(P_f\). The second equivalence shows that the adversary cannot tell whether the voter lied.

The coercion of a voter is defined as that the adversary is able to communicate with the coerced voter during the voting procedure. This is modelled by adding to the bribery behaviour the ability for the adversary to prepare information for the coerced voter to use. Formally, the coerced behaviour of \(P_v\) is given by \(P_v c_{\text{out}, \text{in}}\) which is defined in Definition 3.18.

**Definition 3.18** (process \(P c_{\text{out}, \text{in}}\) [DKR09]). Let \(P\) be a plain process and \(c_{\text{out}, \text{in}}\) be channel names. We define \(P c_{\text{out}, \text{in}}\) as follows:

- \(0 c_{\text{out}, \text{in}} \equiv 0,\)
- \((P \mid Q)^{c_{\text{out}, \text{in}}} \equiv P^{c_{\text{out}, \text{in}}} \mid Q^{c_{\text{out}, \text{in}}}\),
- \((\nu n.P)^{c_{\text{out}, \text{in}}} \equiv \nu n.\text{out}(c_{\text{out}, n}).P^{c_{\text{out}, \text{in}}}\) when \(n\) is a name of base type,
- \((\nu n.P)^{c_{\text{out}, \text{in}}} \equiv \nu n.P^{c_{\text{out}, \text{in}}}\) otherwise,
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- \((\text{in}(v, x). P)_{\text{c}_{\text{out}}, \text{c}_{\text{in}}} \doteq \text{in}(v, x). \text{out}(\text{c}_{\text{out}}, x). P_{\text{c}_{\text{out}}, \text{c}_{\text{in}}}) \) when \( x \) is a variable of base type,

- \((\text{in}(v, x). P)_{\text{c}_{\text{out}}, \text{c}_{\text{in}}} \doteq \text{in}(v, x). P_{\text{c}_{\text{out}}, \text{c}_{\text{in}}}) \) otherwise,

- \((\text{out}(v, M). P)_{\text{c}_{\text{out}}, \text{c}_{\text{in}}} \doteq \text{out}(v, M). P_{\text{c}_{\text{out}}, \text{c}_{\text{in}}}) ,

- \((! P)_{\text{c}_{\text{out}}, \text{c}_{\text{in}}} \doteq ! P_{\text{c}_{\text{out}}, \text{c}_{\text{in}}}) ,

- \((\text{if} M = E \text{ then } P \text{ else } Q)_{\text{c}_{\text{out}}, \text{c}_{\text{in}}} \doteq \text{if} M = E \text{ then } P_{\text{c}_{\text{out}}, \text{c}_{\text{in}}} \text{ else } Q_{\text{c}_{\text{out}}, \text{c}_{\text{in}}}) .

Coercion-resistance is defined similarly to receipt-freeness. The difference is that in coercion, the coerced voter’s vote may be decided by the adversary, since the adversary may prepare information for the coerced voter. For the simplicity of modelling, the adversary’s ability of providing information for the coerced voter is separated from the ability to distinguish two processes. The adversary’s ability to provide information is modelled as a context \( C[.] := \nu_{\text{c}_{\text{out}}}. \nu_{\text{c}_{\text{in}}}. (\_ | P) \) where \( P \) models the adversary’s behaviour, \( \text{c}_{\text{out}} \) and \( \text{c}_{\text{in}} \) are the two communication channels between the adversary and the coerced voter. What the adversary wants the coerced voter to vote is modelled by an equivalence \( C_v[\{P_{v A}\{?/vote\}\} | P_{v B}\{a/vote\}] \approx_{\ell} C_v[(P_{v A}\{c/vote\})^{\text{chc}} | P_{v B}\{a/vote\}] . \) The right side of the equivalence decides the vote. On the left side, symbol “?” is used to represent any candidate, as no matter what the symbol is replaced with, it does not affect the vote.

**Definition 3.19** (coercion-resistance [DKR09]). A voting protocol \( P_{\text{vote}} \) satisfies coercion-resistance if there exists a closed plain process \( P_f \) such that for any \( C[.] := \nu_{\text{c}_{\text{out}}}. \nu_{\text{c}_{\text{in}}}. (\_ | P) \) satisfying \( \hat{n} \cap \text{fn}(C[.]) = \emptyset \) and \( C_v[C[P_{v A}\{?/vote\}] | P_{v B}\{a/vote\}] \approx_{\ell} C_v[(P_{v A}\{c/vote\})^{\text{chc}} | P_{v B}\{a/vote\}] , \) we have

- \( C[|P_f\rangle_{\text{out}(\text{chc}, \_)} \approx_{\ell} P_{v A}\{a/vote\} \)

- \( C_v[C[P_{v B}\{?/vote\}^{\text{c}_{\text{out}}, \text{c}_{\text{in}}} | P_{v j}\{a/vote\}] \approx_{\ell} C_v[C[P_f] | P_{v j}\{c/vote\}] \)

Intuitively, a protocol satisfies coercion-resistance, if for any successful coercion (the coerced voter follows the adversary’s instructions and successfully votes the candidate desired by the adversary), there exists a process \( P_f \) in which the coerced voter can diverge from the coerced behaviour and vote a different candidate; in addition, the adversary cannot tell whether the coerced voter genuinely follows his instruction.
Enforced privacy in e-auctions

Besides e-voting, enforced privacy requirements have also been identified in the e-auction domain. Unlike in the e-voting domain, enforced privacy has not been formally defined in e-auctions. In this chapter, we briefly introduce e-auctions, and then discuss privacy and enforced privacy requirements in the e-auction domain. Next we formalise privacy and enforced privacy in e-auctions. Finally we verify the formalised privacy properties of an e-auction protocol in a case study.

4.1 Introduction to e-auctions

Auctions are ways to negotiate exchange of goods and services. We use e-auctions to refer to auctions over the Internet. A typical (e-)auction works as follows: a seller offers items to bid, then bidders submit bids, finally auctioneers decide the winner. Compared to the traditional auctions, where bidders attend the auction in person, e-auctions attract more participants, as users with the Internet can join an auction. Real-life examples are websites like eBay, eBid, Yahoo!auctions and so on.

There are different types of (e-)auctions. For instance, depending on whether the bids are public, there are sealed-bid auctions and open-bid auctions.

- **Sealed-bid auctions**: There are two phases in an auction: the bidding phase and the opening phase. Bidders can only submit bids in the bidding phase. All bids are sealed in the bidding phase and opened in the opening phase.
- **Open-bid auctions**: Bids are broadcast to all participants.

Other criteria to classify (e-)auctions exist. For example, depending on the bidding price increases or decreases, there are English auctions (A bid needs to be higher than the previous one; the winning bid is the final bid) and Dutch auctions (The bidding price decreases until a bid is submitted); depending on the calculation of payment, there are first-price auctions (The winner pays the price he bid, the highest price) and Vickrey auctions (The winner pays the second highest price). Different auctions are suitable for different types of negotiations, e.g., English auctions are often used in real estate, Dutch auctions are often used in flower selling, and Vickrey auctions are favoured by economists as that Vickrey auctions are better at encouraging bidders to express their real estimation on the value of the items to bid on [Tre07].

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This chapter is based on published work [FAST10]
Chapter 4 Enforced privacy in e-auctions

4.2 Privacy and enforced privacy in e-auctions

Many security issues have been identified in e-auctions, such as, a bidder may falsely claim or forge bids, a seller may not deliver goods, the auctioneer may corrupt with other bidders [Tre05]. Beside security issues, an important problem with existing e-auction systems is privacy. The link between a bidder and his bids needs to be protected as such information can be used to target a bidder with unsolicited junk mails or other malicious purposes, e.g., bid shielding. A major challenge of designing a protocol is to ensure the functionality of the protocol. In addition to that, a challenge for designing a privacy preserving e-auction protocol is that too much anonymity may allow bidders to repudiate bids: if nothing can be learned about the bids, the winning bidder can simply not claim to have won, and no one can identify him. On the other hand, insufficient anonymity may allow bidders to be profiled.

Depending on different types of auctions, privacy may have varying levels. For instance, in sealed-bid auctions, all bids are sealed until the winner is determined, thus, if auctioneers can decide the winners without knowing the non-winning bidder’s bids, sealed-bid auctions can offer bidding-price secrecy for non-winning bidders; while in open-bid auctions, all the bids are published. Some auctions require that the auctioneer cannot link a bidder to his bids, whereas some others do not. The arguments of this are made according to the following lines. In Vickery auctions, a bidder’s bid reflects the bidder’s valuation of the item bid on. Knowing a bidder’s bid, an auctioneer knows the bidder’s valuation. Since the winning bidder pays the second highest price, the auctioneer could enter a bid just under the bidder’s valuation, to increase the auction’s revenue [Tre07]. Contrarily in English auctions, a bidder’s previous bids reveal less information of the bidder’s future bid, thus, that the auctioneer knows the link between a bidder and his previous bids is less harmful [Tre07]. In general, sealed-bid e-auctions require that the non-winning bidder’s bidder-bid relation should be kept secret.

In addition to the above privacy notions, a stronger privacy notion – enforced privacy – has also been identified. In sealed-bid e-auctions, a bidder may be coerced to bid a low price, so that the coercer can win an auction with an unreasonably low price. The phenomenon that a coercer tries to control the winning price by coercion is call bid-rigging. Note that the traditional auctions do not suffer from bid-rigging, as the bidders do not have receipts on submitting a bid [HGP09]. Inspired by the requirement of receipt-freeness in e-voting that a voter should not be able to prove his vote to a voter-buyer, the requirement of receipt-freeness for fighting against bid-rigging has been identified [SM00].

In general, the following two privacy notions are required in sealed-bid e-auctions:

**Bidding-price-secrecy for non-winning bidders:** An e-auction protocol preserves bidding-price-secrecy for non-winning bidders if the adversary cannot determine the bidding price of any non-winning bidder.

---

1A dishonest bidder submits a higher price to deter other bidders with lower valuations, when it approaches the close time of the auction, the dishonest bidder withdraws his bid in order to win with another lower bid from him.
Receipt-freeness for non-winning bidders: An e-auction protocol is receipt-free for non-winning bidders if a non-winning bidder cannot prove how he bids to the adversary.

4.3 Formalisation of privacy notions

We formalise the above two privacy notions using the applied pi calculus in the context of sealed-bid e-auctions. An e-auction protocol normally involves two roles: bidders and auctioneers. An e-auction protocol with \( n_b \) bidders and \( n_a \) auctioneers can be modelled as:

\[
P_{\text{bid}} := \nu \text{chandata}.(P_K \mid P_{b1} \mid \cdots \mid P_{bn} \mid P_{a1} \mid \cdots \mid P_{an}),
\]

where \( P_{bi} \) is an instance of a bidder process, \( P_{aj} \) is an instance of an auctioneer process, \( P_K \) is the key distribution process, and \( \text{chandata} \) models private data and private channels. The corresponding context in where two bidder processes are replaced with a hole is formalised as:

\[
C_b[.] := \nu \text{chandata}.(P_K \mid P_{b1} \mid \cdots \mid P_{b_{n_b-2}} \mid - \mid P_{a1} \mid \cdots \mid P_{an}).
\]

4.3.1 Bidding-price-secrecy

Bidding-price-secrecy for non-winning bidders can be formalised in two levels: standard bidding-price-secrecy and strong bidding-price-secrecy. Standard bidding-price-secrecy is formalised as the adversary cannot derive the bidding price of a non-winning bidder. Strong bidding-price-secrecy is formalised as the adversary cannot even distinguish between the case when a bidder bids for price \( a \) and the case when the bidder bids for price \( c \). In other words, the adversary cannot tell whether a bidder changes his bidding price from \( a \) to \( c \).

Formalisation similar to strong bidding-price-secrecy has been used, e.g., vote-privacy [DKR09]: a process in which voter \( v_A \) votes for \( a \) \((P_{vA\{a/vote\}})\) and voter \( v_B \) votes for \( c \) \((P_{vB\{c/vote\}})\) is observationally equivalent to a process where \( v_A \) votes for \( c \) \((P_{vA\{c/vote\}})\) and \( v_B \) votes for \( a \) \((P_{vB\{a/vote\}})\). The idea is that even if all other voters reveal how they voted, the adversary cannot deduce the votes of voter \( v_A \) and voter \( v_B \), given voter \( v_A \) and voter \( v_B \) counterbalance each other. Different from privacy in voting where the voting result is published, in sealed-bid e-auction protocols, normally a non-winning bidder’s bidding price is not published. Therefore, we do not need a counterbalancing process. Instead, we need a process in which a bidder bids for a higher price so that non-winning bids are not revealed in the opening phase. Therefore, strong bidding-price-secrecy is formalised as follows:

**Definition 4.1** (strong bidding-price-secrecy for non-winning bidders). An auction protocol \( P_{\text{bid}} \), with a bidder sub-process represented as \( P_b \), satisfies strong bidding-price-secrecy for non-winning bidders, if for all possible bidders \( b_A \) and \( b_B \) we have:

\[
C_b[P_{bA\{a/p_b\}} \mid P_{bB\{d/p_b\}}] \approx_c C_b[P_{bA\{c/p_b\}} \mid P_{bB\{d/p_b\}}]
\]

with \( a < d \) and \( c < d \).
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The context $C_b[.]$ is used to capture the assumption made on the checked protocol, usually it includes the other honest participants in the protocol. The process $P_{bA}$ is a non-winning bidder process executed by bidder $b_A$. The process $P_{bB}$ is a bidder process in which the bidder $b_B$ bids for a higher price $d$. The intuition is that the adversary cannot determine whether a non-winning bidder bids for price $a$ or price $c$, provided there exists another bidder who bids for a higher price $d$.

4.3.2 Receipt-freeness

Similar to receipt-freeness in e-voting, when modelling receipt-freeness for non-winning bidders in e-auctions, we also need to model the situation in which a bidder wants to provide his secret information to the adversary. We use the definition in e-voting (Definition 3.15) directly in our model. Intuitively, a bidder, who is willing to share information with the adversary, sends any input of base type, any freshly generated names of base type to the adversary through a fresh public channel $\text{chc}$. Note that public channels are under the adversary’s control.

Now, we can define receipt-freeness for sealed-bid e-auction protocols. Again, we need a bidder process $P_{bB}$ in which bidder $b_B$ bids for a higher price $d$, so that non-winning bids are not revealed. Intuitively, if a non-winning bidder has a strategy to cheat the adversary, and the adversary cannot tell the difference between whether the bidder cheats or not, then the protocol is receipt-free.

Definition 4.2 (receipt-freeness for non-winning bidders). An auction protocol $P_{bid}$, with a bidder sub-process $P_b$, satisfies receipt-freeness for non-winning bidders, if there exists a closed plain process $P_f$ such that:

1. $P_f^{\text{out}(\text{chc},.)} \approx\ell P_{bA}\{c/p_b\}$,

2. $C_b[P_{bA}\{a/p_b\}^{\text{chc}} | P_{bB}\{d/p_b\}] \approx\ell C_b[P_f | P_{bB}\{d/p_b\}]$

with $a < d$ and $c < d$.

Process $P_f$ is a bidder process in which bidder $b_A$ bids for price $c$ but communicates with the adversary and tells the adversary he bids for price $a$. Process $P_{bA}\{c/p_b\}$ is a bidder process in which bidder $b_A$ bids for price $c$. Process $P_{bA}\{a/p_b\}^{\text{chc}}$ is a bidder process in which bidder $b_A$ bids for price $a$ and shares his secret with the adversary. Process $P_{bB}$ is a bidder process in which bidder $b_B$ bids for a higher price $d$. The first equivalence says that ignoring the outputs bidder $b_A$ makes on the channel $\text{chc}$ to the adversary, $P_f$ looks like a normal process in which $b_A$ bids for price $c$. The second equivalence says that the adversary cannot tell the difference between the situation in which $b_A$ obeys the adversary’s commands and bids for price $a$, and the situation in which $b_A$ pretends to cooperate but actually bids for price $c$, provided there is a bidding process $P_{bB}$ that bids higher, ensuring that bidding processes $P_{bA}$ and $P_f$ are not winners. Receipt-freeness is a stronger property than bidding-price-secrecy, for the same reason as receipt-freeness in e-voting is stronger than vote-privacy (as shown [DKR09]).
4.4 Case study: the AS02 protocol

After receipt-freeness has been identified in sealed-bid e-auctions. Abe and Suzuki proposed the first protocol which aims to prevent bid-rigging – the AS02 protocol [AS02]. In this section, we analyse both bidding-price-secrecy and receipt-freeness for non-winning bidders in the AS02 protocol. ProVerif code is available at [1].

4.4.1 Introduction

This protocol is a sealed-bid e-auction protocol. The protocol involves \( n \) bidders \( b_1, \ldots, b_n \) and \( k \) auctioneers \( a_1, \ldots, a_k \). The list of all prices allowed for bids is published before the protocol. During the protocol, each bidder sends a commit for every price in the price list: a ‘bid’ if he wants to bid that price, and a ‘non-bid’ otherwise. Auctioneers work together to open the commitments of all bidders from the highest price down until the winning bid(s) is/are found.

4.4.2 Physical assumptions

In order to ensure privacy of bidders, the protocol has two physical assumptions:

- **a1**: a bidding booth for the bidders, and
- **a2**: a one-way untappable channel from every bidder to every auctioneer.

The bidding booth enables a bidder to privately submit a bid free from control or observation of the adversary. The untappable channels ensure no adversary can see messages sent.

4.4.3 Settings

Before starting the protocol, one auctioneer publishes an increasing price list \( p_1, \ldots, p_m \), a message \( M_{yes} \) for “I bid”, a message \( M_{no} \) for “I do not bid”, a generator \( g \) of subgroup of \( Z_p^* \) with order \( q \), where \( q, p \) are large primes with \( p = 2q + 1 \).

4.4.4 Description of the protocol

The protocol consists of two phases: bidding and opening.

*Bidding phase.* A bidder in the bidding booth chooses a secret key \( x \), publishes his public key \( h = g^x \) with a predetermined signature. Then the bidder chooses a series of random numbers \( r_1, \ldots, r_m \) as secret seeds, one random number for each price, and decides a price \( p_b \) to bid for. Then he generates either a “bid” or

---

\(^2\)The protocol does not specify how to resolve the case where there are fewer bidding items than winners.
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**Figure 4.1: The AS02 protocol.**

A “non-bid” bit-commitment for each price $p_\ell$ ($1 \leq \ell \leq m$), using the following formula:

$$Commit_\ell = \begin{cases} 
g^{M_\ell \cdot h_{r_\ell}} & \text{if } p_\ell = p_b \quad (a \text{ bid for price } p_\ell) \\
g^{M_\ell \cdot h_{r_\ell}} & \text{if } p\ell \neq p_b \quad (a \text{ non-bid for price } p_\ell) 
\end{cases}$$

Next, the bidder publishes the sequence of the bit-commitments with his signature. Then he proves to each auctioneer that he knows the secret key $log_g h = x$ and the discrete logs ($log_g Commit_1, \ldots, log_g Commit_m$) using interactive zero-knowledge proofs. Finally, he computes $t$-out-of-$k$ secret shares $r_{i\ell}$ for each secret seed $r_\ell$ and each auctioneer $a_i$, and then sends the signed secret share $r_{i\ell}$ over the one-way untappable channel to the auctioneer $a_i$ (see Figure 4.1).  

Opening phase. Auctioneers together iterate the following steps for each price $p_\ell = p_m, p_{m-1}, \ldots, p_1$ until the winning bid is determined.

Each auctioneer $a_i$ publishes secret shares $r_{i\ell}$ (the $\ell$-th secret share of a bidder sent to auctioneer $a_i$) of all bidders. For each bidder, all auctioneers work together to

\[3t \text{ is a threshold, } k \text{ is the number of auctioneers, it means only more than } t \text{ auctioneers together can reconstruct the secret seeds.}\]
reconstruct the secret seed $r_\ell$, and check for each bidder whether

$$Commit_\ell = g^{M_{\text{yes}}} h^{r_\ell}.$$ 

If there exist some bidders for which the above equivalences are satisfied, the auctioneers finish checking the current price and then stop. In this case, the price $p_\ell$ is the winning price, those bidders are winning bidders. If there is no equivalence existing, which means there is no bidder bidding for the price $p_\ell$, the auctioneers repeat the above process on the next lower price.

### 4.4.5 Claimed privacy properties

The authors claim the following properties: bidding-price-secrecy and receipt-freeness for non-winning bidders. Intuitively, the bidding price of each bidder is sealed in the bidding phase, and only the winning bidder’s bidding price is revealed in the opening phase, thus the adversary does not know the bidding price for non-winning bidders, thus standard bidding-price-secrecy is satisfied. The strong bidding-price-secrecy is satisfied mainly due to the random number used in calculating the bit-commitments.

**Informal reasoning of receipt-freeness.** We use $M$ to represent either $M_{\text{yes}}$ or $M_{\text{no}}$, the formula for computing $Commit_\ell$ is of the following form:

$$Commit_\ell = g^M \cdot h^{r_\ell} = g^M \cdot (g^{x})^{r_\ell} = g^{M+xr_\ell},$$

since $h = g^x$. Thus, $\log Commit_\ell = M + xr_\ell$. By using interactive zero-knowledge proofs, a bidder is proved to know his secret key $x$ and discrete logs $\log Commit_\ell$. An interesting property of chameleon bit-commitments is that if the bidder bids for price $p_\ell$,

$$\log Commit_\ell = M_{\text{yes}} + xr_\ell$$

he can calculate a fake $r'_\ell$ such that:

$$\log Commit_\ell = M_{\text{no}} + xr'_\ell \quad \text{and} \quad r'_\ell = (M_{\text{yes}} + xr_\ell - M_{\text{no}})/x.$$ 

Using the fake $r'_\ell$, the bidder can show that the bit-commitment $Commit_\ell$ is opened as message $M_{\text{no}}$, which means bidder did not bid for price $p_\ell$. Using the same method, a bidder can open a ‘no’ bit-commitment as a ‘yes’ bit-commitment. Thus, the commit leaks no information concerning the bid, thus the bidder cannot prove how he bid, e.g. receipt-freeness is satisfied.

### 4.5 Modelling AS02

We use the applied pi calculus to model the AS02 protocol. We use two simplifications:

- **s1:** one honest auctioneer; and
- **s2:** perfect zero knowledge proofs.
In the protocol, auctioneers are cooperating to find the winning bid. It takes at least \( t \) auctioneers to decide the winner, thus guaranteeing \( t\)-out-of-\( k \) secrecy. As we focus on bidder privacy, we need to consider only one honest auctioneer. Thus, we simplify the model to have only one honest auctioneer. The AS02 protocol uses interactive zero knowledge proofs to guarantee that each bidder knows his secret key and the discrete logs of bit-commitments. However, the details of these proofs are left unspecified, and thus we did not include them in the model. We simply assume that each bidder knows his secret key and discrete logs of bit-commitments.

In addition, the AS02 does not specify how the auctioneers tell the signed public key from the signed commitments generated by the same bidder. In order for the auctioneer to distinguish the two messages, in our modelling,

- **s3**: we use a symbol \( k \) in the signed public key messages.

**Signature and equational theory.** The signatures and the equational theory model cryptographic primitives used in the protocol. We fix a list of bidders \((b_1, \ldots, b_n)\) and an ordered list of prices \((p_1, \ldots, p_m)\), which are modelled as functions with arity 0. We define function `nextbidder` to find the next bidder in the bidder list, and function `nextprice` to find the next lower price in the price list. Function `checksign` is used to check whether the public signature key is the right one for the signed message, and we use function `getmsg` to get the original message from a signed message. Particularly, chameleon bit-commitments are modelled as a function `commit` with arity 3 (a random number, public key of the bidder and message \( M \) either \( M_{yes} \) or \( M_{no} \)). The relevant properties of chameleon bit-commitments are captured in the following equational theory.

\[
\begin{align*}
\text{commit}(r, \text{pk}(sk_b), M_{yes}) &= E \text{commit}(f(r), \text{pk}(sk_b), M_{no}) \\
\text{commit}(r, \text{pk}(sk_b), M_{no}) &= E \text{commit}(f(r), \text{pk}(sk_b), M_{yes}) \\
\text{open}(\text{commit}(r, \text{pk}(m), r, \text{pk})) &= E m
\end{align*}
\]

Constants \( M_{no} \) and \( M_{yes} \) represent “I do not bid” and “I bid”, respectively. The parameter \( \text{pk}(sk_b) \) is the public key of a bidder, and \( r \) is the secret seed the bidder chooses. Function \( f(r) \) returns the fake secret seed of a secret seed \( r \). We can model the function \( f \) by just giving one parameter - the real secret seed. Because we assume that each bidder knows his secret key and discrete logs of bit-commitments, he can compute the fake secret seed for each real secret seed, as explained in the previous section. The first equivalence means that if a bidder chooses a secret seed \( r \), bids for a price, and calculates the bit-commitment \( \text{commit}(r, \text{pk}(sk_b), M_{yes}) \), he can compute a fake secret seed \( f(r) \), and by using this fake secret seed, the bit-commitment can be opened as message \( M_{no} \), which means “I do not bid”. The second equivalence shows that the opposite situation also holds. A bidder can also open a bit-commitment as if he bids for that price, when actually he does not. All functions defined in this model are shown in Figure 4.2 and the equational theory is shown in Figure 4.3. Recall that `fun` is used to denote function in ProVerif, and `reduc` and `equation` are used to denote the equational theory in ProVerif.

**Main process.** For each bidder \( b_j \), the main process (see Figure 4.4) generates two private channels \( \text{privch}_{b_j}(m1) \) and \( \text{privcha}_{b_j}(m2) \). These channel are used for
### 4.5 Modelling AS02

<table>
<thead>
<tr>
<th>fun b_1/0, ..., fun b_n/0, fun p_1/0, ..., fun p_m/0, fun M_{yes}/0, fun M_{no}/0, fun true/0, fun pk/1, fun commit/3, fun sign/2, fun f/1, fun k/0.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Figure 4.2: Functions.</strong></td>
</tr>
</tbody>
</table>

| reduc nextbidder(b_1) = b_2. |
| reduc nextbidder(b_{n-1}) = b_n. |
| reduc nextbidder(b_n) = ⊥. |
| reduc nextprice(p_m) = p_{m-1}. |
| reduc nextprice(p_2) = p_1. |
| reduc nextprice(p_1) = ⊤. |
| reduc checksign(pk(sk), sign(m, sk)) = true. |
| reduc getmsg(sign(m, sk)) = m. |
| equation commit(r, pk(sk_{b}), M_{yes}) = commit(f(r), pk(sk_{b}), M_{no}). |
| equation commit(r, pk(sk_{b}), M_{no}) = commit(f(r), pk(sk_{b}), M_{yes}). |
| reduc open(commit(r, pk, m), r, pk) = m |
| **Figure 4.3: Equational theory.** |

**Key distribution process.** This process generates and distributes keying material modelling a PKI – public key infrastructure (Figure 4.5). This process first generates n secret keys (k1). Each bidder b_j has one secret key ssk_{b_j} for signing messages. Each secret key corresponds to a public key (k2-k4). Each secret key is assigned to a bidder process by being sent to the bidder over the private channel privch_{b_j}. The corresponding public key is sent to the auctioneer over the private channel privcha_{b_j} (k6) and is published over the pub-
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\[ P_{AS02} := \]

\begin{align*}
m1. & \quad \nu_{\text{privch}}_{b_1}, \nu_{\text{privch}}_{b_2}, \ldots, \nu_{\text{privch}}_{b_n}. \\
m2. & \quad \nu_{\text{privch}}_{a_1}, \nu_{\text{privch}}_{a_2}, \ldots, \nu_{\text{privch}}_{a_n}. \\
m3. & \quad \nu_{\text{untapch}}_{b_1}, \nu_{\text{untapch}}_{b_2}, \ldots, \nu_{\text{untapch}}_{b_n}. \\
m4. & \quad \nu_{\text{sync}}_{b_1}, \nu_{\text{sync}}_{b_2}, \ldots, \nu_{\text{sync}}_{b_n}. \\
m5. & \quad (P_K \mid \text{let } p_b = p_{b_1} \text{ in let untapch} = \text{untapch}_{b_1} \text{ in} \\
m6. & \quad \text{let privch} = \text{privch}_{b_1} \text{ in let } \text{ch} = \text{ch}_{b_1} \text{ in } P_b) \mid \\
m7. & \quad \cdots \mid \text{let } p_b = p_{b_n} \text{ in let untapch} = \text{untapch}_{b_n} \text{ in} \\
m8. & \quad \text{let privch} = \text{privch}_{b_n} \text{ in let } \text{ch} = \text{ch}_{b_n} \text{ in } P_b) \mid P_a)
\end{align*}

Figure 4.4: The main process.

lic channel \(ch_{b_j}\) such that the adversary knows the keys (k7). Therefore, only a bidder knows his own secret key, and everyone including the adversary knows each bidder’s public key. Sending each public key to the auctioneer over a private channel, models the following protocol setting: There are fix number of bidders in sealed-bid auctions, and the auctioneer knows each bidder’s public signing key as predetermined knowledge.

\[ P_K := \]

\begin{align*}
k1. & \quad \nu_{s\text{sk}}_{b_1}, \nu_{s\text{sk}}_{b_2}, \ldots, \nu_{s\text{sk}}_{b_n}. \\
k2. & \quad \text{let } spk_{b_1} = pk(s\text{sk}_{b_1}) \text{ in} \\
k3. & \quad \ldots \\
k4. & \quad \text{let } spk_{b_n} = pk(s\text{sk}_{b_n}) \text{ in} \\
k5. & \quad \text{out}(\text{privch}_{b_1}, s\text{sk}_{b_1}) \mid \cdots \mid \text{out}(\text{privch}_{b_n}, s\text{sk}_{b_n}) \mid \\
k6. & \quad \text{out}(\text{privch}_{a_1}, spk_{b_1}) \mid \cdots \mid \text{out}(\text{privch}_{a_n}, spk_{b_n}) \mid \\
k7. & \quad \text{out}(ch_{b_1}, spk_{b_1}) \mid \cdots \mid \text{out}(ch_{b_n}, spk_{b_n}))
\end{align*}

Figure 4.5: The key distribution process.

**Bidder process.** The applied pi calculus process for a bidder \(P_b\) is given in Figure 4.6. First, a bidder receives his secret signature key from his private channel (b1). Next, the bidder generates his secret key \(s\text{sk}_b\), signs the corresponding public key and publishes the signed message (b2). To indicate that this message contains a key, we add \(k\) into the message (see s3). In addition, the bidder chooses a series of random numbers \(r_1, \ldots, r_m\) as secret seeds (b3). The bidder then computes each bit-commitment \(cmt^{b_\ell}\) as described in Section 4.4.4. For each price, the bidder computes a commitment: if the price is the bidding price, then the bidder commits ‘yes’ with \(M_{\text{yes}}\), otherwise, the bidder commits ‘no’ with \(M_{\text{no}}\) (b4-b9). Finally, the bidder publishes the series of bit-commitments \(cmt^{b_1}, \ldots, cmt^{b_m}\) with his signature (b10), and sends the signed series of secret seeds to the auctioneer through the untappable channel (b11). As we assume there is only one honest auctioneer in the model, we do not need to model secret shares.

**Auctioneer process.** During the bidding phase, the auctioneer launches \(n\) copies of sub-process \text{readinfo} to gather information from each bidder \(b_j\) (a1).
\[ P_b := \]
\[\begin{align*}
&b1. & \text{in}(privch, ssk_b). \\
&b2. & \nu sk_b. \text{out}(ch, \text{sign}((pk(sk_b), k), ssk_b)). \\
&b3. & \nu r_1, \ldots, \nu r_m. \\
&b4. & \text{if } p_1 = p_b \\
&b5. & \text{then let } \text{cmtp}_1 = \text{commit}(r_1, pk(sk_b), M_{yes}) \text{ in} \\
&b6. & \text{else let } \text{cmtp}_1 = \text{commit}(r_1, pk(sk_b), M_{no}) \text{ in} \\
& & \ldots \\
&b7. & \text{if } p_m = p_b \\
&b8. & \text{then let } \text{cmtp}_m = \text{commit}(r_m, pk(sk_b), M_{yes}) \text{ in} \\
&b9. & \text{else let } \text{cmtp}_m = \text{commit}(r_m, pk(sk_b), M_{no}) \text{ in} \\
&b10. & \text{out}(\text{untapch}, \text{sign}(\langle r_1, \ldots, r_m \rangle, ssk_b)). \\
&b11. & \text{out}(\text{untapch}, \text{sign}(\langle r_1, \ldots, r_m \rangle, ssk_b))
\end{align*}\]

Figure 4.6: The bidder process.

In details, the auctioneer collects public signature key \(spk_b\) (r1) and the signed committing public key \(\text{signedpk}\) (supposed to be \(\text{sign}((pk(sk_b), k), ssk_b))\) (r2). The auctioneer verifies whether the committing public key is signed with the right signature (r3) and obtain the committing public key from \(\text{signedpk}\) (r4). Next, the auctioneer reads in the signed commitments \(\text{signedcommit}_{b_j}\) of bidder \(b_j\) (r5) and verifies the signature (r6). If the commitments are correctly signed, the auctioneer obtains the series of bit-commitments \(\text{cmtp}_{b_1}, \ldots, \text{cmtp}_{b_m}\) (r7), then the auctioneer reads in the signed secret seeds \(sr\) from the untappable channel of bidder \(b_j\) (r8). The auctioneer verifies the signature (r9). If the secret seeds are correctly signed, the auctioneer obtains the secret seeds \(ss_{b_1}, \ldots, ss_{b_m}\) (r10). Finally, the auctioneer sends the signal that information collecting for bidder \(b_j\) has finished, over the channel \(\text{synch}\) (r9). In addition, the collected information (the committing public key, the commitments, the secret seeds) is sent to the sub-process in which the winning bidder is determined.

Next the auctioneer needs to synchronise with all bidders (a2). The auctioneer process is not allowed to continue until all bidders reach the end of the bidding phase. In the opening phase, the auctioneer evaluates \(\text{cmtp}_{b_j} = \text{commit}(ss_{b_j}, pk_{b_j}, M_{yes})\) for each bidder (a3, a7, a12). If the two values are equivalent for the first bidder \(b_1\) (a3), bidder \(b_1\) has bid for that price, otherwise, bidder \(b_1\) has not bid for that price. When bidder \(b_1\) has bid for that price, the auctioneer publishes the bidder together with the price over the public channel \(\text{winnerch}\) (a4), then the auctioneer checks the evaluation for the next bidder (if exists) (a7). Once the auctioneer has evaluated for every bidder (a5) and has determined the set of winning bidders (a4), he stops the process (a6). When bidder \(b_1\) has not bid for that price, the auctioneer checks the evaluation for the next bidder (if exists) (a12). Once the auctioneer has evaluated for every bidder and no winner has been found (a8), the auctioneer repeats the evaluation steps for each bidder at the next lower price (a11). If the next lower price does not exist (a9), the process stops (a10) and no bidder has bid for any price. In a similar way, the sub-process \(\text{checknextb}_{b_i}^{p_j}\) is used to evaluate the bid of a bidder \(b_i\) at price \(p_j\), if there are already some winners before bidder \(b_i\). And the sub-process \(\text{checknextb}_{b_i}^{p_j}\) is used to check the next
bidder at price \( p_j \), if there is no winner before that bidder. We use \( \bot \) and \( \top \) to represent the end of the bidder list and price list, respectively.

In the sub-process \( \text{checknextbidder}_b^p \), the auctioneer checks whether the bidder \( b_i \) has bid for price \( p_j \) (n1). If the bidder \( b_i \) has bid for \( p_j \), \( b_i \) is a winning bidder. The auctioneer publishes the winning bidder \( b_i \) and the winning price \( p_j \) (n2). Note that since there already exists one or more winning bidders, \( b_i \) is not the first winner. The auctioneer checks whether the bidder \( b_i \) is the last bidder (n3). If \( b_i \) is the last bidder, the auctioneer has found all winning bidders, thus stops the opening process (n4); otherwise, the auctioneer checks the evaluation for the next bidder at the same price (i.e., whether the next bidder is also a winner) (n5).

In the sub-process \( \text{checknextbnp}_b^p \), the auctioneer first checks whether the bidder \( b_i \) has bid for price \( p_j \) (p1). If the bidder \( b_i \) has bid for \( p_j \), \( b_i \) is a winner. The auctioneer publishes the bidder \( b_i \) and the winning price \( p_j \) (p2). Since there is no winning bidder found before, \( b_i \) is the first winner. Then the auctioneer checks whether the bidder \( b_i \) is the last bidder (p3). If \( b_i \) is the last bidder, bidder \( b_i \) is the only winner. Since the auctioneer has found all winners, he stops the opening process (p4). Otherwise, the auctioneer checks whether the next bidder is also a winner (p5). Note that since there is already a winner \( b_i \), the auctioneer use the process \( \text{checknextbidder}_b^p \). If the bidder \( b_i \) has not bid for \( p_j \), the auctioneer checks whether the bidder is the last bidder (p6). If \( b_i \) is the last bidder, since there is no bidder bid for price \( p_j \) before \( b_i \) and \( b_i \) has not bid for \( p_j \), there is no bidder bid for price \( p_j \). Thus, the auctioneer checks the evaluations for every bidder at the next lower price \( p_{j-1} \). To do so, the auctioneer first checks whether \( p_{j-1} \) is the bottom (whether \( p_j \) is already the lowest price in the price list) (p7).

![Figure 4.7: The auctioneer process.](image)
4.6 Analysis of AS02

After modelling the protocol in the previous section, we formally analyse bidding-price-secrecy and receipt-freeness for bidders. In the AS02 protocol, the winning bid is published, and thus bidding-price-secrecy and receipt-freeness for the winning bidders are not satisfied. Particularly, if all bidders bid for the same price, then all bidders are winners, i.e., no bidder is a non-winning bidder, thus bidding-price-secrecy is not satisfied in this case. From here on, when we refer to bidding-price-secrecy and receipt-freeness, we mean only with respect to non-winning bidders.
4.6.1 Bidding-price-secrecy

In general, bidding-price-secrecy can be formalised in two levels: standard bidding-price-secrecy and strong bidding-price-secrecy. Standard bidding-price-secrecy is defined as no matter how the adversary interacts with the protocol, he cannot derive a non-winning bidder’s bidding price. Thus, it aims to keep the price secret. However, since the AS02 protocol publishes the bidding price list, the adversary initially knows all the prices. No matter which price a bidder bids for, the bidding price is not a secret to the adversary. Therefore, a bidder’s bidding price is not a secret. In fact, what the AS02 protocol aims to protect is the link between bidders and the price he bids, instead of the price itself. Therefore, bidding-price-secrecy of the AS02 protocol is captured by strong bidding-price-secrecy.

Strong bidding-price-secrecy ensures the anonymity of the link between a non-winning bidder and the price he bids for. It is formalised as that the adversary cannot distinguish between the case when a bidder bids for price a and the case when the bidder bids for price c. This property is formally defined in Definition 4.1.
In the verification, we assume all the participants in the context are honest. Thus, the context $C_{AS02}[,\ldots]$ is defined as the auction process $P_{AS02}$ with a hole (c9) instead of two bidder processes (see Figure 4.11). We denote the two bidder process as $P_{b\,A}$ and $P_{b\,B}$. Sub-process c5 to c8 models the other $n - 2$ bidder processes. To verify strong bidding-price-secrecy is to verify the following equivalence

\[
C_{AS02}[(\text{let } p_b = a \text{ in let untapch} = \text{untapch}_{b\,d} \text{ in }
\text{let privch} = \text{privch}_{b\,d} \text{ in let } ch = \text{ch}_{b\,d} \text{ in } P_b)] \\
\approx_{\ell} C_{AS02}[(\text{let } p_b = c \text{ in let untapch} = \text{untapch}_{b\,d} \text{ in }
\text{let privch} = \text{privch}_{b\,d} \text{ in let } ch = \text{ch}_{b\,d} \text{ in } P_b)]
\]

where $a, c, d$ are from the list $p_1, \ldots, p_m$ with $a < d$ and $c < d$.

Strong secrecy properties can be verified, using ProVerif, by querying noninterf. Note that ProVerif is sensitive to evaluations of statements in the if-then-else constructs. ProVerif reports false attacks when directly querying the following predicate: noninterf $p_b$ among $p_1, \ldots, p_{d-1}$. To be able to check noninterf of a bidding price in ProVerif, we modify the bidder process by replacing if-then-else constructions with choices of a list of variables $vp_1, \ldots, vp_{n-1}$. For example, lines (b4-b6) in Figure 4.6 is changed to “let cmtpi = commit(r_i, pk(sk_b), vp_i) in”. Each variable $vp_i$ corresponds to a price $p_i$ and can be assigned to two possible values, either $M_{yes}$ or $M_{no}$. If the variable is assigned $M_{yes}$, the bidder bids that price, otherwise, not. By querying “noninterf $vp_i$ among $(M_{yes}, M_{no}), \ldots, vp_{n-1}$ among $(M_{yes}, M_{no})$”, the variable $vp_i$ is replaced with $M_{yes}$ or $M_{no}$, resulting into different versions of the bidder process with different bidding prices (possibly bidding for multi-prices). ProVerif gives a positive result, which means that these process versions are all observationally equivalent. In this way, we prove that the protocol satisfies strong bidding-price-secrecy.

### 4.6.2 Receipt-freeness

Receipt-freeness is formally defined in Definition 4.2. To prove the AS02 protocol satisfies receipt-freeness, we need to find a process $P_f$ which satisfies both equivalences in the definition of receipt-freeness. The process $P_f$ is shown in Figure 4.12. The two equivalences are shown as follows (eq1 and eq2).

\[
eq_1 (\text{let untapch} = \text{untapch}_{b\,d} \text{ in }
\text{let privch} = \text{privch}_{b\,d} \text{ in let } ch = \text{ch}_{b\,d} \text{ in } P_f^{\text{out}(ch,c)}) \\
\approx_{\ell} (\text{let } p_b = c \text{ in let untapch} = \text{untapch}_{b\,d} \text{ in }
\text{let privch} = \text{privch}_{b\,d} \text{ in let } ch = \text{ch}_{b\,d} \text{ in } P_b),
\]
According to the properties of chameleon bit-commitments, the bidder can send a sequence of fake secret seeds to the adversary, and sends the series of real secret seeds to the auctioneer through an untappable channel. The adversary opens the bit-commitments as the bidder bids for price a, using the fake secret seeds he received, while the auctioneer opens the same bit-commitments as the bidder bids for price c, using the secret seeds the auctioneer received through an untappable channel. Thus, the bidder could execute the process $P_f$ as shown in Figure 4.12 to lie to the adversary. The bidder in this process communicates with the adversary through channel $\text{chc}$, sending the adversary his secret signature key $\text{ssk}_b$ (f1) and his secret key $\text{sk}_b$ (f2). Later the bidder sends the auctioneer $r_1, \ldots, r_m$ through an untappable channel (f14), and send the adversary the same list except changing $r_a$ and $r_c$ to $f(r_a)$ and $f(r_c)$, respectively (f5). The untappable channel ensures the adversary cannot learn anything about the differences.

**Proof sketch:** To prove the first equivalence, we can simply consider $P_f\setminus_{\text{out}(\text{chc}, \cdot)}$ as process $P_f$ without communication on the channel $\text{chc}$. Since the process $P_f\setminus_{\text{out}(\text{chc}, \cdot)}$ works exactly the same as the process $P_b\{c/p_b\}$, the first equivalence (eq1) is satisfied. For detailed proofs, see Appendix A. To show the second equivalence (eq2), we need to consider all the transitions of each side. On both
4.6 Analysis of AS02

Besides, the process $P_K$ only distributes keys, and all the bidder processes in the context follow the same process. For the sake of simplicity, we ignore the outputs in the process $P_K$ and those bidder processes in the context. During the bidding phase the auctioneer process only reads information and synchronises on the private channel $synch$. There is no output on public channels in the auctioneer process. We denote the sequence of names $sk_b, r_1, \ldots, r_m, bsk_b, br_1, \ldots, br_m$ by $\nu n (sk_b, r_1, \ldots, r_m$ are bound names in the non-winning bidder process, and $bsk_b, br_1, \ldots, br_m$ are bound names in the winning bidder process $P_{b}$). After the key distribution, we want to see whether the behaviour of the process $P_b\{a/p_b\}^{chc} | P_{b}B\{d/p_b\}$ is observationally equivalent to $P_f | P_{b}B\{d/p_b\}$. For this purpose, we need to consider all possible executions of these two processes. Here, we consider a particular execution and only show the interesting part of the two frames after each step of execution by
the two processes. Let \( P := P_{b_A}(a/p_b)^{che} \mid P_{b_B}(d/p_b) \) and \( Q := P_f \mid P_{b_B}(d/p_b) \), we have their labelled transitions as shown in Figure 4.13.

The frames we obtained at the end of \( P \) and \( Q \) are statically equivalent. In particular, as the adversary knows the bit-commitments the bidder submits, the public key of the bidder, and the secret seeds, the adversary can open all the commitments of the bidder. The only functions the adversary can use are getmsg and open, to get extra information. By applying these two functions, the adversary can additional get the public key of the bidder represented as \( x_{msg} = \text{getmsg}(x_3, x_1) \) and a series of opened messages from bit-commitments. Since \( x_3 \) and \( x_1 \) are the same for both \( P \) and \( Q \), \( x_{msg} \) is the same for both processes as well. Particularly, \( P_{b_A}(a/p_b) \) bids for price \( a \). The adversary opens the commitments \( cmt^{pc} = \text{commit}(r_c, pk(s_b), M_{yes}) \) and \( cmt^{pe} = \text{commit}(r_c, pk(s_b), M_{no}) \) as follows:

\[
\text{open}(cmt^{pc}, r_a, pk(s_b)) = E M_{yes} \quad \text{open}(cmt^{pe}, r_c, pk(s_b)) = E M_{no}
\]

For the process \( Q \), the process \( P_f \) bids for price \( c \). The adversary has a series of secret seeds, in which two of them are fake: \( f(r_a) \) and \( f(r_c) \). According to the equational theory of chameleon bit-commitments (see Section 4.5), the adversary opens \( cmt^{pc} = \text{commit}(r_a, pk(s_b), M_{no}) = E \text{commit}(f(r_a), pk(s_b), M_{yes}) \) and \( cmt^{pe} = \text{commit}(r_c, pk(s_b), M_{yes}) = E \text{commit}(f(r_c), pk(s_b), M_{no}) \) as follows:

\[
\text{open}(cmt^{pc}, f(r_a), pk(s_b)) = E M_{yes} \quad \text{open}(cmt^{pe}, f(r_c), pk(s_b)) = E M_{no}
\]

All other secret seeds and bit-commitments are the same in both \( P \) and \( Q \), hence the adversary gets the same series of opened messages for both \( P \) and \( Q \) as well.

Next, we consider the opening phase, the auctioneer process is the only active process. According to the protocol, the auctioneer process stops after finding the winning bids. Therefore, non-winning bids are not revealed. Since we have assumed the auctioneer is honest, the information that the auctioneer process reveals is the opened bit-commitments of all bidders at prices no lower than the winning price, and the winning bidders. Only the winning bid is opened as \( M_{yes} \), others are opened as \( M_{no} \). Due to the existence of a higher bid \( (d \text{ in the process } P_{b_B}(d/p_b)) \) on both sides of the equivalence, the bid made by the bidder \( b_A \) will never be published, hence the information the auctioneer process reveals is the same. Now, we can conclude that the protocol satisfies receipt-freeness. \( \square \)

### 4.7 Conclusions

In this chapter, we first discussed privacy and enforced privacy issues in the e-auction domain. An enforced privacy property, receipt-freeness for non-winning bidders, is required in sealed-bid e-auctions. To formally define this property, we first formalised the classical privacy property, strong bidding-price-secrecy for non-winning bidders. On top of that, the enforced privacy property was formalised. As the requirement for privacy is different in e-auction and e-voting, the privacy, strong bidding-price-secrecy for non-winning bidders (Definition 4.1), is formalised differently from vote-privacy (Definition 3.14). Due to this difference, the enforced privacy property, receipt-freeness for non-winning bidders (Definition 4.2), is formalised differently from receipt-freeness in e-voting (Definition 3.17). In addition,
we only model the cooperating bidder forwarding information to the adversary in e-auctions, because of the setting – the bidding booth in which bidders cannot communicate with the adversary. In situations without this setting, a similar property like coercion-resistance can be defined in e-auctions. Finally, the proposed formalisations of strong bidding-price-secrecy and receipt-freeness for non-winning bidders are validated in the case study. The AS02 protocol is modelled in the applied pi calculus, and we successfully verified that it does indeed satisfy bidding-price-secrecy and receipt-freeness for non-winning bidders. The protocol achieves this mainly based on the use of chameleon bit-commitments and an untappable channel, much like Okamoto used in his e-voting protocol \cite{oka96}.

\textit{Future directions}. The case study focused on a protocol relying on chameleon bit-commitments and an untappable channel. It would be interesting to use the proposed formalisation to verify correctness of enforced privacy claims of an auction protocol that uses other cryptographic primitives, such as the auction protocol by Chen et al. \cite{CLK03}, that relies on homomorphic encryption.

One of the main motivation for studying enforced privacy in auctions was the similarity between auction protocols and voting protocols. (For example, in both, roles can be divided into two types: participants and authorities.) Having successfully lifted the concept of enforced privacy to a similar domain, we are now ready to take matters one step further and study enforced privacy in a more complex setting.
Enforced privacy in e-health

In the previous chapter, we formalised enforced privacy in the e-auction domain. In this chapter, we study enforced privacy in the e-health domain. We identify and formalise enforced privacy in e-health in a similar way as in e-voting and e-auctions. Additionally, in e-health, it is required that pharmacists should not be able to prove a doctor’s prescription behaviour to the adversary. We formalise a privacy property capturing this requirement. Furthermore, we formalise a privacy property capturing the conjunction of enforced privacy and pharmacist revealing information to the adversary. Finally, we verify the formalised privacy properties of an e-health protocol in a case study.

5.1 Introduction to e-health

E-health systems are health care systems using distributed electronic devices which communicate via the network, typically the Internet. E-health systems aim to support secure sharing of information and resources across different health care settings and workflows among different health care providers. The services of such systems for the general public are intended to be more secure, more effective, more efficient and more timely.

An e-health system normally consists of at least patients and doctors. Doctors prescribe medicine to patients according to examinations. In some e-health systems, this procedure can be done entirely using electronic devices. For instance, in home-care systems, patients do examinations at home. The examination results are sent to doctors via the network (automatically or manually). Then a doctor sends the prescription back. Whereas, in some other e-health systems, only storing information, such as patient’s medical records, is digitalised. Consequently, e-health systems may have different privacy requirements. For instance, ensuring privacy in systems, which only involve electronic devices for storing data, mainly requires local protection. In e-health systems involving using open networks for communication, adversaries from the network need to be taken into consideration.

In addition, e-health systems may involve more roles. Once a patient obtains a prescription from doctors, the patient needs to get medicine according to the prescription. Therefore, pharmacists are involved. During this procedure, a pharmacist may have access to some private information, for example, prescriptions. In some systems, a pharmacist is even allowed to change one type of medicine to another type with similar functions. And in emergency cases, the pharmacist needs to be able to contact the doctor who has prescribed certain medicine. However,

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This chapter is based on published work [FHIES11] and [ESORICS12]
pharmacists, in general, are not always trustworthy. Hence, pharmacists may not be reliable about not revealing private information. In fact, pharmacists are allowed to offer certain information to others in particular cases, like doing research. Therefore, pharmacists may influence privacy in e-health.

Furthermore, depending on systems, even more other parties can be involved. For instance, medical administration and social insurance, which are normally assumed to trustworthy, and nurses, who need to access private information of patients. In some e-health systems, emergency situations are considered. In emergency situations, privacy becomes subtle since patient’s information is normally revealed.

Hence, we can see that roles and relations in e-health systems are complex compared to e-voting and e-auction systems. In e-voting and e-auctions, there is a natural division into two types of roles: participants (voters, bidders) and authorities (who run the election/auction). E-health systems have to deal with a far more complex constellation of roles. This may cause complexity of privacy in e-health.

5.2 Privacy and enforced privacy in e-health

Due to the sensitive nature of health care data, privacy is one of the foremost challenges raised by adopting electronic storage and communication. In health care, it is often necessary to collect information for statistical study, like, collecting locations of patients having a certain type of disease to study the correspondence between locations and the disease. Thus, one privacy challenge is to maintain user anonymity even if information is revealed for statistical study. In addition, given the sensitive nature of health care data, handling this data must meet strict privacy requirements. Traditionally, data in health care (e.g., patient records) is stored on paper files. Privacy is relatively easily satisfied by controlling access to the physical documents. Those who had access could be considered trusted not to violate privacy of the data. With the advent of e-health systems – systems that digitally store and exchange health care data – health care data is open to inference by adversaries controlling the network or manipulating digital devices.

Indeed, privacy in e-health has been recognised as an important requirement necessary for adoption by the general public [MRS06, KAB09]. Moreover, due to the complexity of e-health systems, existing privacy control techniques from domains such as e-voting (e.g., [DKR09]) and e-auctions (e.g., [FAST10]) does not carry over straightforwardly. E-health systems involve more complex roles. Each of these roles has access to different private information and has different privacy concerns. As existing approaches from other domains are not properly equipped to handle such a diverse array of roles, privacy must be tailored to the e-health domain. In addition, roles may not be trustworthy, although some of them may be able to access sensitive data of others, for example, pharmacists normally have access to prescriptions. Thus, third parties may influence a target participant’s privacy. Therefore, we shall consider privacy taking into account of third parties’ influences.

Depending on the protected role, privacy in e-health can be classified into patient privacy and doctor privacy. Privacy of other roles, e.g., pharmacists, insurance companies, medical administrations, is beyond our consideration, as they are public
5.2 Privacy and enforced privacy in e-health

entities.

5.2.1 Patient privacy

The importance of patient privacy in e-health is traditionally seen as vital to establishing a good doctor-patient relationship. This is even more pertinent with the emergence of the Electronic Patient Record [And96]. As in most of the literature, a necessary early stage of e-health is to transform the paper-based healthcare process into a digital process. The most important changes in this stage are made to patient information processing, mainly health care records. To properly express privacy requirements for such patient records, privacy policies are considered the de facto standard. There are three main approaches to implement these requirements of patient privacy: access control, architectural design, and the use of cryptography.

Patient privacy by access control. To preserve privacy of electronic health care records, one necessary part is to limit access to these records to allowed parties. The need for access control is supported by several privacy threats to personal health information listed by Anderson [And96]. Many access control approaches designed for patient privacy can be found in the literature, including access rules proposed by Anderson [And96], consent-based access rules [Lou98], role-based access control (RBAC) [RCHS03], organisation based access control [KBM+03], etc.

Patient privacy by architectural design. E-health systems cater to a number of different roles, including doctors, patients, pharmacists, insurers, etc. Each such role has its own sub-systems or components. As such, e-health systems can be considered as a large network of systems, including administrative system components, laboratory information systems, radiology information systems, pharmacy information systems, and financial management systems. Diligent architectural design is an essential step to make such a complex system function correctly. Since privacy is important in e-health systems, keeping privacy in mind when designing the architecture of such systems is a promising path towards ensuring privacy [SV09]. Examples of architectures taking privacy in mind include the architecture of wireless sensor networks in e-health [KLS+10], the architecture for e-health systems proposed by Maglogiannis et al. [MKDH09], the architecture for cross-institutional image sharing in e-health [CHCK07], etc.

Cryptographic approaches to patient privacy. Cryptography is a necessary tool for privacy in e-health systems, especially communications between components of systems [BB96]. For example, Van der Haak et al. [vWB+03] use digital signatures and public-key authentication (for access control) to satisfy legal requirements for cross-institutional exchange of electronic patient records. Ateniese et al. [ACdD03] use pseudonyms to preserve patient anonymity, and enable a user to transform statements concerning one of his pseudonyms into statements concerning one of his other pseudonyms (e.g., transforming a prescription for the pseudonym used with his doctor to a prescription for the pseudonym used with the pharmacist). Layouni
et al. \cite{LVS+09} consider communication between health monitoring equipment at a patient’s home and the health care centre. They propose a protocol using wallet-based credentials (a cryptographic primitive) to let patients control when and how much identifying information is revealed by the monitoring equipment. More recently, De Decker et al. \cite{dDLVV08} propose a health care system for communication between insurance companies and administrative bodies as well as patients, doctors and pharmacists. Their system relies on various cryptographic primitives to ensure privacy, including zero-knowledge proofs, signed proofs of knowledge (a signature scheme which uses zero-knowledge proofs to sign a message), and bit-commitments. Their system is explained in more detail in Section 5.4.

5.2.2 Doctor privacy

A relatively understudied privacy aspect is that of doctor privacy. Matyáš \cite{Mat98} investigates the problem of enabling analysis of prescription information while ensuring doctor privacy. His approach is to group doctors, and release the data per group, hiding who is in the group. He does not motivate a need for doctor privacy, however. Two primary reasons for doctor privacy have been identified in the literature: (1) \cite{ACdD03} to safeguard doctors against administrators setting specific efficiency metrics on their performance (e.g., requiring the cheapest medicine be used, irrespective of the patient’s needs). To address this, Ateniese et al. \cite{Ad02,ACdD03} propose an anonymous prescription system that uses group signatures for privacy; (2) \cite{dDLVV08} to prevent a pharmaceutical company from bribing a doctor to prescribe their medicine. A typical scenario can be described as follows. A pharmaceutical company seeks to persuade a doctor to favour a certain kind of medicine by bribing or coercing. To prevent this, a doctor should not be able to prove which medicine he is prescribing to this company (in general, to the adversary). This implies that doctor privacy must be enforced by e-health systems. De Decker et al. also note that preserving doctor privacy is not sufficient to prevent bribery: pharmacists could act as go-betweens, revealing the doctor’s identity to the briber, as pharmacists often have access to prescriptions, and thus know something about the prescription behaviour of a doctor. This leads us to formulate the requirement of independence of prescribing-privacy: no third party should be able to help the adversary link a doctor to his prescription.

Observations

In the above overview, we observe that current approaches to privacy in e-health mostly focus on patient privacy and try to solve it as an access control or authentication problem. Doctor privacy is also required, but research on doctor privacy is in its infancy. We believe that doctor privacy is as important as patient privacy and should be studied in more depth. It is clear from the analysis that privacy in e-health systems needs to be addressed at different layers: access control ensures privacy at the service layer; privacy by architecture design addresses privacy concerns at the system/architecture layer; use of cryptography guarantees privacy at the communication layer. Since e-health systems are complex \cite{TGC09} and rely on correct communications between many sub-systems, we strongly advocate to
study privacy in e-health as a communication problem. In fact, message exchanges in communication protocols may leak information which leads to a privacy breach. Classical privacy notions which are well-studied in the literature, attempt to ensure that privacy can be enabled. However, enabling privacy is far from enough. In many cases, a system must enforce user privacy instead of allowing the user to pursue it. To avoid doctor bribery, we take into account enforced privacy for doctors. In addition, we consider that one party’s privacy may depend on another party (e.g., in the case of a pharmacist revealing prescription behaviour of a doctor). Our opinion is that offering privacy is insufficient if privacy can be reduced in such ways.

In summary, we focus on the following privacy notions for doctors in communication protocols in the e-health domain.

**prescribing-privacy:** A protocol preserves prescribing-privacy if the adversary cannot link a doctor to his prescriptions.

**enforced prescribing-privacy:** A protocol satisfies enforced prescribing-privacy if a doctor cannot prove his prescriptions to the adversary.

**independency of prescribing-privacy:** A protocol preserves independency of prescribing-privacy if third parties cannot help the adversary to link a doctor to the doctor’s prescriptions.

**independency of enforced prescribing-privacy:** A protocol ensures independency of enforced prescribing-privacy if a doctor cannot prove his prescriptions to the adversary given that third parties sharing information with the adversary.

### 5.3 Formalisation of privacy notions

In order to formally verify privacy notions of a protocol, the first step is to give precise definitions of the privacy notions. The privacy notions in the previous section are formalised in the applied pi calculus. These notions focus on protecting doctor’s prescription behaviour against bribery. Such kinds of privacy notions have not been studied formally so far. In the end, we briefly show the definitions of anonymity, strong anonymity, untraceability and strong untraceability for e-health protocols, as such notions have been formally studied in the literature (e.g., [SS96, vMR08, BHM08, KT09, ACRR10, KTV10]), which can be lifted to the e-health domain.

In the following discussions, we model an e-health protocol \( P_{eh} \) as an \( n \)-role well-formed [ACRR10] protocol of the form:

\[
P_{eh} := \nu \text{chandata.init.} (!R_1 | \ldots | !R_n).
\]

Unlike in e-voting and e-auctions where the number of participates is determined, in e-health, participants need not to be predetermined. Essentially, this formalisation allows us to model an unbounded number of users (modelled by the exclamation
mark in front of each role) and represent each user as an instance of a role. In particular, we have a doctor role \( R_{dr} \) of the form:

\[
R_{dr} := \nu \text{Id}_{dr}.\text{init}_{dr}!P_{dr},
\]

where

\[
P_{dr} := \nu \text{presc}.\text{main}_{dr}.
\]

We focus on the behaviour of a doctor, since we aim to formalise privacy notions for doctors. Each doctor is associated with an identity and can execute an infinite number of sessions (modelled by the exclamation mark in front of \( P_{dr} \)). Within each session, the doctor will create a prescription. Processes \( \text{init} \) and \( \text{init}_{dr} \) model the initialisation of the protocol and the doctor role. Process \( P_{dr} \) models a session of the doctor role. Furthermore, we use \( C_{ch}[\_] \) to denote a context (a process with a hole) consisting of honest users,

\[
C_{ch}[\_] := \nu \text{chandata}.\text{init}.(\!|R_1| \ldots \!|R_n| \_.)
\]

Finally, \( \text{Id}_{dr} \) and \( \text{presc} \) are free variables; \( d_A \) and \( d_B \) are free names, representing doctor identities known to the adversary; and \( p_A \) and \( p_B \) are two free names, representing two different prescriptions.

### 5.3.1 Prescribing-privacy

Prescribing-privacy aims to protect doctors’ prescription behaviour, which can be captured by the unlinkability of a doctor and his prescriptions. Considering that doctors’ prescriptions are revealed eventually, e.g., in the DLV08 e-health protocol, unlinkability is modelled as indistinguishability when two honest users swap their actions (or items), e.g., see the formalisation of vote-privacy [5.14]. Thus, prescribing-privacy is modelled as the equivalence of two doctor processes: in the first process, an honest doctor \( d_A \) prescribes \( p_A \) in one of his sessions and another honest doctor \( d_B \) prescribes \( p_B \) in one of his sessions; in the second one, \( d_A \) prescribes \( p_B \) and \( d_B \) prescribes \( p_A \).

**Definition 5.1** (prescribing-privacy). A well-formed e-health protocol \( P_{ch} \) with a doctor role \( R_{dr} \), satisfies prescribing-privacy if for all possible doctors \( d_A \) and \( d_B \) (\( d_A \neq d_B \)) we have

\[
C_{ch}[\langle \text{init}_{dr}\{d_A/\text{Id}_{dr}\},(\!|P_{dr}\{d_A/\text{Id}_{dr}\} | \text{main}_{dr}\{d_A/\text{Id}_{dr},p_A/\text{presc}\}) \rangle | \\
\langle \text{init}_{dr}\{d_B/\text{Id}_{dr}\},(\!|P_{dr}\{d_B/\text{Id}_{dr}\} | \text{main}_{dr}\{d_B/\text{Id}_{dr},p_B/\text{presc}\}) \rangle |

\approx_{!} C_{ch}[\langle \text{init}_{dr}\{d_A/\text{Id}_{dr}\},(\!|P_{dr}\{d_A/\text{Id}_{dr}\} | \text{main}_{dr}\{d_A/\text{Id}_{dr},p_A/\text{presc}\}) \rangle | \\
\langle \text{init}_{dr}\{d_B/\text{Id}_{dr}\},(\!|P_{dr}\{d_B/\text{Id}_{dr}\} | \text{main}_{dr}\{d_B/\text{Id}_{dr},p_B/\text{presc}\}) \rangle |
\]

where \( p_A \) and \( p_B \) (\( p_A \neq p_B \)) are two prescriptions.

Process \( \langle \text{init}_{dr}\{d_A/\text{Id}_{dr}\},(\!|P_{dr}\{d_A/\text{Id}_{dr}\} | \text{main}_{dr}\{d_A/\text{Id}_{dr},p_A/\text{presc}\}) \rangle \) models an instance of a doctor, with identity \( d_A \). The sub-process \( \text{main}_{dr}\{d_A/\text{Id}_{dr},p_A/\text{presc}\} \) models a prescribing session in which \( d_A \) prescribes \( p_A \) for a patient. The sub-process \( \!|P_{dr}\{d_A/\text{Id}_{dr}\} \) models other prescribing sessions of \( d_A \). Similarly, process \( \langle \text{init}_{dr}\{d_B/\text{Id}_{dr}\},(\!|P_{dr}\{d_B/\text{Id}_{dr}\} | \text{main}_{dr}\{d_B/\text{Id}_{dr},p_B/\text{presc}\}) \rangle \) models another doctor \( d_B \). On the right hand of the equivalence, the two doctors, \( d_A \) and \( d_B \), swap their prescriptions, \( p_A \) and \( p_B \).
5.3.2 Enforced prescribing-privacy

De Decker et al. [dDLVV08] identify the need to prevent a pharmaceutical company from bribing a doctor to favour their medicine. Hence, doctor’s prescribing-privacy should be enforced by protocols to prevent doctor bribery. This means that intuitively, even if a doctor collaborates, the adversary cannot be certain that the doctor has followed his instructions. Bribed users cannot be modelled as part of the adversary, as they are not trusted by the adversary. Inspired by the formalisation of receipt-freeness in e-voting [3.17] and e-auction [4.2], we define enforced prescribing-privacy to be satisfied if there exists a process where the bribed doctor does not follow the adversary’s instruction (e.g., prescribing a particular medicine), which is indistinguishable from a process where she does.

Modelling this notion necessitates modelling a doctor who genuinely reveals all her private information to the adversary. This is achieved by process transformation \( P^{\text{chc}} \), which transforms a plain process \( P \) into one which shares all private information over the channel \( \text{chc} \) with the adversary (see Definition 3.15). In addition, we also use the transformation \( P^{\text{out}(\text{chc}.)} \) (see Definition 3.16). This models a process \( P \) which erases all outputs on channel \( \text{chc} \). Formally, \( P^{\text{out}(\text{chc}.)} := \nu \text{chc}.(P | \text{!ln(chc, x)}) \).

**Definition 5.2** (enforced prescribing-privacy). A well-formed e-health protocol \( P_{ch} \) with a doctor role \( R_{dr} \), satisfies enforced prescribing-privacy for a doctor \( d_{A} \), if there exist processes \( \text{init}_{dr}' \) and \( P_{dr}' \), such that:

1. \( C_{ch}((\text{init}_{dr}'(d_{A}/Id_{dr}).(!P_{dr}(d_{A}/Id_{dr}) | P_{dr}'(d_{A}/Id_{dr})))) | (\text{init}_{dr}(d_{A}/Id_{dr}).(!P_{dr}(d_{A}/Id_{dr}) | \text{main}_{dr}(d_{A}/Id_{dr}, P_{A}/\text{presc})))) \approx_{\ell} C_{ch}(((\text{init}_{dr}(d_{A}/Id_{dr}))^{\text{chc}}.(!P_{dr}(d_{A}/Id_{dr}) | (\text{main}_{dr}(d_{A}/Id_{dr}, P_{A}/\text{presc}))^{\text{chc}})) | (\text{init}_{dr}(d_{B}/Id_{dr}).(!P_{dr}(d_{B}/Id_{dr}) | \text{main}_{dr}(d_{B}/Id_{dr}, P_{B}/\text{presc})))));\)

2. \( \text{init}_{dr}'(d_{A}/Id_{dr}).(!P_{dr}(d_{A}/Id_{dr}) | P_{dr}'(d_{A}/Id_{dr})) \text{out}(\text{chc}.) \approx_{\ell} \text{init}_{dr}(d_{A}/Id_{dr}).(\text{main}_{dr}(d_{A}/Id_{dr}, P_{A}/\text{presc}));\)

where \( \text{init}_{dr}'(d_{A}/Id_{dr}).(!P_{dr}(d_{A}/Id_{dr}) | P_{dr}'(d_{A}/Id_{dr})) \) is a closed plain process, \( \text{chc} \) is a fresh channel name, \( P_{A} \) and \( P_{B} \) \( (P_{A} \neq P_{B}) \) are two prescriptions, and \( d_{B} \) \( (d_{A} \neq d_{B}) \) is a doctor identity.

In the definition, the process \( \text{init}_{dr}'(d_{A}/Id_{dr}).(!P_{dr}(d_{A}/Id_{dr}) | P_{dr}'(d_{A}/Id_{dr})) \) models the process in which the doctor \( d_{A} \) lies to the adversary about one of his prescriptions. The real prescription behaviour of \( d_{A} \) is modelled by the second equivalence. The first equivalence shows that the adversary cannot distinguish whether \( d_{A} \) lied, given a counter-balancing doctor \( d_{B} \).

The difference between this formalisation and receipt-freeness in e-voting and in e-auctions is that in this definition only a part of the doctor process (the initiation sub-process and a prescribing session) shares information with the adversary. In e-voting, each voter only votes once. In the contrast, a doctor prescribes multiple times for various patients. As patients and situations of patients vary, a doctor cannot prescribe medicine from the bribing pharmaceutical company all the time. Therefore, only part of the doctor process shares information with the adversary. We model only one bribed prescribing session, as that if a doctor can cheat the adversary in one session, he can easily cheat in multiple similar sessions. This definition can be easily extended to model multiple prescribing sessions being bribed.
5.3.3 Independence of prescribing-privacy

Usually, e-health systems have to deal with a complex constellation of roles: doctors, patients, pharmacists, insurance companies, medical administration, etc. Each of these roles has access to different private information and has different privacy concerns. An untrusted role may be bribed to reveal private information to the adversary such that the adversary can break another roles’ privacy. De Decker et al. [dDLVV08] note that pharmacists may have sensitive data which can be revealed to the adversary to break a doctor’s prescribing-privacy. To prevent a party (not a doctor) to do this, e-health protocols are required to satisfy **independency of prescribing-privacy**, meaning that even if another party \( R_i \) reveals their information (i.e., \( R_i^{\text{chc}} \)), the adversary should not be able to break a doctor’s prescribing-privacy.

**Definition 5.3** (independency of prescribing-privacy). A well-formed e-health protocol \( P_{ch} \) with a doctor role \( R_{dr} \), satisfies prescribing-privacy independent of role \( R_i \), if for all possible doctors \( d_A \) and \( d_B \) (\( d_A \neq d_B \)) we have

\[
C_{ch}[[ R_i^{\text{chc}} | \left( \text{init}_{dr}\{ d_A/Id_{dr} \}, (!P_{dr}\{ d_A/Id_{dr} \}) \right) | \left( \text{main}_{dr}\{ d_A/Id_{dr}, p_A/\text{presc} \} \right)] | \left( \text{init}_{dr}\{ d_B/Id_{dr} \}, (!P_{dr}\{ d_B/Id_{dr} \}) \right) | \left( \text{main}_{dr}\{ d_B/Id_{dr}, p_B/\text{presc} \} \right)] \approx \ell C_{ch}[[ R_i^{\text{chc}} | \left( \text{init}_{dr}\{ d_A/Id_{dr} \}, (!P_{dr}\{ d_A/Id_{dr} \}) \right) | \left( \text{main}_{dr}\{ d_A/Id_{dr}, p_A/\text{presc} \} \right)] | \left( \text{init}_{dr}\{ d_B/Id_{dr} \}, (!P_{dr}\{ d_B/Id_{dr} \}) \right) | \left( \text{main}_{dr}\{ d_B/Id_{dr}, p_B/\text{presc} \} \right)]
\]

where \( p_A \) and \( p_B \) (\( p_A \neq p_B \)) are two prescriptions, and \( R_i \) is a non-doctor role.

Note that we assume a worst situation in which role \( R_i \) genuinely cooperates with the adversary. For example, the pharmacist forwards all information obtained from channels hidden from the adversary. The equivalence shows that no matter how role \( R_i \) cooperates with the adversary, the adversary cannot link a doctor to the doctor’s prescriptions.

5.3.4 Independence of enforced prescribing-privacy

We have discussed two situations where a doctor’s prescription behaviour can be revealed when either the doctor or another different party cooperates with the adversary. It is natural to consider the conjunction of these two, i.e., a situation in which the adversary coerces both a doctor and another party (not a doctor). Since the adversary obtains more information, this constitutes a stronger attack on doctor’s prescribing-privacy. To address this problem, we define **independency of enforced prescribing-privacy**, which is satisfied when a doctor’s prescribing-privacy is preserved even if both the doctor and another party reveal their private information to the adversary.

**Definition 5.4** (independency of enforced prescribing-privacy). A well-formed e-health protocol \( P_{ch} \) with a doctor role \( R_{dr} \), satisfies enforced prescribing-privacy independent of role \( R_i \) for a doctor \( d_A \), if there exist processes \( \text{init}'_{dr} \) and \( P'_{dr} \), such
5.3 Formalisation of privacy notions

5.3.5 Anonymity and strong anonymity

Anonymity is a privacy notion that protects users’ identities. We model anonymity as indistinguishability of processes initiated by two different users.

**Definition 5.5** (doctor anonymity). A well-formed e-health protocol \( P_{eh} \) with a doctor role \( R_{dr} \) satisfies doctor anonymity for a doctor \( d_A \) if there exists another doctor \( d_B \), such that

\[
C_{eh}[\text{init}_{dr}\{d_A/Id_{dr}\}!P_{dr}\{d_A/Id_{dr}\}] \approx_{\ell} C_{eh}[\text{init}_{dr}\{d_B/Id_{dr}\}!P_{dr}\{d_B/Id_{dr}\}].
\]

A stronger notion of anonymity is defined in [ACRR10], capturing the situation that the adversary cannot even find out whether a user (with identity \( d_A \)) has participated in a session of the protocol or not.

**Definition 5.6** (strong doctor anonymity [ACRR10]). A well-formed e-health protocol \( P_{eh} \) with a doctor role \( R_{dr} \) satisfies strong doctor anonymity, if

\[
P_{eh} \approx_{\ell} \nu chandata.init.(!R_1 \mid \ldots \mid !R_n \mid (\text{init}_{dr}\{d_A/Id_{dr}\}!P_{dr}\{d_A/Id_{dr}\})).
\]

Similarly, we can define anonymity and strong anonymity for patient and other roles in an e-health protocol, by replacing the doctor role with a different role.

5.3.6 Untraceability and strong untraceability

Untraceability is a notion preventing the adversary from tracing a user, meaning that he cannot tell whether two executions are initiated by the same user.

**Definition 5.7** (doctor untraceability). A well-formed e-health protocol \( P_{eh} \) with a doctor role \( R_{dr} \) satisfies doctor untraceability if, for any two doctors \( d_A \) and \( d_B \neq d_A \),

\[
C_{eh}[\text{init}_{dr}\{d_A/Id_{dr}\}!(P_{dr}\{d_A/Id_{dr}\} \mid P_{dr}\{d_A/Id_{dr}\})] \approx_{\ell} C_{eh}[(\text{init}_{dr}\{d_A/Id_{dr}\}P_{dr}\{d_A/Id_{dr}\}) \mid (\text{init}_{dr}\{d_B/Id_{dr}\}P_{dr}\{d_B/Id_{dr}\})].
\]
A stronger notion of untraceability is proposed in \cite{ACRR10} that captures the adversary’s inability to distinguish the situation where one user executes the protocol multiple times from no user executing the protocol more than once.

**Definition 5.8** (strong doctor untraceability \cite{ACRR10}). A well-formed e-health protocol $P_{eh}$ with a doctor role $R_{dr}$ being the $j$-th role, satisfies strong doctor untraceability, if

$$P_{eh} \approx_{\ell} vchandata.init.(\mid R_1 \mid \ldots \mid R_{j-1} \mid \mid R_{j+1} \mid \mid R_n \mid (\nu Id_{dr}.init_{dr}.P_{dr})).$$

Similarly, we can define untraceability and strong untraceability for patient and other roles in a protocol, by replacing the doctor role with a different role.

### 5.4 Case study: the DLV08 protocol

In this section, we apply the above formal definitions for doctor privacy in a case study as a validation of the definitions. We choose to analyse the DLV08 e-health protocol proposed by De Decker et al. since it claims enforced privacy for doctors. However, the analysis is not restricted to doctor privacy. We provide a rather complete analysis of the protocol including patient anonymity, patient untraceability, patient/doctor information secrecy and patient/doctor authentication as well. ProVerif code is available at \cite{1}.

#### 5.4.1 Introduction

The DLV08 protocol is a complex health care protocol for the Belgium situation \cite{DLVV08}, which captures most aspects of the current Belgian health care practice and aims to provide a strong guarantee of privacy for patients and doctors. The protocol involves five roles: doctor, patient, pharmacist, medicine prescription administrator (MPA) and health insurance institute (HII); and it works as follows: a doctor prescribes medicine to a patient; next the patient obtains medicine from a pharmacist according to the prescription; following that, the pharmacist forwards the prescription to his MPA, the MPA checks the prescription and refunds the pharmacist; finally, the MPA sends invoices to the patient’s HII and is refunded. As we do not focus on properties such as revocability and reimbursement, we do not consider the other two roles: public safety organisation (PSO) and social security organisation (SSO).

#### 5.4.2 Cryptographic primitives

To ensure security and privacy properties, the DLV08 protocol employs several special cryptographic primitives, besides the classical ones, like encryption. We briefly introduce the following special cryptographic primitives.

*Bit-commitments.* The bit-commitments scheme consists of two phases, committing phase and opening phase. On the committing phase, a message sender makes
a commitment on a message. It can be considered as putting the message into a box, and sending the box to the receiver. Later in the opening phase, the sender sends the key of the box to the receiver. The receiver opens the box and obtains the message.

**Zero-knowledge proofs.** A zero-knowledge proof is a cryptographic scheme which can be used for one party (prover) to prove to another party (verifier) that a statement is true, without leaking secret information of the prover. A zero-knowledge proof scheme can be interactive or non-interactive. We consider the non-interactive zero-knowledge proofs in this protocol.

**Digital credentials.** A digital credential is like a certificate, which can be used to prove that the owner qualifies some requirements. Unlike some paper certificates such as passport which gives out the owner’s identity, a digital credential could be used to authenticate the owner anonymously. For example, a digital credential can be used to prove that a driver is old enough to drive without showing the age of the driver.

**Anonymous authentication.** Anonymous authentication is a scheme for authenticating a user anonymously. The procedure of an anonymous authentication is actually a zero-knowledge proof, with the digital credential being the public information of the prover. In the scheme, a user’s digital credential is used as the public key in the public key authentication structure. A verifier can check whether a message is signed correctly by the prover, while the verifier cannot identify the prover. Thus, this ensures anonymous authentication.

**Verifiable encryptions.** A verifiable encryption is a zero-knowledge proof as well. A prover encrypts a message, and uses zero-knowledge proofs to prove that the encrypted message satisfies some properties without showing the original message.

**Signed proofs of knowledge.** Signed proofs of knowledge is using proofs of knowledge as a digital signature scheme (for details see [Bra00]). Intuitively, a prover signs a message using some secret information, which can be considered as a secret signing key. And the prover uses proofs of knowledge to convince the verifier that he has the secret signing key corresponding to the public key.

### 5.4.3 Settings

Every participant of the protocol is equipped with some initial information.

- A doctor has an identity ($Id_{dr}$), a pseudonym ($Pnym_{dr}$), and an anonymous doctor credential ($Cred_{dr}$) issued by trusted authorities.

- A patient has an identity ($Id_{pt}$), a pseudonym ($Pnym_{pt}$), an HII ($Hii$), a social security status ($Sss$), a health expense account ($Acc$) and an anonymous patient credential ($Cred_{pt}$) issued by trusted authorities.
Pharmacists, MPA, and HII are public entities, each of which has an identity \((\text{Id}_{\text{ph}}, \text{Id}_{\text{mpa}}, \text{Id}_{\text{hii}})\), a secret key \((\text{sk}_{\text{ph}}, \text{sk}_{\text{mpa}}, \text{sk}_{\text{hii}})\) and an authorised public key certificate \((\text{pk}_{\text{ph}}, \text{pk}_{\text{mpa}}, \text{pk}_{\text{hii}})\) issued by trusted authorities.

### 5.4.4 Description of the protocol

The DLV08 protocol consists of four sub-protocols: doctor-patient sub-protocol, patient-pharmacist sub-protocol, pharmacist-MPA sub-protocol, and MPA-HII sub-protocol. We describe the sub-protocols one by one.

**Doctor-patient sub-protocol**

The doctor authenticates himself to a patient using the authorised doctor credential. The patient verifies the doctor credential. If the verification passes, the patient authenticates himself to the doctor using the patient credential, sends the bit-commitments on his identity to the doctor, and proves to the doctor that the identity used in the credential is the same as in the bit-commitments. After verifying the patient credential, the doctor generates a prescription, computes a prescription identity, computes the doctor bit-commitments. Then the doctor combines these computed messages with the received patient bit-commitments; signs these messages using a signed proof of knowledge, which proves that the doctor’s pseudonym used in the doctor credential is the same as in the doctor bit-commitments. Together with the proof, the doctor sends the open information of the doctor bit-commitments. The communication in the doctor-patient sub-protocol is shown as a message sequence chart (MSC) in Figure 5.1.

**Patient-Pharmacist sub-protocol**

The pharmacist authenticates himself to the patient. The patient verifies the authentication and obtains, from the authentication, the pharmacist’s identity and the pharmacist’s MPA. Then the patient anonymously authenticates himself to the pharmacist, and proves his social security status. Next, the patient computes verifiable encryptions \(v_{c1}, v_{c2}, v_{c3}, v_{c3}', v_{c4}, v_{c5}\), where

- \(v_{c1}\) encrypts the patient’s HII using the MPA’s public key and proves that the HII encrypted in \(v_{c1}\) is the same as the one in the patient’s credential.
- \(v_{c2}\) encrypts the doctor’s pseudonym using the MPA’s public key and proves that the doctor’s pseudonym encrypted in \(v_{c2}\) is the same as the one in the doctor commitment embedded in the prescription.
- \(v_{c3}\) encrypts the patient’s pseudonym using the public safety organisation’s public key and proves that the pseudonym encrypted in \(v_{c3}\) is the same as the one in the patient’s commitment.
- \(v_{c3}'\) encrypts the patient’s HII using the social security organisation’s public key and proves that the content encrypted in \(v_{c3}'\) is the same as the HII in the patient’s credential.
5.4 Case study: the DLV08 protocol

Id\(_{pt}\), Pnym\(_{pt}\), Hii, Sss, Acc

\hspace{1cm} pt

\hspace{1cm} anonymous authentication (Auth\(_{dr}\))

\hspace{1cm} verify authentication, commit on Id\(_{pt}\) (Comt\(_{pt}\))

\hspace{1cm} anonymous authentication (Auth\(_{pt}\)), commitment (Comt\(_{pt}\)).

\hspace{1cm} PtProof (zk: link between Auth\(_{pt}\), Comt\(_{pt}\))

\hspace{1cm} \text{verify authentication, commit on Pnym\(_{dr}\)},
\hspace{1cm} \text{prescribe medicine presc,}
\hspace{1cm} \text{compute prescription identity}

\hspace{1cm} \text{PrescProof (spk: prove the link between Auth\(_{dr}\), Comt\(_{dr}\), presc and the patient),}
\hspace{1cm} \text{open information to commitment}

\hspace{1cm} \text{verify PrescProof,}
\hspace{1cm} \text{open Comt\(_{dr}\)}

\hspace{1cm} \text{Id\(_{dr}\), Pnym\(_{dr}\)}

\hspace{1cm} \text{\[DLV08\]} I. Doctor-Patient sub-protocol

Figure 5.1: Doctor-Patient sub-protocol.

- \(vc_4\) encrypts the patient’s pseudonym using the MPA’s public key and proves that the patient’s pseudonym encrypted in \(vc_4\) is the same as the one in the patient’s credential.

- \(vc_5\) encrypts the patient’s pseudonym using his HII’s public key and proves that the patient’s pseudonym encrypted in \(vc_5\) is the same as the one in the patient’s credential.

- \(c_5\) encrypts \(vc_5\) using the MPA’s public key.

The patient sends the received prescription to the pharmacist and proves to the pharmacist that the patient’s identity in the prescription is the same as in the patient credential. The patient sends \(vc_1, vc_2, vc_3, vc'_3, vc_4, c_5\) as well. The pharmacist verifies the correctness of all the received messages. If every message is correctly formatted, the pharmacist charges the patient, and delivers the medicine. Then the pharmacist generates an invoice and sends it to the patient. The patient computes a receipt \(Receipt\text{Ack}\): signing a message (consists of the prescription identity, the pharmacist’s identity, \(vc_1, vc_2, vc_3, vc'_3, vc_4, vc_5\)) using a signed proof of knowledge and proving that he can construct the patient credential. This receipt proves that the patient has received his medicine. The pharmacist verifies the correctness of the receipt. The communication in the patient-Pharmacist sub-protocol is shown in Figure 5.2.
Pharmacist-MPA sub-protocol

The pharmacist and the MPA first authenticate each other using public key authentication. Then the pharmacist sends the received prescription and the receipt ReceiptAck, together with $vc_1, vc_2, vc_3, vc'_3, vc_4, c_5$, to the MPA. The MPA verifies correctness of the received information. Then, the MPA decrypts $vc_1, vc_2, vc_3, vc_4, c_5$, which provide the patient’s HII, the doctor’s pseudonym, the patient’s pseudonym, and $vc_5$. The communication in the pharmacist-MPA sub-protocol is shown in Figure 5.3.

MPA-HII sub-protocol

The MPA and the patient’s HII first authenticate each other using public key authentication. Then the MPA sends the receipt ReceiptAck to the patient’s HII as well as the verifiable encryption $vc_5$ which encrypts the patient’s pseudonym.
with the patient’s HII’s public key. The patient’s HII checks the correctness of ReceiptAck, decrypts $vc_5$ and obtains the patient’s pseudonym. From the patient pseudonym, the HII obtains the identity of the patient; then updates the patient’s account and pays the MPA. The MPA pays the pharmacist when he receives the payment. The communication in the MPA-HII sub-protocol is shown in Figure 5.4.

### 5.4.5 Claimed privacy properties

The DLV08 protocol is claimed to satisfy the following properties:

- **Secrecy of patient and doctor information:** No other party should be able to know a patient or a doctor’s information, unless the information is intended to be revealed in the protocol.

- **Authentication:** All parties should properly authenticate each other.

- **Patient anonymity:** No party should be able to determine a patient’s identity.

- **Patient untraceability:** Prescriptions issued to the same patient should not be linkable to each other.

- **Prescribing-privacy:** The protocol protects a doctor’s prescription behaviour.

- **Enforced prescribing-privacy:** The protocol prevents bribery between doctors and pharmaceutical companies.

- **Independency of prescribing-privacy:** Pharmacists should not be able to provide evidence to pharmaceutical companies about doctors’ prescription.
Figure 5.4: MPA-HII sub-protocol.

5.5 Modelling DLV08

We model the DLV08 protocol in the applied pi calculus. Since the description of the protocol is not clear in some details, before modelling the protocol, a few ambiguities need to be settled. Next we explain the modelling of a few cryptographic primitives, since security and privacy rely heavily on these cryptographic primitives in the protocol. Then, we illustrate the modelling of the protocol.

5.5.1 Underspecification of the DLV08 protocol

The DLV08 protocol leaves the following issues unspecified:

- **a1** whether a zero-knowledge proof is transferable;
- **a2** whether an encryption is probabilistic;
- **a3** whether a patient/doctor uses a fresh identity and/or pseudonym for each session;
- **a4** whether credentials are freshly generated in each session;
- **a5** what a patient’s social security status is and how it can be modified;
- **a6** how many HIIs exist and whether a patient can change his HII;
- **a7** whether a patient/doctor can obtain a credential by requesting one;
- **a8** what type of communication channels are used (public or untappable).

To be able to discover potential flaws on privacy, we make the following (weakest) assumptions in our modelling of the DLV08 protocol:
5.5 Modelling DLV08

- **s1** the zero-knowledge proofs used are non-interactive and transferable;
- **s2** encryptions are not probabilistic;
- **s3** a patient/doctor uses the same identity and pseudonym in every session;
- **s4** a patient/doctor has the same credential in every session;
- **s5** a patient’s social security status is the same in every session;
- **s6** there are many HIIs, different patients may have different HIIs, and a patient’s HII is fixed and cannot be changed;
- **s7** a patient/doctor’s credential can be obtained by requesting one;
- **s8** the communication channels are public.

5.5.2 Modelling cryptographic primitives

The cryptographic primitives are modelled in the applied pi calculus using function symbols and equational theory. All functions and equational theory are shown in Figure 5.5, Figure 5.6 and Figure 5.7.

**Bit-commitments.** The bit-commitments scheme is modelled as two functions: function `commit`, modelling the committing phase, and function `open`, modelling the opening phase. Function `commit` creates a commitment with two parameters: a message `m` and a random number `r`. A commitment can only be opened with the correct opening information `r`, thus reveals the message `m`.

\[
\text{fun} \quad \text{commit}/2. \\
\text{reduc} \quad \text{open}(\text{commit}(m, r), r) = m.
\]

**Zero-knowledge proofs.** Non-interactive zero-knowledge proofs can be modelled as function `zk(secrets, pub_info)` (a function with two parameters: a tuple of secret information `secrets`, and a tuple of public information `pub_info`) inspired by BMU08. The verifying information and the secret information satisfies a relation. Since the secret information is only known by the prover, only the prover can construct the zero-knowledge proof. To verify a zero-knowledge proof is to check whether the relation between the secret formation and the verifying formation is satisfied. The verification of a zero-knowledge proof is modelled as function `Vfy-zk(zk(secrets, pub_info), verif_info)`, in which two parameters are: a zero-knowledge proof to be verified `zk(secrets, pub_info)` and the verification information `verif_info`. Compared to that in BMU08, we define each zero-knowledge specifically, as only a limited number of zero-knowledge proofs are used in the protocol. We specify each verification rule as in Figure 5.7. Since the `pub_info` and `verif_info` happens to be the same in all the zero-knowledge proofs verifications in this protocol, the generic structure of verification rule is as

\[
\text{Vfy-zk}(\text{zk}(\text{secrets}, \text{pub_info}), \text{pub_info}) = \text{true},
\]
where \texttt{true} is a constant. The specific function to check a zero-knowledge proof of type $z$ is denoted as $\text{Vfy-zk}_z$, e.g., verification of a patient’s anonymous authentication is modelled by function $\text{Vfy-zk}_{\text{Auth}_{pt}}$.

**Digital credentials.** A digital credential is issued by trusted authorities. We assume the procedure of issuing a credential is perfect, which means that the adversary cannot forge a credential nor obtain one by impersonation. We model digital credentials as a private function (declared by key word \texttt{private fun} in ProVerif) which is only usable by honest users. In the DLV08 protocol, a credential can have several attributes; we model these as parameters of the credential function.

\[
\text{private fun} \, \text{drcred}/2. \quad \text{private fun} \, \text{ptcred}/5.
\]

There are two credentials in the DLV08 protocol: a doctor credential which is modelled as $\text{Cred}_{dr} := \text{drcred}(\text{Pnym}_{dr}, \text{Id}_{dr})$, and a patient credential which is modelled as $\text{Cred}_{pt} := \text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hii}, \text{Sss}, \text{Acc})$.

**Anonymous authentication.** The procedure of anonymous authentication is a zero-knowledge proof using the digital credential as public information. The anonymous authentication of a doctor is modelled as

\[
\text{Auth}_{dr} := \text{zk}((\text{Pnym}_{dr}, \text{Id}_{dr}), \text{drcred}(\text{Pnym}_{dr}, \text{Id}_{dr}))
\]

and the verification of the authentication is modelled as

\[
\text{Vfy-zk}_{\text{Auth}_{dr}}(\text{Auth}_{dr}, \text{drcred}(\text{Pnym}_{dr}, \text{Id}_{dr})).
\]

The equational theory for the verification is

\[
\text{reduc} \quad \text{Vfy-zk}_{\text{Auth}_{dr}}(\text{zk}((\text{Pnym}_{dr}, \text{Id}_{dr}), \text{drcred}(\text{Pnym}_{dr}, \text{Id}_{dr})), \\
\text{drcred}(\text{Pnym}_{dr}, \text{Id}_{dr})) = \text{true}.
\]

The verification implies that the creator of the authentication is a doctor, because only doctors can use the function \text{drcred}, and thus create a valid proof. The adversary can observe a credential \text{drcred}(\text{Pnym}_{dr}, \text{Id}_{dr}), but does not know secrets \text{Pnym}_{dr}, \text{Id}_{dr}, and thus cannot forge a valid zero-knowledge proof. If the adversary forges a zero-knowledge proof with fake secret information \text{Pnym}_{dr}', and \text{Id}_{dr}', the fake zero-knowledge proof will not pass verification. For the same reason, a validated proof proves that the credential belongs to the creator of the zero-knowledge proof. Similarly, an anonymous authentication of a patient is modelled as

\[
\text{Auth}_{pt} := \text{zk}((\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hii}, \text{Sss}, \text{Acc}), \\
\text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hii}, \text{Sss}, \text{Acc}))
\]

and the verification rule is modelled as

\[
\text{reduc} \quad \text{Vfy-zk}_{\text{Auth}_{pt}}(\text{zk}((\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hii}, \text{Sss}, \text{Acc}), \\
\text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hii}, \text{Sss}, \text{Acc}), \\
\text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hii}, \text{Sss}, \text{Acc})) = \text{true}.
\]
Verifiable encryptions. A verifiable encryption is modelled as a zero-knowledge proof. The encryption is embedded in the zero-knowledge proof as public function. The receiver can obtain the cipher text from the proof. For example, a patient wants to prove that he encrypted a secret $s$ using a public key $k$ to a pharmacist, while the pharmacist does not know the corresponding secret key for $k$. The pharmacist cannot open the cipher text to test whether it uses the public key $k$ to encrypt. However, the zero-knowledge proof can prove that the cipher text is encrypted using $k$, while not revealing $s$. The general structure of the verification of a verifiable encryption is

$$\text{Vfy-venc}(\text{zk}(\text{secrets}, (\text{pub info}, \text{cipher})), \text{verif info}) = \text{true},$$

where $\text{secrets}$ is private information, $\text{pub info}$ and $\text{cipher}$ consist public information, $\text{verif info}$ is the verification information.

Signed proofs of knowledge. A signed proof of knowledge is a scheme which signs a message, and proves a property of the signer. For the DLV08 protocol, this proof only concerns equality of attributes of credentials and commitments (e.g. the identify of this credential is the same as the identity of that commitment). To verify a signed proof of knowledge, the verifier must know which credentials/commitments are considered. Hence, this information must be obtainable from the proof, and thus is included in the model. In general, a signed proof of knowledge is modelled as function

$$\text{spk}(\text{secrets}, \text{pub info}, \text{msg}),$$

which models a signature using private value(s) $\text{secrets}$ on the message $\text{msg}$, with public information $\text{pub info}$ as settings. What knowledge is proven, depends on the specific instance of the proof and is captured by the verification functions for the specific proofs. For example, to prove that a user knows a) all fields of a (simplified) credential, b) all fields of a commitment to an identity, and c) that the commitment concerns the same identity as the commitment, he generates the following proof:

$$\text{spk}(\text{Id}_{pt}, \text{Pnym}_{pt}, r_{pt}), \quad (\text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}), \text{commit}(\text{Id}_{pt}, r_{pt})), \quad (\text{msg}).$$

These proofs are verified by checking that the signature is correct, given the signed message and the verification information. E.g., the above example proof can be verified as follows:

$$\text{reduce \ Vfy-spk( spk((Id_{pt}, Pnym_{pt}, r_{pt}), (ptcred(Id_{pt}, Pnym_{pt}), commit(Id_{pt}, r_{pt})), msg ), (signed\_message*) \quad (\text{verify info*)} \quad (message*) ) = \text{true}.}$$
Figure 5.5: Functions.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>fun</code></td>
<td><code>true/0.</code>, <code>hash/3.</code>, <code>pk/1.</code></td>
</tr>
<tr>
<td><code>fun</code></td>
<td><code>enc/2.</code>, <code>commit/2.</code>, <code>sign/2.</code></td>
</tr>
<tr>
<td><code>private fun</code></td>
<td><code>dcred/2.</code>, <code>ptcred/5.</code>, <code>zk/2.</code></td>
</tr>
<tr>
<td><code>fun</code></td>
<td><code>spk/3.</code>, <code>invoice/1.</code>, <code>key/1.</code></td>
</tr>
<tr>
<td><code>fun</code></td>
<td><code>host/1.</code></td>
</tr>
</tbody>
</table>

Figure 5.6: Equational theory part I: non-zero-knowledge part.

_reduc_ dec(enc(m, pk(sk)), sk) = m.
_reduc_ open(commit(x, y), y) = x.
_reduc_ Vfy-sign(sign(x, y), pk(y)) = true.
_reduc_ getsignmsg(sign(x, y), pk(y)) = x.
_reduc_ getpublic(zk(x, y)) = y.
_reduc_ getmsg(spk(x, y, z)) = z.
_reduc_ getSpkVinfo(spk(x, y, z)) = y.
_equation_ key(host(x)) = x.
_equation_ host(key(x)) = x.

Other cryptographic primitives. Hash functions, encryptions and signing messages are modelled by functions `hash`, `enc`, and `sign`, respectively (Figure 5.5). Correspondingly, decryption and retrieving the message from a signature are modelled as functions `dec` and `getsignmsg` (Figure 5.6). Functions `getpublic`, `getSpkVinfo` and `getmsg` model retrieving public information from a zero-knowledge proof, from a signed proof of knowledge, and obtaining the message from a signed proof of knowledge, respectively (Figure 5.6).

5.5.3 Modelling the DLV08 protocol

Modelling the doctor-patient sub-protocol. This sub-protocol is used for a doctor, whose steps are labelled `di` in Figure 5.8, to prescribe medicine for a patient, whose steps are labelled `ti` in Figure 5.9.

First, the doctor anonymously authenticates to the patient using credential `Cred_{dr}` (`d1`). The patient reads in the doctor authentication (`t1`), obtains the doctor credential (`t2`), and verifies the authentication (`t3`). If the verification in step (`t3`) succeeds, the patient anonymously authenticates himself to the doctor using his credential (`t5`, the first `zk` function), generates a nonce `r_{pt}` (`t4`), computes a commitment with the nonce as opening information, and proves that the patient identity used in the patient credential is the same as in the commitment, thus linking the patient commitment and the patient credential (`t5`, the second `zk`).

The doctor reads in the patient authentication as `recv_Auth_{pt}` and the patient proof as `recv_PtProof` (`d2`), obtains the patient credential from the patient authentication (`d3`), obtains the patient commitment `c_{Comt_{pt}}` and the patient credential from the patient proof, tests whether the credential matches the one embedded in the patient authentication (`d4`), then verifies the authentication (`d5`) and the patient
5.5 Modelling DLV08

\[ \text{reduc } \text{Vfy-zk}_{\text{Auth}}(\text{zk}((\text{Pnym}_{dr}, \text{Id}_{dr}), \text{drcr}d(\text{Pnym}_{dr}, \text{Id}_{dr}))), \]
\[ \text{drcr}d(\text{Pnym}_{dr}, \text{Id}_{dr})) = \text{true}. \]

\[ \text{reduc } \text{Vfy-zk}_{\text{Auth}}(\text{zk}((\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc})), \]
\[ \text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc})), \]
\[ \text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc})) = \text{true}. \]

\[ \text{reduc } \text{Vfy-zk}_{\text{PtProof}}(\text{zk}((\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc}), \]
\[ \text{commit}(\text{Id}_{pt}, \text{r}_{pt}), \]
\[ \text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc})), \]
\[ \text{commit}(\text{Id}_{pt}, \text{r}_{pt}), \]
\[ \text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc})) = \text{true}. \]

\[ \text{reduc } \text{Vfy-spk}_{\text{PrescProof}}(\text{spk}((\text{Pnym}_{dr}, \text{r}_{dr}, \text{Id}_{dr}), \]
\[ \text{commit}(\text{Pnym}_{dr}, \text{r}_{dr}), \text{drcr}d(\text{Pnym}_{dr}, \text{Id}_{dr})), \]
\[ \text{presc, PrescriptID}, \text{commit}(\text{Pnym}_{dr}, \text{r}_{dr}), \]
\[ \text{commit}(\text{Id}_{pt}, \text{r}_{pt}))), \]
\[ \text{drcr}d(\text{Pnym}_{dr}, \text{Id}_{dr}), \text{presc, PrescriptID}, \]
\[ \text{commit}(\text{Pnym}_{dr}, \text{r}_{dr}), \text{commit}(\text{Id}_{pt}, \text{r}_{pt})) = \text{true}. \]

\[ \text{reduc } \text{Vfy-spk}_{\text{PtSsk}}(\text{spk}((\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc}, \text{r}_{pt}), \]
\[ \text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc}), \text{Sss}), \]
\[ \text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc}, \text{Sss}))) = \text{true}. \]

\[ \text{reduc } \text{Vfy-venc}_{\text{Hi}}(\text{zk}((\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc}), \]
\[ \text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc}), \]
\[ \text{enc}(\text{Hi}, \text{p}_{k_{x}}))), \]
\[ \text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc}, \text{enc}(\text{Hi}, \text{p}_{k_{x}}), \text{p}_{k_{x}})) = \text{true}. \]

\[ \text{reduc } \text{Vfy-venc}_{\text{DrnymMpa}}(\text{zk}((\text{Pnym}_{dr}, \text{r}_{dr}), \]
\[ \text{spk}((\text{Pnym}_{dr}, \text{r}_{dr}, \text{Id}_{dr}), \]
\[ \text{commit}(\text{Pnym}_{dr}, \text{r}_{dr}), \text{drcr}d(\text{Pnym}_{dr}, \text{Id}_{dr})), \]
\[ \text{presc, PrescriptID}, \]
\[ \text{commit}(\text{Pnym}_{dr}, \text{r}_{dr}), \text{c}_{\text{ph-Comt}_{pt}})), \]
\[ \text{enc}(\text{Pnym}_{dr}, \text{p}_{k_{x}}))), \]
\[ \text{spk}((\text{Pnym}_{dr}, \text{r}_{dr}, \text{Id}_{dr}), \]
\[ \text{commit}(\text{Pnym}_{dr}, \text{r}_{dr}), \text{drcr}d(\text{Pnym}_{dr}, \text{Id}_{dr})), \]
\[ \text{presc, PrescriptID}, \]
\[ \text{commit}(\text{Pnym}_{dr}, \text{r}_{dr}), \text{c}_{\text{ph-Comt}_{pt}})), \]
\[ \text{enc}(\text{Pnym}_{dr}, \text{p}_{k_{x}}), \text{p}_{k_{x}}) = \text{true}. \]

\[ \text{reduc } \text{Vfy-venc}_{\text{Drnym}}(\text{zk}((\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc}), \]
\[ \text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc}, \text{enc}(\text{Pnym}_{pt}, \text{p}_{k_{x}}))), \]
\[ \text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc}, \text{enc}(\text{Pnym}_{pt}, \text{p}_{k_{x}}), \text{p}_{k_{x}})) = \text{true}. \]

\[ \text{reduc } \text{Vfy-spk}_{\text{ReceiptAck}}(\text{spk}((\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc}, \]
\[ \text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc}, \]
\[ (\text{e}_{\text{PrescriptID}}, \text{c}_{\text{pt-Id}_{ph}}, \text{v}_{c_{1}}, \text{v}_{c_{2}}, \text{v}_{c_{3}}, \text{v}_{c_{4}}, \text{v}_{c_{5}})), \]
\[ \text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hi}, \text{Sss}, \text{Acc}, \]
\[ (\text{e}_{\text{PrescriptID}}, \text{c}_{\text{pt-Id}_{ph}}, \text{v}_{c_{1}}, \text{v}_{c_{2}}, \text{v}_{c_{3}}, \text{v}_{c_{4}}, \text{v}_{c_{5}})) = \text{true}. \]

Figure 5.7: Equational theory part II: zero-knowledge part.
\[
P_{dr} :=
\]
1. out(ch, zk((Pnym_{dr}, Id_{dr}), drcred(Pnym_{dr}, Id_{dr}))).
2. in(ch, (rcv_{Auth_{pt}}, rcv_{PtProof})).
3. let c_Cred_{pt} = getpublic(rcv_{Auth_{pt}}) in
4. let (c_{Comt_{pt}} = c_Cred_{pt}) = getpublic(rcv_{PtProof}) in
5. if \( \text{Vfy-zk}_{Auth_{pt}}(rcv_{Auth_{pt}}, c_Cred_{pt}) = \text{true} \) then
6. if \( \text{Vfy-zk}_{PtProof}(rcv_{PtProof}, (c_{Comt_{pt}}, c_Cred_{pt})) = \text{true} \) then
7. \( \nu \text{presc}. \)
8. \( \nu r_{dr}. \)
9. let PrescriptID = hash(presc, c_{Comt_{pt}}, commit(Pnym_{dr}, r_{dr})) in
10. out(ch, spk((Pnym_{dr}, r_{dr}, Id_{dr}),
\quad (commit(Pnym_{dr}, r_{dr}), drcred(Pnym_{dr}, Id_{dr})),
\quad (presc, PrescriptID, commit(Pnym_{dr}, r_{dr}), c_{Comt_{pt}})),
\quad r_{dr})).

Figure 5.8: The doctor process \( P_{dr} \).

proof (d6). If the verification in the previous item succeeds, the doctor generates a prescription presc (d7), generates a nonce \( r_{dr} \) (d8), computes a prescription identity PrescriptID (d9), and computes a commitment Comt_{dr} using the nonce as opening information (d10). Note that a medical examination of the patient is not part of the DLV08 protocol. Next, the doctor signs the message \( (\text{presc}, \text{PrescriptID}, \text{Comt}_{dr}, c_{\text{Comt}_{pt}}) \) using a signed proof of knowledge. This proves the pseudonym used in the credential Cred_{dr} is the same as in the commitment Comt_{dr}, thus linking the prescription to the credential. The doctor sends the signed proof of knowledge together with the open information of the doctor commitment \( r_{dr} \) (d10).

The patient reads in the prescription as \( rcv_{PrescProof} \) and the opening information of the doctor commitment (t6), obtains the prescription c_presp, prescription identity c_PrescriptID, doctor commitment c_Comt_{dr}, and tests the patient commitment signed in the receiving message (t7). Then the patient verifies the signed proof of prescription (t8). If the verification succeeds, the patient obtains the doctor’s pseudonym c_Pnym_{dr} by opening the doctor commitment r_{dr} (d10).

**Modelling the patient-pharmacist sub-protocol.** This sub-protocol is used for a patient, whose steps are labelled ti in Figure 5.10, to obtain medicine from a pharmacist, whose steps are labelled hi in Figure 5.11.

First, the pharmacist authenticates to the patient using a public key authentication (h1). Note that the pharmacist does not authenticate anonymously, and that the pharmacist’s MPA identity is embedded. The patient reads in the pharmacist authentication \( rcv_{Auth_{ph}} \) (t10) and verifies the authentication (t11). If the verification succeeds, the pharmacist obtains the pharmacist’s MPA identity from the authentication (t12), thus obtains the public key of MPA (t13). Then the patient anonymously authenticates himself to the pharmacist, and proves his social security status using the proof PtAuthSss (t14). The patient generates a nonce which will be used as a message in a signed proof of knowledge (t15), and
computes verifiable encryptions \(vc_1, vc_2, vc_3, \ \hat{vc}_3, \ \hat{vc}_4, \ \hat{vc}_5\) (t16-t21). These divulge the patient’s HII, the doctor’s pseudonym, and the patient’s pseudonym to the MPA, the patient’s pseudonym to the HII, and the patient pseudonym and HII to the social safety organisation, respectively. The patient encrypts \(vc_5\) with MPA’s public key as \(c_5\) (t22). The patient computes a signed proof of knowledge

\[
PtSpk = \text{spk}((\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hii}, \text{Sss}, \text{Acc}), \\
(\text{ptcred}((\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hii}, \text{Sss}, \text{Acc}), \\
\text{zk}((\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hii}, \text{Sss}, \text{Acc}), \\
(\text{commit}(\text{Id}_{pt}, r_{pt}), \\
(\text{ptcred}(\text{Id}_{pt}, \text{Pnym}_{pt}, \text{Hii}, \text{Sss}, \text{Acc})))))
\]

which proves that the patient identity embedded in the prescription is the same as in his credential. In the prescription, this identity is contained in a commitment. For simplicity, we model the proof using the commitment instead of the prescription. The link between commitment and prescription is ensured when the proof is verified (h10).

The patient sends the prescription \(rcv_{PrescProof}\), the signed proof \(PtSpk\), and \(vc_1, vc_2, vc_3, \ \hat{vc}_3, \ \hat{vc}_4, \ c_5\) to the pharmacist (t23). The pharmacist reads in the authentication \(rcv_{PtAuthSss}\) (h2), obtains the patient credential and his social security status (h3), verifies the authentication (h4). If the verification succeeds, the pharmacist reads in the patient’s prescription \(rcv_{ph_{PrescProof}}\), the signed proof of knowledge \(rcv_{ph_{PtSpk}}\), the verifiable encryptions \(rcv_{vc_1}, rcv_{vc_2}, rcv_{vc_3}, rcv_{vc_4}, rcv_{vc_5}\,\hat{vc}_3, \ \hat{vc}_4\) and cipher text \(rcv_{c_5}\) (h5); and verifies \(rcv_{ph_{PrescProof}}\) (h6-h8), \(rcv_{ph_{PtSpk}}\) (h9-h10), and \(rcv_{vc_1}, rcv_{vc_2}, rcv_{vc_3}, rcv_{vc_4}\,\hat{vc}_3, \ \hat{vc}_4\) (h11-h20). If all the verifications succeed, the pharmacist charges the patient, and delivers the medicine (neither are modelled as they are out of DLV08’s scope). Then the pharmacist generates an invoice with the prescription identity embedded in it and sends the invoice to the patient (h21).

The patient reads in the invoice (t24), computes a receipt: a signed proof of knowledge \(ReceiptAck\) which proves that he receives the medicine (t25); and sends
the signed proof of knowledge to the patient (t26). The pharmacist reads in the receipt \texttt{rcv.ReceiptAck} (h22) and verifies its correctness (h23).

\textit{Modelling the pharmacist-MPA sub-protocol.} The pharmacist-MPA sub-protocol is used for the pharmacist, whose steps are labelled \texttt{hi} in Figure 5.13 to report the received prescriptions to the MPA, whose steps are labelled \texttt{mi} in Figure 5.12.

As the pharmacist mostly forwards the information supplied by the patient, this protocol greatly resembles the patient-pharmacist protocol described above. Each
\[P_{ph,p1} :=\]
\[
h1. \quad \text{out}(\text{ch}, \text{sign}(\text{Id}_{\text{ph}}, c_{\text{ph}}. \text{Id}_{\text{mpa}}, \text{sk}_{\text{ph}})).
\]
\[
h2. \quad \text{in}(\text{ch}, \text{rcv}_{\text{PtAuthSss}}).
\]
\[
h3. \quad \text{let } (c_{ph}. \text{Cred}_{pt}, c_{ph}. \text{Sss}) = \text{getpublic}(\text{rcv}_{\text{PtAuthSss}}) \text{ in}
\]
\[
h4. \quad \text{if } Vfy-zk_{\text{PtAuthSss}}(\text{rcv}_{\text{PtAuthSss}}, (c_{ph}. \text{Cred}_{pt}, c_{ph}. \text{Sss})) = \text{true then}
\]
\[
h5. \quad \text{in}(\text{ch}, (\text{rcv}_{\text{ph}}. \text{PrescriptProof}, \text{rcv}_{\text{ph}}. \text{PtSpk},
\]
\[
\text{rcv}_{\text{vc}1}, \text{rcv}_{\text{vc}2}, \text{rcv}_{\text{vc}3}, \text{rcv}_{\text{vc}4}, \text{rcv}_{\text{vc}5})).
\]
\[
h6. \quad \text{let } (c_{ph}. \text{Comt}_{dr}, c_{ph}. \text{Comt}_{pt}) = \text{getSpkVinfo}(\text{rcv}_{\text{ph}}. \text{PrescriptProof}) \text{ in}
\]
\[
h7. \quad \text{let } (c_{ph}. \text{presc}, c_{ph}. \text{PrescriptID}) = \text{getmsg}(\text{rcv}_{\text{ph}}. \text{PrescriptProof}) \text{ in}
\]
\[
h8. \quad \text{if } Vfy-spk_{\text{PrescriptID}}(\text{rcv}_{\text{ph}}. \text{PrescriptProof}, (c_{ph}. \text{Cred}_{pt}, c_{ph}. \text{presc},
\]
\[
\text{c}_{\text{ph}}. \text{PrescriptID}, c_{\text{ph}}. \text{Comt}_{dr}, c_{\text{ph}}. \text{Comt}_{pt})) = \text{true then}
\]
\[
h9. \quad \text{let } c_{msg} = \text{getmsg}(\text{rcv}_{\text{ph}}. \text{PtSpk}) \text{ in}
\]
\[
h10. \quad \text{if } Vfy-spk_{\text{PtSpk}}(\text{rcv}_{\text{ph}}. \text{PtSpk},
\]
\[
(c_{ph}. \text{Cred}_{pt}, c_{ph}. \text{Comt}_{pt}, c_{msg})) = \text{true then}
\]
\[
h11. \quad \text{let } (= c_{ph}. \text{Cred}_{pt}, c_{Enc}_{1}) = \text{getpublic}(\text{rcv}_{\text{vc}1}) \text{ in}
\]
\[
h12. \quad \text{if } Vfy-venc_{\text{Hii}}(\text{rcv}_{\text{vc}1}, (c_{ph}. \text{Cred}_{pt}, c_{Enc}_{1}, \text{rcv}_{\text{ph}}. \text{pk}_{\text{mpa}})) = \text{true then}
\]
\[
h13. \quad \text{let } (= \text{rcv}_{\text{ph}}. \text{PrescriptProof}, c_{Enc}_{2}) = \text{getpublic}(\text{rcv}_{\text{vc}2}) \text{ in}
\]
\[
h14. \quad \text{if } Vfy-venc_{\text{DrnymMpa}}(\text{rcv}_{\text{vc}2}, (\text{rcv}_{\text{ph}}. \text{PrescriptProof},
\]
\[
\text{c}_{\text{Enc}_{2}}, \text{rcv}_{\text{ph}}. \text{bk}_{\text{mpa}})) = \text{true then}
\]
\[
h15. \quad \text{let } (= c_{ph}. \text{Cred}_{pt}, c_{Enc}_{3}) = \text{getpublic}(\text{rcv}_{\text{vc}3}) \text{ in}
\]
\[
h16. \quad \text{if } Vfy-venc_{\text{ptnym}}(\text{rcv}_{\text{vc}3}, (c_{ph}. \text{Cred}_{pt}, c_{Enc}_{3}, \text{pk}_{\text{sso}})) = \text{true then}
\]
\[
h17. \quad \text{let } (= c_{ph}. \text{Cred}_{pt}, c_{Enc}_{4}) = \text{getpublic}(\text{rcv}_{\text{vc}4}) \text{ in}
\]
\[
h18. \quad \text{if } Vfy-venc_{\text{Hii}}(\text{rcv}_{\text{vc}4}, (c_{ph}. \text{Cred}_{pt}, c_{Enc}_{4}, \text{pk}_{\text{sso}})) = \text{true then}
\]
\[
h19. \quad \text{let } (= c_{ph}. \text{Cred}_{pt}, c_{Enc}_{5}) = \text{getpublic}(\text{rcv}_{\text{vc}5}) \text{ in}
\]
\[
h20. \quad \text{if } Vfy-venc_{\text{ptnym}}(\text{rcv}_{\text{vc}5},
\]
\[
(c_{ph}. \text{Cred}_{pt}, c_{Enc}_{5}, \text{rcv}_{\text{ph}}. \text{bk}_{\text{mpa}})) = \text{true then}
\]
\[
h21. \quad \text{out}(\text{ch}, \text{inv}(c_{ph}. \text{PrescriptID})).
\]
\[
h22. \quad \text{in}(\text{ch}, \text{rcv}_{\text{ReceiptAck}}).
\]
\[
h23. \quad \text{if } Vfy-spk_{\text{ReceiptAck}}(\text{rcv}_{\text{ReceiptAck}}, (c_{ph}. \text{Cred}_{pt}, c_{ph}. \text{PrescriptID},
\]
\[
\text{Id}_{\text{ph}}, \text{rcv}_{\text{vc}1}, \text{rcv}_{\text{vc}2}, \text{rcv}_{\text{vc}3}, \text{rcv}_{\text{vc}4}, \text{rcv}_{\text{vc}5})) = \text{true then } 0
\]

Figure 5.11: The pharmacist process in the patient-pharmacist sub-protocol \(P_{ph,p1}\).

Step is modelled in details as follows: The pharmacist authenticates himself to his MPA by sending his identity and the signed identities of the pharmacist and the MPA (h24). The MPA stores this authentication in \(rcv_{\text{mpa}}. \text{Auth}_{\text{ph}}\), and stores the pharmacist’s identity in \(c_{\text{mpa}}. \text{Id}_{\text{ph}}\) (m1). From the pharmacist’s identity, the MPA obtains the pharmacist’s public key (m2). Then the MPA verifies the pharmacist’s authentication against the pharmacist’s public key (m3). If the verification succeeds, according to the corresponding rule in the equational theory,
and the MPA verifies that he is indeed the pharmacist’s MPA (m4), the MPA then authenticates itself to the pharmacist by sending the signature of his identity (m5). The pharmacist reads in the MPA’s authentication in \( rcv\_mpa\_Auth\_mpa \) (h25), and verifies the authentication (h26). If the verification succeeds, the pharmacist sends the following to the MPA: prescription \( rcv\_mpa\_PrescProof \), received receipt \( rcv\_ReceiptAck \), and verifiable encryptions \( rcv\_vc_1, rcv\_vc_2, rcv\_vc_3, rcv\_vc_4, rcv\_vc_5 \) (h27). The MPA reads in the information (m6) and ver-
\[ P_{\text{mpa}} {p_2} := \]

- **h24.** out(ch, (\(\text{sign}(\langle \text{Id}_{\text{mpa}}, c_{\text{mpa}}-\text{Id}_{\text{mpa}} \rangle, \text{sk}_{\text{mpa}} \rangle, \text{Id}_{\text{mpa}})\))
- **h25.** in(ch, \(\text{recv}_{\text{Auth}_{\text{mpa}}} \))
- **h26.** if Vfy-sign(\(\text{recv}_{\text{Auth}_{\text{mpa}}}, \text{recv}_{\text{ph}}-\text{pk}_{\text{mpa}}\)) = true then
  - **h27.** out(ch, (\(\text{recv}_{\text{vc}_1}, \text{recv}_{\text{vc}_2}, \text{recv}_{\text{vc}_3}, \text{recv}_{\text{vc}_4}, \text{recv}_{\text{vc}_5}, \text{recv}_{\text{ReceiptAck}}\))

Figure 5.13: The pharmacist process in the pharmacist-MPA sub-protocol \(P_{\text{ph}} {p_2} \).

Modelling the MPA-HII sub-protocol. This protocol covers the exchange of information between the pharmacist’s MPA, whose steps are labelled \(m_i\) in Figure 5.14 and the patient’s HII, whose steps are labelled \(i_i\) in Figure 5.15.

\[ P_{\text{mpa}} {p_2} := \]

- **m25.** out(ch, (\(\text{sign}(\langle \text{Id}_{\text{mpa}}, \text{sk}_{\text{mpa}} \rangle, \text{Id}_{\text{mpa}})\))
- **m26.** in(ch, \(\text{recv}_{\text{mpa}}-\text{Auth}_{\text{hii}} \))
- **m27.** let \(c_{\text{mpa}}-\text{pk}_{\text{hii}} = \text{key}(c_{\text{mpa}}-\text{Hii})\) in
- **m28.** if Vfy-sign(\(\text{recv}_{\text{mpa}}-\text{Auth}_{\text{hii}}, c_{\text{mpa}}-\text{pk}_{\text{hii}}\)) = true then
  - **m29.** if getsignmsg(\(\text{recv}_{\text{mpa}}-\text{Auth}_{\text{hii}}, c_{\text{mpa}}-\text{pk}_{\text{hii}}\)) = \(c_{\text{mpa}}-\text{Hii}\) then
    - **m30.** out(ch, (\(\text{recv}_{\text{mpa}}-\text{ReceiptAck}, \text{dec}(\text{recv}_{\text{mpa}}-c_5, \text{sk}_{\text{mpa}})\))
    - **m31.** in(ch, \(\text{recv}_{\text{mpa}}-\text{Invoice} \))

Figure 5.14: The MPA process in the MPA-HII sub-protocol \(P_{\text{mpa}} {p_2} \).

The MPA sends his identity to the HII and authenticates to the HII using public key authentication \(m25\). The HII stores the MPA’s identity in \(\text{recv}_{\text{hii}}-\text{Id}_{\text{mpa}}\) and stores the authentication in \(\text{recv}_{\text{hii}}-\text{Auth}_{\text{mpa}}\) \(i1\). From the MPA’s identity, the HII obtains the MPA’s public key \(i2\). Then the HII verifies the MPA’s authentication \(i3\). If the verification succeeds, the HII authenticates to the MPA using public key authentication \(i4\). The MPA stores the authentication in \(\text{recv}_{\text{mpa}}-\text{Auth}_{\text{hii}}\) \(m26\). Then the MPA obtains the HII’s public key from the HII’s identity \(m27\) and verifies the HII’s authentication \(m28\). If the verification succeeds, and the MPA verifies that the authentication is from the intended HII \(m29\), the MPA sends the receipt \(\text{recv}_{\text{mpa}}-\text{PrescProof}\) and the patient pseudonym encrypted for the HII – verifiable encryption \(\text{recv}_{\text{mpa}}-\text{vc}_5 = \text{dec}(\text{recv}_{\text{mpa}}-c_5, \text{sk}_{\text{mpa}})\) \(m30\). The HII receives the receipt as \(\text{recv}_{\text{hii}}-\text{ReceiptAck}\) and the encrypted patient pseudonym for the HII as \(c_{\text{hii}}-\text{vc}_5\) \(i5\). The HII verifies the above two pieces of information \(i6-i10\). If the verifications succeed, the HII decrypts the encryption \(c_{\text{hii}}-\text{Enc}_5\) which is embedded in \(c_{\text{hii}}-\text{vc}_5\), and obtains the patient’s pseudonym \(i11\). Finally, the HII sends an invoice of the prescription identity to the MPA \(i12\). The MPA stores the invoice.
\[
P_{\text{hii}} := 
\begin{align*}
\text{i1}. & \quad \text{in}(\text{ch}, (r_{\text{hii}}\text{Auth}_{\text{mpa}}, r_{\text{hii}}\text{Id}_{\text{mpa}})) \\
\text{i2}. & \quad \text{let } c_{\text{hii}} \cdot p_{\text{mpa}} = \text{key}(r_{\text{hii}}\text{Id}_{\text{mpa}}) \text{ in} \\
\text{i3}. & \quad \text{if } V{\text{f}}\cdot\text{sign}(r_{\text{hii}}\text{Auth}_{\text{mpa}}, c_{\text{hii}} \cdot p_{\text{mpa}}) = \text{true} \text{ then} \\
\text{i4}. & \quad \text{out}(\text{ch}, \text{sign}(\text{Id}_{\text{hii}}, \text{sk}_{\text{hii}})) \\
\text{i5}. & \quad \text{in}(\text{ch}, (r_{\text{hii}}\text{ReceiptAck}, c_{\text{hii}} \cdot vc5)) \\
\text{i6}. & \quad \text{let } c_{\text{hii}} \cdot C_{\text{red}} \cdot pt = \text{getSpkVinfo}(r_{\text{hii}}\text{ReceiptAck}) \text{ in} \\
\text{i7}. & \quad \text{let } (c_{\text{hii}} \cdot \text{PrescriptID}, c_{\text{hii}} \cdot \text{Id}_{\text{ph}}, c_{\text{hii}} \cdot vc1, c_{\text{hii}} \cdot vc2, c_{\text{hii}} \cdot vc3, c_{\text{hii}} \cdot vc\prime, c_{\text{hii}} \cdot vc\prime\prime, c_{\text{hii}} \cdot vc\prime\prime\prime) = \text{getmsg}(r_{\text{hii}}\text{ReceiptAck}) \text{ in} \\
\text{i8}. & \quad \text{if } V{\text{f}}\cdot\text{spk}_{\text{ReceiptAck}}(r_{\text{hii}}\text{ReceiptAck}, (c_{\text{hii}} \cdot \text{Cred}_{\text{pt}}, c_{\text{hii}} \cdot \text{PrescriptID}, c_{\text{hii}} \cdot \text{Id}_{\text{ph}}, c_{\text{hii}} \cdot vc1, c_{\text{hii}} \cdot vc2, c_{\text{hii}} \cdot vc3, c_{\text{hii}} \cdot vc\prime, c_{\text{hii}} \cdot vc\prime\prime, c_{\text{hii}} \cdot vc\prime\prime\prime)) = \text{true} \text{ then} \\
\text{i9}. & \quad \text{let } (= c_{\text{hii}} \cdot \text{Cred}_{\text{pt}}, c_{\text{hii}} \cdot \text{Enc}_{\text{pt}}) = \text{getpublic}(c_{\text{hii}} \cdot vc5) \text{ in} \\
\text{i10}. & \quad \text{if } V{\text{f}}\cdot\text{venc}_{\text{Ptnym}}(c_{\text{hii}} \cdot vc5, (c_{\text{hii}} \cdot \text{Cred}_{\text{pt}}, c_{\text{hii}} \cdot \text{Enc}_{\text{pt}}, p_{\text{k}_{\text{hii}}})) = \text{true} \text{ then} \\
\text{i11}. & \quad \text{let } c_{\text{hii}} \cdot \text{Pnym}_{\text{pt}} = \text{dec}(c_{\text{hii}} \cdot \text{Enc}_{\text{pt}}, \text{sk}_{\text{hii}}) \text{ in} \\
\text{i12}. & \quad \text{out}(\text{ch}, \text{invoice}(c_{\text{hii}} \cdot \text{PrescriptID})) 
\end{align*}
\]

Figure 5.15: The HII process \(P_{\text{hii}}\).

In \(r_{\text{mpa}}\text{Invoice}\) (m31). Afterwards, the HII pays the MPA and updates the patient account. As before, handling payment and storing information are beyond the scope of the DLV08 protocol and therefore, we do not model this stage.

Composition. Finally, we compose the two parts of patient processes, the two parts of pharmacist processes and the two parts of the MPA processes. Since the DLV08 protocol works as four sub-protocols executing in order, as shown in Figure 5.16, we simply connect each two parts to compose them. We compose \(P_{\text{pt}}\cdot p_{1}\) and \(P_{\text{pt}}\cdot p_{2}\) to obtain \(P_{\text{pt}}\) (as shown in Figure 5.17), compose \(P_{\text{ph}}\cdot p_{1}\) and \(P_{\text{ph}}\cdot p_{2}\) to obtain \(P_{\text{ph}}\) (as shown in Figure 5.18), and compose \(P_{\text{mpa}}\cdot p_{1}\) and \(P_{\text{mpa}}\cdot p_{2}\) to obtain \(P_{\text{mpa}}\) (as shown in Figure 5.19).

The protocol. The DLV08 protocol is modelled as the five roles \(R_{\text{dr}}, R_{\text{pt}}, R_{\text{ph}}, R_{\text{mpa}},\) and \(R_{\text{hii}}\) running in parallel.

\[
P_{\text{DLV08}} := \nu m. \text{init}.(\!R_{\text{pt}} \!\mid \!R_{\text{dr}} \!\mid \!R_{\text{ph}} \!\mid \!R_{\text{mpa}} \!\mid \!R_{\text{hii}}) \\
\text{init} := \text{let } p_{k_{\text{ssso}}} = p(k_{\text{sk}_{\text{ssso}}}) \text{ in } \text{out}(\text{ch}, p_{k_{\text{ssso}}})
\]

where \(\nu m\) represents global secrets \(\text{sk}_{\text{ssso}}\) and private channels \(\text{ch}_{\text{hp}}, \text{ch}_{\text{mp}}, \text{ch}_{\text{phpt}},\) process \(\text{init}\) initialises the settings of the protocol. Each role is of the form \(R_i := \text{init}_i \cdot P_i\).

In summary, the DLV08 protocol is composed as shown in Figure 5.20. In the protocol model, the roles \(R_{\text{dr}}, R_{\text{hii}}, R_{\text{pt}}, R_{\text{ph}}, R_{\text{mpa}}\) are shown as in Figure 5.21, Figure 5.22, Figure 5.17, Figure 5.18 and Figure 5.19 respectively.
Figure 5.16: Overview of the DLV08 protocol.

\[ R_{pt} := \nu Id_{pt}. \]
\[ \nu Pnym_{pt} \cdot \nu Sss \cdot \nu Acc. \]
\[ \text{in}(ch_{hp}, Hii). \text{let} \ c_{pt-pk_{hii}} = \text{key}(Hii) \text{ in} \]
\[ \text{!}(\text{in}(ch_{phpt}, rcv_{pk_{ph}}). \]
\[ \text{let} \ rtv_{pt-pk_{ph}} = \text{rtv}_{pk_{ph}} \text{ in let} \ Id_{ph} = \text{host}(\text{rtv}_{pk_{ph}}) \text{ in} \]
\[ (\ldots) \]
\[ \text{let} \ c.Pnym_{dr} = \text{open}(c, Comt_{dr}, rcv_{r_{dr}}) \text{ in} \]
\[ \text{in}(ch, rcv_{Auth_{ph}}). \]
\[ \ldots \]

Figure 5.17: The process for role patient \( R_{pt} \).
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Figure 5.18: The process for role pharmacist $R_{ph}$.

Figure 5.19: The process for role MPA $R_{mpa}$.

Figure 5.20: The process for the DLV08 protocol.

Figure 5.21: The process for role doctor $R_{dr}$.

5.6 Analysis of DLV08

In this section, We analyse the claimed properties, including secrecy of patient and doctor information, authentication, (strong) patient and doctor anonymity,
5.6 Analysis of DLV08

\[
R_{\text{hii}} := \nu \text{sk}_{\text{hii}}.
\]

\[
\begin{align*}
&\text{let } \text{pk}_{\text{hii}} = \text{pk}(\text{sk}_{\text{hii}}) \text{ in} \\
&\text{let } \text{Id}_{\text{hii}} = \text{host}(\text{pk}_{\text{hii}}) \text{ in} \\
&(\text{out}(\text{ch}, \text{pk}_{\text{hii}}) \mid \text{out}(\text{ch}_{\text{hp}}, \text{Id}_{\text{hii}}) \mid !(P_{\text{hii}}))
\end{align*}
\]

Figure 5.22: The process for role HII \(R_{\text{hii}}\).

(strong) patient and doctor untraceability, (enforced) prescribing-privacy and independence of (enforced) prescribing-privacy of the DLV08 protocol. The two properties, doctor anonymity and untraceability, are not required by the protocol but are still interesting to analyse.

5.6.1 Secrecy of patient and doctor information

The DLV08 protocol is claimed to satisfy the following requirement: any party involved in the prescription processing workflow should not know the information of a patient and a doctor unless the information is intended to be revealed in the protocol. In [DLVV08], this requirement is considered as an access control requirement. We argue that ensuring the requirement with access control is not enough when a party is dishonest. Since it is not clearly described whether the involved parties are honest, assuming a party is dishonest by observing the network and manipulating the protocol, we found that the party may obtain information which he should not access.

We analyse secrecy of patient and doctor information, respecting to a dishonest party who has the Dolev-Yao adversary ability. Private information of patients and doctors, which needs to be protected, is as follows: patient identity (\(\text{Id}_{\text{pt}}\)), doctor identity (\(\text{Id}_{\text{dr}}\)), patient pseudonym (\(\text{Pnym}_{\text{pt}}\)), doctor pseudonym (\(\text{Pnym}_{\text{dr}}\)), a patient’s social security status (\(\text{Sss}\)), a patient’s health insurance institute (\(\text{Hii}\)). Although it is not required, the health expense account \(\text{Acc}\) of a patient has also been verified.

**Verification result.** We query the standard secrecy of the set of private information using ProVerif [Bla01]. The verification result (see Table 5.1) shows that a patient’s identity, pseudonym, health expense account, health insurance institute and identity of a doctor (\(\text{Id}_{\text{pt}}, \text{Pnym}_{\text{pt}}, \text{Hii}, \text{Acc}, \text{Id}_{\text{dr}}\)) satisfy standard secrecy; a patient’s social security status \(\text{Sss}\) and a doctor’s pseudonym \(\text{Pnym}_{\text{dr}}\) do not satisfy standard secrecy. The \(\text{Sss}\) is revealed by the proof of social security status from the patient to the pharmacist. The \(\text{Pnym}_{\text{dr}}\) is revealed by the revealing of both the commitment of the patient’s pseudonym and the open key to the commitment during the communication between the patient and the doctor.

To fix secrecy of a patient’s social security status, it requires that the proof of social security status only reveals the status to the pharmacist. Since how a social security status is represented and what the pharmacist needs to verify, are not clear, we cannot give explicit suggestions. However, if the social security status is a number, and the pharmacist only needs to verify that the number is higher than a certain threshold, the patient can prove it using zero-knowledge proof without
checked Security property | initial model | cause(s) | improvement
--- | --- | --- | ---
Secrecy of $\text{Id}_{pt}$ | ✓ | | 
Secrecy of $\text{Pnym}_{pt}$ | ✓ | | 
Secrecy of $\text{Sss}_{pt}$ | × | revealed session key | 
Secrecy of $\text{Hii}_{pt}$ | ✓ | | 
Secrecy of $\text{Acc}_{pt}$ | ✓ | | 
Secrecy of $\text{Id}_{dr}$ | ✓ | | 
Secrecy of $\text{Pnym}_{dr}$ | × | revealed session key | 

Table 5.1: Verification results of secrecy for patients and doctors.

revealing the number; if the pharmacist needs to verify the exact value of the status, one way to fix its secrecy is that the pharmacist and the patient agree on a session key and the status is encrypted using the key. Similarly, a way to fix the secrecy of $\text{Pnym}_{dr}$ is to encrypt the open information using the agreed session key.

### 5.6.2 Patient and doctor authentication

The protocol claims that all parties should be able to properly authenticate each other. Compared to authentications between public entities, pharmacists, MPA and HII, we focus on authentications between patients and doctors, as patients and doctors use anonymous authentication. Authentications between patients and pharmacists are sketched as well.

The authentication from a patient to a doctor is defined as when the doctor finishes his process and believes that he prescribed medicine for a patient, then the patient did ask the doctor for prescription. Similarly, the authentication from a doctor to a patient is defined as when the patient believes that he visited a doctor, the doctor did prescribe medicine for the patient.

Authentications are modelled as correspondence properties. To verify the authentication of a patient, we add an event $\text{EndDr}(c_{\text{Cred}_{pt}}, c_{\text{Comt}_{pt}})$ at the end of the doctor process, meaning the doctor believes that he prescribed medicine for a patient who has a credential $c_{\text{Cred}_{pt}}$ and committed $c_{\text{Comt}_{pt}}$; and add an event $\text{StartPt}(ptcred(Id_{pt}, \text{Pnym}_{pt}, \text{Hii}, \text{Sss}, \text{Acc}), commit(Id_{pt}, r_{pt}))$ in the patient process, meaning that the patient did ask for a prescription. Then we query $\text{ev(inj)}: \text{EndDr}(x, y) => \text{ev(inj)}: \text{StartPt}(x, y)$, meaning that when the event $\text{EndDr}$ is executed, there is an (unique) event $\text{StartPt}$ has been executed before. Similarly, to authenticate a doctor, we add to the patient process an event $\text{EndPt}(c_{\text{Cred}_{dr}}, c_{\text{Comt}_{dr}}, c_{\text{Presc}}, c_{\text{PrescriptID}})$ at the end, and add an event $\text{StartDr}(drcred(Pnym_{dr}, Id_{dr}), commit(Pnym_{dr}, r_{dr}, presc, PrescriptID))$ in the doctor process, then query $\text{ev(inj)}: \text{EndPt}(x, y, z, t) => \text{ev(inj)}: \text{StartDr}(x, y, z, t)$.

**Verification result.** The queries are verified using the tool ProVerif [Bla02]. The verification result shows that the doctor authentication, both injective and non-injective, succeed; the non-injective patient authentication succeeds and injective patient authentication fails. The failure is caused by the a replay attack from the adversary. That is, the adversary can impersonate a patient by replaying old messages from the patient. This authentication flaw leads to termination of the
successive procedure, the patient-pharmacist sub-process. We verified the authentications between patients and pharmacists as well. Non-injective patient authentication succeeds, and injective patient authentication fails. This means that the messages received by a pharmacist are from a patient, but not necessarily from the patient who the pharmacist is communicating with. Neither non-injective nor injective pharmacist authentication succeeds. This means that the adversary can record messages from a pharmacist and pretend to be that pharmacist. The verification results are summarised in Table 5.2.

Table 5.2: Verification results of authentication of patients and doctors.

The reason that injective patient authentications to doctors and pharmacists fail is that they suffer from replay attack. To fix them, one way is to add a challenge step from the doctor or pharmacist to the patient, and when the patient authenticates to the doctor or pharmacist, the patient includes the challenge in the proofs. This assures that the proof is freshly generated. Thus it prevents the adversary replaying old messages. The reason that the (injective and non-injective) authentication from a pharmacist to a patient fails is that the adversary can generate the invoice to replace the one from the real pharmacist. To fix the flaw, one way is for the pharmacist to sign the invoice.

5.6.3 (Strong) patient and doctor anonymity

The DLV08 protocol claims that no party should be able to determine the identity of a patient. We define (strong) patient anonymity to capture the requirement. Note that in the original paper of the DLV08 protocol, the terminology of the privacy notion for capturing this requirement is patient untraceability. Our definition of untraceability (Definition 5.7) has different meaning from theirs (for details, see Section 5.6.4). Also note that the satisfaction of standard secrecy of patient identity does not fully capture this requirement, as the adversary can still guess about it.

Patient and doctor anonymity. Doctor anonymity is defined as in Definition 5.5
Patient anonymity can be defined in a similar way by replacing the role of doctor with the role of patient.

\[ C_{ch}[\text{\init}_\text{pt}\{t_\text{A}/\text{Id}_\text{pt}\}!P_{\text{pt}}\{t_\text{A}/\text{Id}_\text{pt}\}] \approx \epsilon \ C_{ch}[\text{\init}_\text{pt}\{t_\text{B}/\text{Id}_\text{pt}\}!P_{\text{pt}}\{t_\text{B}/\text{Id}_\text{pt}\}] \]
To verify doctor/patient anonymity, is to check the satisfiability of the corresponding equivalence between processes in the definition. This is done by modelling the two processes on two sides of the equivalence as bi-processes, and verify the bi-process using ProVerif. Recall that a bi-process models two processes sharing the same structure and differing only in terms or destructors. The two processes are written as one process with choice-constructors which tells ProVerif the spots where the two processes differ. For example, $\text{choice}[x, y]$ means that the first process uses $x$ to replace $\text{choice}[x, y]$ while the second process uses $y$.

The bi-process for verifying doctor anonymity is

$$
\nu m. \text{init}(!R_{pt} | !R_{dr} | !R_{ph} | !R_{mpa} | !R_{hii} | \nu \text{Pnym}_{dr}. \text{let Id}_{dr} = \text{choice}[d_{A}, d_{B}] \text{ in } !P_{dr}),
$$

and the bi-process for verifying patient anonymity is

$$
\nu m. \text{init}(!R_{pt} | !R_{dr} | !R_{ph} | !R_{mpa} | !R_{hii} | \text{let Id}_{pt} = \text{choice}[t_{A}, t_{B}] \text{ in } 
\nu \text{Pnym}_{pt}. \nu \text{Sss}. \nu \text{Acc}. \text{in}(\text{ch}_{hp}, \text{Hii}). \text{let } c_{pt}. p_{hii} = \text{key}(\text{Hii}) \text{ in } !P_{pt}).
$$

**Strong patient and doctor anonymity.** Strong doctor anonymity is defined as in Definition 5.6. By replacing the role of doctor with the role of patient, we obtain the definition of strong patient anonymity.

The bi-process for verifying strong doctor anonymity is

$$
\text{free } d_{A}; 
\nu m. \text{init}(!R_{pt} | !R_{dr} | !R_{ph} | !R_{mpa} | !R_{hii} | 
\nu d_{B}. \nu \text{Pnym}_{dr}. \text{let Pnym}_{dr} = \text{nPnym}_{dr} \text{ in } !\text{(let Id}_{dr} = \text{choice}[d_{B}, d_{A}] \text{ in } P_{dr}))),
$$

and the bi-process for verifying strong patient anonymity is

$$
\text{free } t_{A}; 
\nu m. \text{init}(!R_{pt} | !R_{dr} | !R_{ph} | !R_{mpa} | !R_{hii} | \nu t_{B}. \nu \text{Pnym}_{pt}. \nu \text{Sss}. \nu \text{Acc}. 
\text{in}(\text{ch}_{hp}, \text{Hii}). \text{let } c_{pt}. p_{hii} = \text{key}(\text{Hii}) \text{ in } !\text{(let Id}_{pt} = \text{choice}[t_{B}, t_{A}] \text{ in } P_{pt}))).
$$

**Verification result.** The bi-processes are verified using ProVerif. The verification results show that patient anonymity and strong patient anonymity are satisfied and neither doctor anonymity nor strong doctor anonymity is not satisfied. For strong doctor anonymity, the adversary can distinguish a process initiated by an unknown doctor and a known doctor. Given a doctor process, where the doctor has identity $d_{A}$, pseudonym $\text{Pnym}_{dr}$, and credential $\text{drcred}(\text{Pnym}_{dr}, d_{A}), \text{Pnym}_{dr}$, and $\text{drcred}(\text{Pnym}_{dr}, d_{A})$ are revealed. We assume that the adversary knows another doctor identity $d_{B}$. The adversary can fake an anonymous authentication by faking the zero-knowledge proof as $ZK((\text{Pnym}_{dr}, d_{B}), \text{drcred}(\text{Pnym}_{dr}, d_{A}))$. If the zero-knowledge proof passes the corresponding verification $\text{Vfy-ZK}_{\text{Auth}_{dr}}$ by the patient, then the adversary knows the doctor process is executed by the doctor $d_{B}$. Otherwise, not. For the same reason, doctor anonymity fails the verification. Both flaws can be fixed by requiring a doctor to generate a new credential in each session ($s4'$).

5.6.4 (Strong) patient and doctor untraceability

Even if a user’s identity is not revealed, the adversary may be able to trace a user by telling whether two executions are done by the same user. The DLV08 protocol
claims that prescriptions issued to the same patient should not be linkable to each other. In other words, the situation in which a patient executes the protocol twice should be indistinguishable from the situation in which two different patients execute the protocol individually. To satisfy this requirement, patient untraceability is required. Note that patient untraceability is claimed as patient unlinkability by the authors of the DL08 protocol, we use different terminology.

**Patient and doctor untraceability.** Doctor untraceability has been defined in Definition 18, and patient untraceability can be defined in a similar style.

The bi-process for verifying doctor untraceability is

\[
\nu\mu\text{init.}(!R_{pt} \mid !R_{dr} \mid !R_{ph} \mid !R_{mpa} \mid !R_{hi} \mid (\nu nPnym_{dr}.\nu wpnym_{dr}).
\]

\[
((\text{let } Id_{dr} = d_A \text{ in let } Pnym_{dr} = nPnym_{dr} \text{ in } P_{dr}) | \n
(\text{let } Id_{dr} = \text{choice\{d_A, d_B\} in let } Pnym_{dr} = \text{choice\{nPnym_{dr}, wpnym_{dr}\} in } P_{dr}))
\]

and the bi-process for verifying patient untraceability is

\[
\nu\mu\text{init.}(!R_{pt} \mid !R_{dr} \mid !R_{ph} \mid !R_{mpa} \mid !R_{hi} \mid \n
(\nu Pnym_{pt}.\nu Sss.\nu Acc.\nu wpnym_{pt}.\nu Sss.\nu Acc).
\]

\[
\text{in}(ch_{hp}, nHi). \text{in}(ch_{hp}, wHi).
\]

\[
\text{let } c_{pt}.npk_{hi} = \text{key}(nHi) \text{ in let } c_{pt}.wpk_{hi} = \text{key}(wHi) \text{ in }
\]

\[
(\text{let } Hi = nHi \text{ in let } c_{pt}.pk_{hi} = c_{pt}.npk_{hi} \text{ in let } Id_{pt} = t_A \text{ in }
\]

\[
\text{let } Pnym_{pt} = nPnym_{pt} \text{ in let } Sss = nSss \text{ in let } Acc = nAcc \text{ in } P_{pt}) | \n
(\text{let } Hi = \text{choice\{nHi, wHi\} in let } c_{pt}.pk_{hi} = \text{choice\{c_{pt}.pk_{hi}, c_{pt}.wpk_{hi}\} in }
\]

\[
(\text{let } Id_{pt} = \text{choice\{t_A, t_B\} in let } Pnym_{pt} = \text{choice\{nPnym_{pt}, wpnym_{pt}\} in }
\]

\[
(\text{let } Sss = \text{choice\{nSss, wSss\} in let } Acc = \text{choice\{nAcc, wAcc\} in } P_{pt}))
\]

**Strong patient and doctor untraceability.** Strong untraceability is modelled as a patient executing the protocol repeatedly is indistinguishable from different patients executing the protocol each once. Strong doctor untraceability is defined as in Definition 21, and strong patient untraceability can be defined in the same manner.

The bi-process for verifying strong doctor untraceability is

\[
\nu\mu\text{init.}(!R_{pt} \mid !R_{ph} \mid !R_{mpa} \mid !R_{hi} \mid (\nu nId_{dr}.\nu Pnym_{dr} !(\nu wId_{dr}.\nu wpnym_{dr}).
\]

\[
(\text{let } Id_{dr} = \text{choice\{nId_{dr}, wId_{dr}\} in let } Pnym_{dr} = \text{choice\{nPnym_{dr}, wpnym_{dr}\} in } P_{dr}))
\]

and the bi-process for verifying strong patient untraceability is

\[
\nu\mu\text{init.}(!R_{dr} \mid !R_{ph} \mid !R_{mpa} \mid !R_{hi} \mid (\nu wpnym_{pt}.\nu Pnym_{pt}.\nu Sss.\nu Acc.\text{in}(ch_{hp}, nHi).
\]

\[
(\nu wId_{pt}.\nu wpnym_{pt}.\nu Sss.\nu Acc).
\]

\[
(\text{let } Id_{pt} = \text{choice\{nId_{pt}, wId_{pt}\} in let } Pnym_{pt} = \text{choice\{nPnym_{pt}, wpnym_{pt}\} in }
\]

\[
(\text{let } Sss = \text{choice\{nSss, wSss\} in let } Acc = \text{choice\{nAcc, wSss\} in }
\]

\[
(\text{in}(ch_{hp}, wHi). \text{let } Hi = \text{choice\{nHi, wHi\} in let } c_{pt}.pk_{hi} = \text{key}(Hi) \text{ in } P_{pt}))
\]

**Verification result.** The bi-processes are verified using ProVerif. The verification results show that the DLV08 protocol does not satisfy patient/doctor untraceability, nor strong untraceability. The strong doctor untraceability fail because the
adversary can distinguish sessions initiated by one doctor and by different doctors. The doctor’s pseudonym is revealed and a doctor uses the same pseudonym in all sessions. Sessions with the same doctor pseudonyms are initiated by the same doctor. For the same reason, doctor untraceability also fails. Both of them can be fixed by requiring a doctor to freshly generate his pseudonym in each session (s3’).

For strong patient untraceability, the adversary can distinguish sessions initiated by one patient (with identical social security statuses) and initiated by different patients (with different social security statuses). Second, the adversary can distinguish sessions initiated by one patient (with identical cipher texts $\text{enc}(\text{Pnym}_pt, \text{pk}_sso)$ and identical cipher texts $\text{enc}(\text{Hii}, \text{pk}_sso)$) and initiated by different patients (with different cipher texts $\text{enc}(\text{Pnym}_pt, \text{pk}_sso)$ and different cipher texts $\text{enc}(\text{Hii}, \text{pk}_sso)$).

Third, since the patient credential is the same in all sessions and is revealed, the adversary can also trace a patient by the patient’s credential. Fourth, the adversary can distinguish sessions using the same HII and sessions using different HIIs. For the same reasons, patient untraceability fails. Both flaws can be fixed by revising the assumptions (s5’, s2’, s4” and s6’).

5.6.5 Prescribing-privacy

Prescribing-privacy has been defined in Definition 5.1. To verify the prescribing-privacy, it is to check the satisfaction of the equivalence in the definition. The bid-process for verifying the equivalence is

$$\text{free } d_A, \text{free } d_B, \text{free } p_A, \text{free } p_B. \nu m.\text{init.}(|!R_{pt} | !R_{dr} | !R_{ph} | (\nu n\text{Pnym}_dr, \nu w\text{Pnym}_dr. \begin{align*}
& \text{let } \text{Id}_dr = \text{choice}[d_A, d_B] \text{ in} \\
& \text{let } \text{Pnym}_dr = \text{choice}[n\text{Pnym}_dr, w\text{Pnym}_dr, ] \text{ in} \\
& \text{let } \text{presc} = p_A \text{ in } \text{main}_dr) | \\
& (\nu n\text{Pnym}_dr, \nu w\text{Pnym}_dr. \begin{align*}
& \text{let } \text{Id}_dr = \text{choice}[d_B, d_A] \text{ in} \\
& \text{let } \text{Pnym}_dr = \text{choice}[n\text{Pnym}_dr, w\text{Pnym}_dr] \text{ in} \\
& \text{let } \text{presc} = p_B \text{ in } \text{main}_dr) \}. \end{align*}
$$

Verification result. The verification, using ProVerif, shows that the DLV08 protocol does not satisfy prescribing-privacy, i.e., the adversary can distinguish whether a prescription is prescribed by doctor $d_A$ or doctor $d_B$. In the prescription proof, a prescription is linked to a doctor credential. And a doctor credential is linked to a doctor identity. Thus, the adversary can link a doctor to his prescription. To break the link, one way is to make sure that the adversary cannot link a doctor credential to a doctor identity. This can be achieved by adding randomness to the credential (s4’).

5.6.6 Enforced prescribing-privacy

The definition of enforced prescribing-privacy is modelled as the existence of a process $P'_dr$, such that the two equivalences in Definition 5.2 are satisfied. Due to the existential quantification, we cannot verify the property directly using ProVerif.
Examining the DLV08 protocol, we find an attack on enforced prescribing-privacy, even after fixing prescribing privacy (with assumption \(s4'\)). A bribed doctor is able to prove to the adversary of his prescription as follows:

1. A doctor communicates with the adversary to agree on a bit-commitment that he will use, which links the doctor to the commitment.

2. The doctor uses the agreed bit-commitment in the communication with his patient. This links the bit-commitment to a prescription.

3. Later, when the patient uses this prescription to get medicine from a pharmacist, the adversary can observe the prescription being used. This proves that the doctor has really prescribed the medicine.

Formally, using ProVerif, we can show that if a doctor reveals all his information to the adversary, the doctor’s prescribing-privacy is broken. To prove that there exist no alternative processes for a doctor to cheat the adversary, we assume that there exists a process \(P'_{dr}\) which satisfies the definition of enforced prescribing-privacy, and then derive some contradiction. A bribed doctor reveals the nonces used in the commitment and the credential to the adversary. Thus, the adversary links a bribed doctor to his commitment and credential. In the prescription proof, a prescription is linked to a doctor’s commitment and credential. Suppose there exists a process \(P'_{dr}\) in which the doctor lies to the adversary that he prescribed \(p_A\), while the adversary observes that the commitment or the credential is linked to \(p_B\). The adversary can detect that the doctor has lied.

**Proof.** Assume there exists a process \(P'_{dr}\) which satisfies the definition of enforced prescribing-privacy. i.e., \(\exists P'_{dr}\) such that

\[
\begin{align*}
&1. \quad C_{ch} \left( \text{init}'_{dr} \{ d_A/IId_{dr} \} . \left( P_{dr} \{ d_A/IId_{dr} \} \ | P'_{dr} \{ d_A/IId_{dr} \} \right) \right) | \\
&\quad \left( \text{init}_{dr} \{ d_B/IId_{dr} \} . \left( P_{dr} \{ d_B/IId_{dr} \} \ | main_{dr} \{ d_B/IId_{dr}, p_A/presc \} \right) \right) \\
&\quad \approx \_C_{ch} \left( \text{init}_{dr} \{ d_A/IId_{dr} \} . \left( P_{dr} \{ d_A/IId_{dr} \} \ | main_{dr} \{ d_A/IId_{dr}, p_A/presc \} \right) \right) | \\
&\quad \left( \text{init}_{dr} \{ d_B/IId_{dr} \} . \left( P_{dr} \{ d_B/IId_{dr} \} \ | main_{dr} \{ d_B/IId_{dr}, p_B/presc \} \right) \right)
\end{align*}
\]

\[
\begin{align*}
&2. \quad init'_{dr} \{ d_A/IId_{dr} \} \!\! \Rightarrow \!\! \left( P'_{dr} \{ d_A/IId_{dr} \} \!\! \Rightarrow \!\! \left( main_{dr} \{ d_A/IId_{dr}, p_A/presc \} \right) \!\! \Rightarrow \!\! \left( \text{out(chc,.)} \right) \!\! \Rightarrow \!\! \left( P_{dr} \{ d_A/IId_{dr} \} \!\! \Rightarrow \!\! \left( main_{dr} \{ d_A/IId_{dr}, p_A/presc \} \right) \!\! \Rightarrow \!\! \left( \text{out(chc,.)} \right) \right) \right).
\end{align*}
\]

According to the definition of labelled bisimilarity and static equivalence, if two processes are labelled bisimilar, we have that if \(M =_E N\) at any state of one process, then there exist some states of the other process, such that \(M =_E N\). Due to the first equivalence, we have that if \(M =_E N\) on the right hand side of the first equivalence, then \(M =_E N\) on the left hand side.

On the right hand side of the first equivalence, there exists an output of a prescription proof \(PrescProof^\tau\), over public channels, from a process initiated by doctor \(d_A\) (process \(main_{dr} \{ d_A/IId_{dr}, p_A/presc \}^{chc}\)). The adversary can obtain the prescription \(p_A\) from \(PrescProof^\tau\).

\[
(p_A, PrescriptID^r, Comt_{dr}^r, c_{Comt_{pt}^r}) = \text{getmsg}(PrescProof^\tau)
\]
On the left hand side of the first equivalence, there should also exist an output of a prescription proof $PrescProof^l$ over public channels, from which the adversary can obtain a prescription $p_A$.

$$(p_A, PrescriptID^l, Comt_{dr}^l, c_{Comt_{pt}^l}) = \text{getmsg}(PrescProof^l)$$

Next, we prove that prescription proof $PrescProof^l$ is the prescription proof in process $init'_{dr}\{d_A/Id_{dr}\}, P'_{dr}\{d_A/Id_{dr}\}$. On the right hand side of the first equivalence, the term $PrescProof^r$ is computed as

$$PrescProof^r = \text{spk}((Pnym^r_{dr}, r'^r_{dr}, d_A, n'^r_{dr}),$$

$$\text{(commit}(Pnym^r_{dr}, r'^r_{dr}), dcred(Pnym^r_{dr}, d_A, n'^r_{dr})),$$

$$(p_A, PrescriptID^r, \text{commit}(Pnym^r_{dr}, r'^r_{dr}), c_{Comt_{pt}^l})$$

where the doctor pseudonym $Pnym^r_{dr}$ and the nonce for doctor commitment $r'^r_{dr}$ and the nonce for doctor credential $n'^r_{dr}$ (used for assumption $s4^*$) are revealed to the adversary on $chc$ channel. On the left hand side of the first equivalence, to be equivalent to the right side, there should also exist sub-processes which output messages on $chc$ channel. Such sub-processes can only be $init'_{dr}\{d_A/Id_{dr}\}$ and $P'_{dr}\{d_A/Id_{dr}\}$, because there is no output on $chc$ in other sub-processes in the left hand side process. Suppose $Pnym^1_{dr}, r^1_{dr}$ and $n^1_{dr}$ are the doctor pseudonym and the nonces sent to the adversary over $chc$ on the left hand side (corresponding to $Pnym^r_{dr}, r'^r_{dr}$ and $n'^r_{dr}$ on the right hand side). Since the process $init'_{dr}\{d_A/Id_{dr}\}, P'_{dr}\{d_A/Id_{dr}\}$ knows $Pnym^1_{dr}, r^1_{dr}, n^1_{dr}$ and $d_A$, the process is able to compute and thus output a prescription proof $PrescProof^l$, where the prescription is $p_A$.

$$PrescProof^l = \text{spk}((Pnym^1_{dr}, r^1_{dr}, d_A, n^1_{dr}),$$

$$\text{(commit}(Pnym^1_{dr}, r^1_{dr}), dcred(Pnym^1_{dr}, d_A, n^1_{dr})),$$

$$(p_A, PrescriptID^l, \text{commit}(Pnym^1_{dr}, r^1_{dr}), c_{Comt_{pt}^l})$$

On the other hand, sub-processes on the left hand side of the first equivalence, except processes $init'_{dr}\{d_A/Id_{dr}\}$ and $P'_{dr}\{d_A/Id_{dr}\}$, cannot generate such a prescription as $PrescProof^l$ with $Pnym^1_{dr}, r^1_{dr}$ and $n^1_{dr}$ shared with the adversary.

1) The sub-processes except doctor sub-processes honestly follow the protocol and thus do not generate a signed proof knowledge which matches the pattern of $PrescProof^l$.

2) The prescription proof $PrescProof^l$ cannot be generated in doctor sub-processes except the sub-processes $init'_{dr}\{d_A/Id_{dr}\}$ and $P'_{dr}\{d_A/Id_{dr}\}$.

- First, the doctor sub-processes do not generate $PrescProof^l$ using outside information. The doctor sub-processes honestly follow the protocol and thus do not purposely read in the secret values ($Pnym^1_{dr}, r^1_{dr}$ and $n^1_{dr}$) from the $chc$ channel to generate $PrescProof^l$. Even the adversary feeds the sub-processes with $Pnym^1_{dr}, r^1_{dr}$ and $n^1_{dr}$ over public channels, the sub-processes cannot generate $PrescProof^l$. Because the honest doctor sub-processes do not use read in values to generate a prescription proof.

- Second, the honest doctor sub-processes can not generate $PrescProof^l$ using their initial knowledge.
Since in honest doctor sub-processes, \( P_{nym, dr} \), \( r_{dr} \), and \( n_{dr} \) are freshly generated in each session of a doctor sub-process, the honest doctor sub-processes do not generate \( \text{PrescProof}^l \) using globally shared information.

* The honest doctor sub-processes can not generate \( \text{PrescProof}^l \) using self-generated information. Suppose an honest doctor sub-process generates \( \text{PrescProof}^l \) using self-generated information. In order to do so, the honest doctor sub-process needs to generates \( P_{nym}^1, r_{dr}^l \), and \( n_{dr}^l \). As we stated earlier, \( \text{init}'_{dr} \{ d_A / Id_{dr} \} \) or \( P_{dr} \{ d_A / Id_{dr} \} \) need to know \( P_{nym}^1, r_{dr}^l \), and \( n_{dr}^l \), so that the three values can be sent to the adversary over \( \text{chc} \). However, there is no way for process \( \text{init}'_{dr} \{ d_A / Id_{dr} \} \) or \( P_{dr} \{ d_A / Id_{dr} \} \) to know \( n_{dr}^l \) from the honest doctor sub-process, because \( n_{dr} \) is never revealed by an honest doctor sub-process. Therefore, the honest doctor processes do not generate \( n_{dr}^l \) and thus cannot generate \( \text{PrescProof}^l \) using self-generated information.

Therefore, the process which outputs the prescription proof \( \text{PrescProof}^l \) on the left hand side of the first equivalence is \( \text{init}'_{dr} \{ d_A / Id_{dr} \} \) and/or \( P_{dr} \{ d_A / Id_{dr} \} \). Thus, the prescription in process \( \text{init}'_{dr} \{ d_A / Id_{dr} \} \). \( P_{dr} \{ d_A / Id_{dr} \} \) is \( p_A \).

However, in the right hand side process of the second equivalence, there is a prescription proof output over public channels from process \( \text{main}_{dr} \{ d_A / Id_{dr}, P_{ph}/\text{presc} \} \). The adversary obtains prescription \( p_B \) from the prescription proof. To satisfy the second equivalence, the left hand side process of the second equivalence (the process \( \text{init}'_{dr} \{ d_A / Id_{dr} \} ^{\text{out(\text{chc})}} \cdot (P_{dr} \{ d_A / Id_{dr} \} ^{\text{out(\text{chc})}}) \)) should also output a prescription proof (denoted as \( \text{PrescProof}^l \)) where the prescription is \( p_B \). Since the process \( \text{init}'_{dr} \{ d_A / Id_{dr} \} ^{\text{out(\text{chc})}} \cdot (P_{dr} \{ d_A / Id_{dr} \} ^{\text{out(\text{chc})}}) \) outputs \( \text{PrescProof}^l \), its corresponding process, \( \text{init}'_{dr} \{ d_A / Id_{dr} \} . P_{dr} \{ d_A / Id_{dr} \} \), in which communication over \( \text{chc} \) is not hidden, should also output \( \text{PrescProof}^l \). Therefore, the prescription in \( \text{init}'_{dr} \{ d_A / Id_{dr} \} . P_{dr} \{ d_A / Id_{dr} \} \) is \( p_B \). Since \( p_A \neq p_B \), contradiction is found.

\[ \Box \]

### 5.6.7 Independence of (enforced) prescribing-privacy

The doctor’s prescribing-privacy independent of the pharmacist is modelled by replacing \( R_i \) with \( R_{ph} \) in Definition 5.3. The bi-process for verifying the property is

\[
\begin{align*}
\text{free } d_A, \text{free } d_B, \text{free } p_A, \text{free } p_B, \\
\nu \mu. \text{init}.(\left\{ R_{ph} \right\} \left\{ R_{dr} \right\}^{\text{chc}} | \left( \nu n P_{nym, dr}. \nu w P_{nym, dr}. \\
\text{let } Id_{dr} = \text{choice}[d_A, d_B] \text{ in} \\
\text{let } P_{nym, dr} = \text{choice}[n P_{nym, dr}, w P_{nym, dr}] \text{ in} \\
\text{let } \text{presc} = p_A \text{ in } \text{main}_{dr} \} | \\
\left( \nu n P_{nym, dr}. \nu w P_{nym, dr}. \\
\text{let } Id_{dr} = \text{choice}[d_B, d_A] \text{ in} \\
\text{let } P_{nym, dr} = \text{choice}[n P_{nym, dr}, w P_{nym, dr}] \text{ in} \\
\text{let } \text{presc} = p_B \text{ in } \text{main}_{dr} \} ) \right) 
\end{align*}
\]

The verification, using ProVerif, shows that the protocol (after fixing the flaw on prescribing-privacy with assumption s4) satisfies this property.
### Table 5.3: Verification results of privacy properties and revised assumptions.

<table>
<thead>
<tr>
<th>checked privacy notion</th>
<th>initial model</th>
<th>cause(s)</th>
<th>improvement</th>
<th>revised model</th>
</tr>
</thead>
<tbody>
<tr>
<td>prescribing-privacy</td>
<td>×</td>
<td>s4</td>
<td>s4’</td>
<td>✓</td>
</tr>
<tr>
<td>enforced presc.-priv.</td>
<td>× (with s4’)</td>
<td>s8’</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>ind. of presc.-priv.</td>
<td>✓ (with s4’)</td>
<td>s8’</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>ind. of enf. presc.-priv.</td>
<td>✓ (with s4’)</td>
<td>s8’</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>patient anonymity</td>
<td>✓</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>strong patient anonymity</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>doctor anonymity</td>
<td>×</td>
<td>s4</td>
<td>s4’</td>
<td>✓</td>
</tr>
<tr>
<td>strong doctor anonymity</td>
<td>×</td>
<td>s4</td>
<td>s4’</td>
<td>✓</td>
</tr>
<tr>
<td>patient untraceability</td>
<td>×</td>
<td>s2, s4, s5, s6</td>
<td>s2’, s4”, s5’, s6’</td>
<td>✓</td>
</tr>
<tr>
<td>strong patient untrace.</td>
<td>×</td>
<td>s2, s4, s5, s6</td>
<td>s2’, s4”, s5’, s6’</td>
<td>✓</td>
</tr>
<tr>
<td>doctor untraceability</td>
<td>×</td>
<td>s3</td>
<td>s3’</td>
<td>✓</td>
</tr>
<tr>
<td>strong doctor untrace.</td>
<td>×</td>
<td>s3</td>
<td>s3’</td>
<td>✓</td>
</tr>
</tbody>
</table>

Similarly, the doctor’s enforced prescribing-privacy independent of pharmacist is defined as replacing $R_i$ with $R_{ph}$ in Definition 5.4. The flaw described in Section 5.6.6 is also applied here. Intuitively, when a doctor can prove his prescription without the pharmacist sharing information with the adversary, the doctor can prove it when the pharmacist genuinely cooperates with the adversary.

The verification results for privacy properties are summarised in Table 5.3.

### 5.7 Addressing the flaws of the DLV08 protocol

To summarise, we modify assumptions in Section 5.5.1 to fix the flaws found in our analysis of the privacy notions.

- **s2’** The encryptions are probabilistic.
- **s3’** A doctor’s pseudonym is freshly generated in every session.
- **s4’** A doctor freshly generates an unpredictable credential in each session. We model this with another parameter (a random number) of the credential. Following this, anonymous authentication using these credentials proves knowledge of the used randomness.
- **s4”** A patient freshly generates a credential in each session.
- **s5’** A patient’s social security status is different in each session.
- **s6’** All patients share the same HII.

The modified protocol was verified again using ProVerif. The verification results show that the protocol with revised assumptions satisfies doctor anonymity, strong doctor anonymity, and prescribing-privacy, as well as untraceability and strong untraceability for both patient and doctor.
To make the protocol satisfies enforced prescribing-privacy, we apply the following assumption on communication channels.

- \textbf{s8}' The communication channels are untappable, except that the communication channels for authentications remain public.

Our model of the protocol is accordingly modified as follows: replacing channel ch in lines d10, t6 with an untappable channel ch$\_dp$, replacing channel ch in lines t23, t26, h5, h22 with an untappable channel ch$\_ptph$, and replacing channel ch in lines t24, h21 with an untappable channel ch$\_ptph$. The untappable channels are modelled as global private channels. We prove that the protocol (with \textbf{s4}' and \textbf{s8}') satisfies enforced prescribing-privacy by showing the existence of a process $P'_{dr}$ (as shown in Figure 5.23) such that the equivalences in Definition 5.2 are satisfied. The Equivalences are verified using ProVerif.

\begin{verbatim}
init_{dr}\{d_{k}/Id_{dr}\};(!P_{dr}\{d_{k}/Id_{dr}\} \mid P'_{dr}\{d_{k}/Id_{dr}\}) :=
let Id_{dr} = d_{k} in \nu Pnym_{dr}.
(!P_{dr} (*the P_{dr} has assumptions s4' and s8')
| (out(chc,Id_{dr}).
out(chc,Pnym_{dr}).
\nu n_{dr}.out(chc,n_{dr}).
(*s4': creating a nonce and adding it in zk and spk*)
out(ch,zk(Pnym_{dr},Id_{dr},n_{dr}),dcred(Pnym_{dr},Id_{dr},n_{dr}))).
in(ch,(rev.Auth_{pt},rev.PtProof)).
out(chc,(rev.Auth_{pt},rev.PtProof)).
let c.Cred_{pt} = getpublic(rev.Auth_{pt}) in
let (c.Comt_{pt},c.Cred_{pt}) = getpublic(rev.PtProof) in
if Vfy-zk_{Auth_{pt}}(rev.Auth_{pt},c.Cred_{pt}) = true then
if Vfy-zk_{PtProof}(rev.PtProof,(c.Comt_{pt},c.Cred_{pt})) = true then
out(chc,p_{k}).
\nu r_{dr}.
out(chc,r_{dr}).
let PrescriptID = hash(p_{B},c.Comt_{pt},commit(Pnym_{dr},r_{dr})) in
out(ch_{dp}.(spk((Pnym_{dr},r_{dr},Id_{dr},n_{dr}),
(commit(Pnym_{dr},r_{dr}),dcred(Pnym_{dr},Id_{dr},n_{dr})),
(p_{B},PrescriptID,commit(Pnym_{dr},r_{dr},c.Comt_{pt})),
(r_{dr})).
out(chc.(spk((Pnym_{dr},r_{dr},Id_{dr},n_{dr}),
(commit(Pnym_{dr},r_{dr}),dcred(Pnym_{dr},Id_{dr},n_{dr})),
(p_{B},hash(p_{k},c.Comt_{pt},commit(Pnym_{dr},r_{dr})),
commit(Pnym_{dr},r_{dr},c.Comt_{pt})),
(r_{dr}))))
\end{verbatim}

Figure 5.23: The doctor process $P'_{dr}$ (using untappable channels).

However, with the above assumptions the DLV08 protocol does not satisfy independency of enforced prescribing-privacy. We first show that $P'_{dr}$ is not sufficient for proving this with ProVerif. Then we prove (analogous to the proof in Section 5.6.6)
that there is no alternative process $P'_{dr}$ which satisfies Definition 5.4. Intuitively, all information sent over untappable channels are received by pharmacists and can be genuinely revealed to the adversary by the pharmacists (do not lie by assumption). Hence, there still exist links between a doctor, his nonces, his commitment, his credential and his prescription, when the doctor is bribed/coerced to reveal the nonces used in the commitment and the credential to the adversary.

**Proof.** Suppose, there exists a process $P'_{dr}$ which satisfies the definition of independency of enforced prescribing-privacy. That is, $\exists P'_{dr}$ such that the two equivalences in Definition 5.4 are satisfied.

1. $\approx_{l} C_{ch} \lhd P_{ph}^{chc} \mid (\text{init}'_{dr}\{d_{A}/Id_{dr}\},(P_{dr}\{d_{A}/Id_{dr}\} \mid P'_{dr}\{d_{A}/Id_{dr}\})) \mid$

\[ (\text{init}_{dr}\{d_{B}/Id_{dr}\},(P_{dr}\{d_{B}/Id_{dr}\} \mid \text{main}_{dr}\{d_{B}/Id_{dr},p_{A}/presc\})))]

2. $\approx_{l} \text{init}'_{dr}\{d_{A}/Id_{dr}\}^{out(chc)}\cdot (P'_{dr}\{d_{A}/Id_{dr}\})^{out(chc)}$

We prove that this assumption leads to contradictions.

According to the definition of labelled bisimilarity and static equivalence, if two processes are labelled bisimilar, we have that if $M = E N$ at any state of one process, then there exist some states of the other process, such that $M = E N$. Due to the first equivalence, we have that if $M = E N$ on the right hand side of the first equivalence, then $M = E N$ on the left hand side of the first equivalence.

In the right hand side process of the first equivalence, the doctor $d_{A}$ computed a prescription proof $\text{PrescProof}^{T}$ in sub-process $(\text{main}_{dr}\{d_{A}/Id_{dr},p_{A}/presc\})^{chc}$. This prescription proof eventually is revealed to the adversary on the chc channel by a pharmacist. The adversary can obtain a prescription $p_{A}$ and a doctor commitment $\text{Comt}^{r}_{dr}$ from it using

$$(p_{A}, \text{PrescriptID}^{r}, \text{Comt}^{r}_{dr}, c_{\text{Comt}^{r}_{dr}}) = \text{getmsg}(\text{PrescProof}^{T}).$$

The adversary can also obtain the doctor credential $\text{Cred}^{r}_{dr}$ and the same doctor commitment $\text{Comt}^{r}_{dr}$ from the prescription proof using

$$(\text{Comt}^{r}_{dr}, \text{Cred}^{r}_{dr}) = \text{getSpkVinfo}(\text{PrescProof}^{T}).$$

In the left hand side process of the first equivalence, there should also exist an output of a prescription proof $\text{PrescProof}^{l}$ on the chc channel by a pharmacist, such that

$$(p_{A}, \text{PrescriptID}^{l}, \text{Comt}^{l}_{dr}, c_{\text{Comt}^{l}_{dr}}) = \text{getmsg}(\text{PrescProof}^{l}),$$

and

$$(\text{Comt}^{l}_{dr}, \text{Cred}^{l}_{dr}) = \text{getSpkVinfo}(\text{PrescProof}^{l}).$$

Furthermore, in the right hand side process of the first equivalence, the term $\text{Comt}^{r}_{dr}$ is computed as

$$\text{Comt}^{r}_{dr} = \text{commit}(\text{Pnym}^{r}_{dr}, r^{r}_{dr}),$$
where $\text{Pnym}_{dr}^r$, $r_{dr}$ are revealed to the adversary on chc channel, and the term $\text{Cred}_{dr}'$ is computed as

$$\text{Cred}_{dr}' = \text{drcred}(d_A, \text{Pnym}_{dr}^r, n_{dr}''),$$

where $\text{Pnym}_{dr}^r$ and $n_{dr}'$ are revealed to the adversary on chc channel. In the left hand side process of the first equivalence, there should also exist sub-processes which output messages on chc channel. Such sub-processes can only be $\text{Pnym}_{dr}^r$, $\text{init}_{dr}'\{d_A/Id_{dr}\}$ and $P_{dr}'\{d_A/Id_{dr}\}$.

Suppose the corresponding doctor pseudonym, the nonce used for doctor commitments and the nonce used in doctor credentials on the left hand side, which are sent to the adversary over chc, are $\text{Pnym}_{dr}^1$, $r_{dr}^l$ and $n_{dr}^l$. The adversary obtains the bribed doctor’s commitment on the left hand side as

$$\text{Comt}_{dr}^l = \text{commit}(\text{Pnym}_{dr}^1, r_{dr}^l),$$

and the bribed doctor’s credential of the left hand side as

$$\text{Cred}_{dr}^l = \text{drcred}(d_A, \text{Pnym}_{dr}^1, n_{dr}^l).$$

According to the protocol and the transformation $P_{chc}^c$, messages output over chc by process $!R_{ph}^{chc}$ cannot be used as $\text{Pnym}_{dr}^1$, $r_{dr}^l$ or $n_{dr}^l$, because the relations between the output messages and the prescription proof $\text{PrescProof}^l$ cannot be satisfied. Thus, the processes that output $\text{Pnym}_{dr}^1$, $r_{dr}^l$ and $n_{dr}^l$ over chc are $\text{init}_{dr}'\{d_A/Id_{dr}\}$ and/or $P_{dr}'\{d_A/Id_{dr}\}$.

Next, we show that $\text{PrescProof}^l$ is indeed the prescription proof generated by the process $\text{init}_{dr}'\{d_A/Id_{dr}\}$. $P_{dr}'\{d_A/Id_{dr}\}$ is able to generate such a prescription proof, with the knowledge of $\text{Pnym}_{dr}^1$, $r_{dr}^l$, $n_{dr}^l$ and $d_A$.

$$\text{PrescProof}^l = \text{spk}((\text{Pnym}_{dr}^1, r_{dr}^l, d_A, n_{dr}^l),$$

$$(\text{commit}(\text{Pnym}_{dr}^1, r_{dr}^l), \text{drcred}(\text{Pnym}_{dr}^1, d_A, n_{dr}^l)),

(p_A, \text{PrescriptID}^l, \text{commit}(\text{Pnym}_{dr}^1, r_{dr}^l), c_{\text{Comt}_{dr}'}.)

Second, no sub-processes, except $\text{init}_{dr}'\{d_A/Id_{dr}\}$ and $P_{dr}'\{d_A/Id_{dr}\}$, can generate a prescription with $\text{Pnym}_{dr}^1$, $r_{dr}^l$ and $n_{dr}^l$, which are shared with the adversary over chc.

- Sub-processes, except doctor sub-processes, on the left hand side (including $!R_{ph}^{chc}$) honestly follow the protocol (or transformed process) and thus do not compute a signed proof of knowledge which matches the pattern of prescription proof $\text{PrescProof}^l$.
- Doctor sub-processes, except $\text{init}_{dr}'\{d_A/Id_{dr}\}$ and $P_{dr}'\{d_A/Id_{dr}\}$, cannot generate a prescription with $\text{Pnym}_{dr}^1$, $r_{dr}^l$ and $n_{dr}^l$. As honest doctor processes, $\text{Pnym}_{dr}^1$, $r_{dr}^l$ and $n_{dr}^l$ are not purposely read in by the processes over chc. Even $\text{Pnym}_{dr}^1$, $r_{dr}^l$ or $n_{dr}^l$ is feed by the adversary over public channels, honest doctor sub-processes will not use the read in values to generate a prescription proof. In addition, the nonce $n_{dr}$ is freshly generated in each session of an honest
Therefore, the prescription proof in $\text{init}'_{dr}\{d_k/Id_{dr}\}.P'_{dr}\{d_k/Id_{dr}\}$ is $\text{PrescProof}'_l$, meaning that the doctor prescribes $p_A$ in process $\text{init}'_{dr}\{d_k/Id_{dr}\}.P'_{dr}\{d_k/Id_{dr}\}$.

However, on the second equivalence, in the right hand side process, the doctor prescribes $p_B$. Thus, on the left hand side, the doctor prescribes $p_B$ in process $\text{init}'_{dr}\{d_k/Id_{dr}\}.P'_{dr}\{d_k/Id_{dr}\}$. Therefore, in corresponding process where $\text{chc}$ is not hidden (process $\text{init}'_{dr}\{d_k/Id_{dr}\}.P'_{dr}\{d_k/Id_{dr}\}$), the doctor prescribes $p_B$, because showing the communication over $\text{chc}$ does not affect the communication between the doctor and his communication partner. Since $p_A \neq p_B$, contradiction is found.

Intuitively, a bribed doctor is linked to the nonces he sent to the adversary. The nonces are linked to the doctor’s prescription in a prescription proof. A doctor’s prescription proof is sent over untappable channels first to a patient and later from the patient to a pharmacist. Malicious pharmacists reveal the prescription proof to the adversary. If a bribed doctor lied about his prescription, the adversary can detect it by checking the doctor’s corresponding prescription proof revealed by the pharmacist. The untappable channel assumption enables the protocol to satisfy enforced prescribing-privacy but not independency of enforced prescribing-privacy because untappable channels enable a bribed doctor to hide his prescription proof and thus allow the doctor to lie about his prescription, however the pharmacist gives the prescription proof away, from which the adversary can detect whether the doctor lied about the prescription.

5.8 Conclusions

In this chapter, we studied enforced privacy in the e-health domain. We identified the requirement that doctor privacy should be enforced to prevent doctor bribery by (e.g.) the pharmaceutical industry. To capture this requirement, we first formalised the classical privacy property prescribing-privacy, and its enforced privacy counterpart, enforced prescribing-privacy. The cooperation between the bribed doctor and the adversary is formalised in the same way as in receipt-freeness in e-voting. However, the formalisation of enforced prescribing-privacy differs from receipt-freeness in e-voting, due to the domain requirement that only part of the doctor’s process needs to share information with the adversary.

Next, we noted that e-health systems involve untrusted third parties, such as pharmacists. Such parties should also not be able to assist an adversary in breaking doctor privacy. To capture this requirement, we formally defined independency of prescribing-privacy. This requirement must hold, even if the doctor is forced...
to help the adversary. To capture that, we formally defined independency of enforced prescribing-privacy. These formalisations were validated in a case study of the DLV08 protocol. The protocol was modelled in the applied pi calculus and verified with the help of the ProVerif tool. In addition to the (enforced) doctor privacy notions, we analysed secrecy, authentication, anonymity and untraceability for both patients and doctors. Ambiguities which may lead to flaws were found and addressed.

The experience gained in the domain-specific studies in e-auctions (Chapter 4) and e-health (this chapter), and the differences between the formalisations enable us to generalise from these formalisations and develop a domain-independent formal framework for verifying enforced privacy properties in the next chapter.
Enforced privacy in the presence of others

In the previous chapters, we proposed domain-specific formalisations of enforced privacy in e-auctions and e-health. However, the adversary’s ability to bribe or coerce does not depend on any specific domain. In order to address the enforced privacy concerns domain-independently, in this chapter, we propose a generic property – enforced privacy: a user’s privacy is preserved even if the user collaborates with the adversary by sharing information.

Enforced privacy only prevents the target user from undoing his own privacy. However, a third party may help to break user privacy (collaboration), e.g., pharmacist may help prove a doctor’s prescription behaviour, revealing your vote may enable the adversary to deduce another voter’s vote. On the other hand, we identify that a third party can help maintain privacy (coalition), e.g., a non-coerced voter (who votes as the adversary desires) can swap receipts with a coerced voter, providing the coerced voter “proof” of compliance while being free to vote as he pleases. Accounting for the privacy effect of third parties is particularly necessary in domains where many non-trusted roles are involved. For example, pharmacists in e-health may be able to help reveal prescription behaviour of doctors. In order to ensure doctor prescribing-privacy, an e-health system must prevent this [DLV08, FHI11]. This requirement has been expressed and formalised in e-health [ESORICS12] and e-voting [DLL11]. We generalise these formalisations as independency of privacy: the help of a set of third parties does not enable the adversary to break a target user’s privacy. To capture the converse situation – the privacy effect of third parties helping the target user by sharing information with the target user, we propose a new notion of coalition privacy: a target user’s privacy is preserved with the help of a set of third parties sharing information with the target user. In particular, we use this notion to also capture the situation where third parties are involved but no information is shared between the target user and third parties. In this case, the mere existence of the third parties can help to create a situation where privacy is preserved.

6.1 Privacy notions

We distinguish between two classes of privacy-affecting behaviour: the target user (collaborating with the adversary or not), and the behaviour of third parties. Third parties may be neutral, collaborating with the adversary (attacking), or collaborating with the target user (defending) – thus we also consider the situation where some are attacking and some are defending. A target user who collaborates with

This chapter is based on published work [ESORICS13]
the adversary is not under the adversary’s direct control, contrary to a compromised user who genuinely shares initial private information with the adversary. A neutral third party, like an honest user, follows the protocol specification exactly. Thus, such a third party neither actively helps nor actively harms the target user’s privacy. A defending third party helps the target user to preserve his privacy. An attacking third party communicates with the adversary to break the target user’s privacy. Note that we do not consider a third party that attacks and defends the target user simultaneously. Given this classification, a target user will find himself one of the following four situations w.r.t. third parties: 1) all are neutral; 2) some are attacking; 3) some are defending; and 4) some are attacking, some are defending. In the latter three cases, the remaining third parties (if any) are considered neutral. Combining the various behaviours of the third parties with those of the target user gives rise to eight privacy properties (see Table 6.1). In the table, privacy properties are abbreviated. The property, data-privacy, where the target user does not collaborate with the adversary and third parties are all neutral, is abbreviated as priv. Abbreviations of other properties can be distinguished by their prefix to priv. In particular, the prefix e indicates that the target user collaborates with the adversary. Prefix i indicates a set of third parties collaborates with the adversary. And prefix c indicates that a set of third parties cooperate with the target user. The properties listed in Table 6.1 cover all combinations of prefixes. Examples of each property listed in Table 6.1 are as follows:

- data-privacy (priv): the adversary cannot link the contents of an encrypted email to the user;
- enforced-privacy (epriv): a voter should not be able to prove to a vote-buyer how he voted;
- independency-of-privacy (ipriv): in e-health the adversary cannot link a doctor to his prescriptions, despite the help of a pharmacist;
- independency-of-enforced-privacy (iepriv): the adversary should not be able to link a doctor to his prescriptions (to prevent bribes), even when both the pharmacist and the doctor are helping him;
- coalition-privacy (cpriv): in location-based services, the user’s real location is hidden amongst the locations of the helping users;
- coalition-enforced-privacy (cepriv): in anonymous routing, a sender remains anonymous if he synchronises with a group of senders, even if he seems to collaborate;

<table>
<thead>
<tr>
<th>target user collaborates with adversary</th>
<th>third parties</th>
<th>some defending</th>
<th>some attacking</th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td>priv</td>
<td>cpriv</td>
<td>cpriv</td>
</tr>
<tr>
<td>yes</td>
<td>epriv</td>
<td>ipriv</td>
<td>iepriv</td>
</tr>
</tbody>
</table>

Table 6.1: Privacy notions
6.2 Formal framework

- coalition-independency-of-privacy (cipriv): the adversary cannot link an RFID chip to its identity, even though some malicious readers are helping the adversary, provided other RFID tags behave exactly as the target one;

- coalition-independency-of-enforced-privacy (ciepriv): in electronic road pricing, other users may hide a user’s route from the adversary, even if the user seems to collaborate and malicious routers relay information on passing cars to the adversary.

The examples above illustrate that similar privacy concerns arise in many different domains – e-voting, e-health, location-based services, RFID, etc. So far, attempts at formalising privacy have usually been domain-specific (e.g., [KR05, vMR08, DKR09, ACRR10, BPT11, DDDS11, DLL12, FAST10, ESORICS12]). We advocate a domain-independent approach to privacy, and develop a formal framework to achieve this in Section 6.2.

6.2 Formal framework

In this section, we present a formal framework which allows us to give domain-independent formalisations. We define a standard form of protocols which is able to represent any protocol. To formally define enforced privacy properties and independency of privacy properties, we model collaboration between users and the adversary. The collaboration allows us to precisely specify which information is shared and how it is shared, thus provides the necessary flexibility for modelling various types of collaboration. To model coalition privacy properties, we propose the notion of coalition in our framework to formally capture the behaviour and shared information among a target user and a set of third parties.

6.2.1 Well-formed protocols

In the applied pi calculus, a protocol is normally modelled as a plain process. For the simplicity of formalising privacy properties, we define a standard form of a protocol [ACRR10] and any protocol can be written in this form.

Definition 6.1 (well-formed protocols). A protocol with \( p \) roles is well-formed if it is a closed plain process \( P_w \) of the form:

\[
P_w := \nu \hat{c}. (\text{genkey} | R_1 | \cdots | R_p)
\]

\[
R_i := \nu \text{id}_i. \nu \text{data}_i. \text{init}_i. !(\nu s. \nu \text{sdata}_i. \text{sinit}_i. \text{main}_i) \quad (\forall i \in \{1, \cdots, p\})
\]

where

1. \( P_w \) is canonical [ACRR10]: names and variables in the process never appear both bound and free, and each name and variable is bound at most once;

2. data is typed, channels are ground, private channels are never sent on any channel;

3. \( \nu \hat{c}, \nu \text{data}_i, \) and \( \nu \text{sdata}_i \) may be null;
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4. $\text{init}_i$ and $\text{sinit}_i$ are sequential processes;

5. $\text{genkey}$, $\text{init}_i$, $\text{sinit}_i$ and $\text{main}_i$ can be any process (possibly null) such that $P_w$ is a closed plain process.

In process $P_w$, $\tilde{c}$ are channel names; $\text{genkey}$ is a sub-process in which shared data (e.g., keys shared between two roles) are generated and distributed; $R_i$ ($1 \leq i \leq p$) is a role. To distinguish instances taking the same role $R_i$, each instance is dynamically associated with a distinct identity $\nu_{\text{id}}$; $\text{data}_i$ is private data of an instance; $\text{init}_i$ models the initialisation of an instance; $(\nu_s, \nu_{\text{data}}, \text{sinit}_i, \text{main}_i)$ models a session of an instance. To distinguish sessions of the same instance, each session is dynamically associated to a distinct identity ($\nu_s$); $\text{sdata}_i$ is private data of a session; $\text{sinit}_i$ models the initialisation of a session; $\text{main}_i$ models the behaviour of a session.

Note that this standard form does not limit the type of protocols we consider. A role may include a number of sub-roles so that a user may take more than one part in a protocol. The identities do not have to be used in the process. All of $\nu_c$, $\nu_{\text{data}}$, and $\nu_{\text{data}}$ may be null and $\text{genkey}$, $\text{init}_i$, $\text{sinit}_i$ and $\text{main}_i$ can be any process (possibly null) such that $P_w$ is a closed plain process. Any process can be written in a canonical form by $\alpha$-conversion \cite{ACRR10}. Thus, any protocol can be written as a well-formed protocol.

Example 6.2 (well-formed protocol). A very simple anonymous proxy service could work via an intermediary TTP: the user sends her service requests to the TTP, which forwards them under its own identity to APP. Each request is distinct from others. In more detail, the system can be formalised in the following form (well-formed):

\[
P := \nu c_p.(\!\!\!\!R_U \parallel R_{\text{TTP}})
\]
\[
R_U := \nu \text{id}.!(\nu \text{req}.\text{out}(c_p, (\text{id}, \text{req})))
\]
\[
R_{\text{TTP}} := \text{in}(c_p, (\text{id}_x, x)).\text{out}(\text{ch}, (\text{id}_x, x))
\]

A user process is $R_U$ and the TTP process is $R_{\text{TTP}}$. $c_p$ is a private channel between a user and TTP which models the assumption that APP does not know the message between a user and TTP. $\text{ch}$ is a public channel over which information is sent to the APP.

6.2.2 Data-privacy

We formally define the property data-privacy that acts as the foundation upon which other properties are built. To do so, we need to make explicit which data is protected. Thus, the property data-privacy always specifies the target data. When there is no ambiguity of the target data, we use data-privacy for short. In process $P_w$, the target data $\tau$ is a bound name which belongs to a role (the target role $R_i$), i.e., $\tau \in \text{bn}(R_i)$. For the sake of simplicity, we (re)write the role $R_i$ in the form of

\[
R_i := \nu \text{id}_i.\nu \tau.\hat{R}_i,
\]

where $\hat{R}_i$ is a plain process which has two variables $\text{id}_i$ and $\tau$. Note that by $\alpha$-conversion we can always transform any role $R_i$ into the above form. When
6.2 Formal framework

\[ \tau \in \text{data}, \]

\[ \hat{R}_t := \nu \text{data}/\tau.\text{init}.!(\nu s, \nu \text{sdata}.s\text{init}.\text{main}). \]

When \( \tau \) is session data in session \( s \), i.e., \( \tau \in \text{sdata}'_s \),

\[ \hat{R}_t := \nu \text{data}.\text{init}.!(\nu s, \nu \text{sdata}.s\text{init}.\text{main}) | (\nu \nu \text{sdata}'/\tau.s\text{init}'.\text{main})). \]

In case that only information in session parties, we require that \( \nu \not\in \text{bn}(P_w) \), \( \nu \text{sdata}'/\tau.s\text{init}'.\text{main} \) is obtained by applying \( \alpha \)-conversion on bound names and variables in the original process \( \nu \text{sdata}/\tau.\text{init}.\text{main} \).

Intuitively, data-privacy w.r.t. \( \tau \) of protocol \( P_w \), is the unlinkability of an honest user taking role \( R_i \) and his instantiation of the target data \( \tau \). An honest user taking role \( R_i \) is modelled as process \( R_i \). We denote a particular user – the target user process, as \( \hat{R}_t \{\text{id}/id_i\} \) where \( R_i := \nu \text{id} \hat{R}_t \), variable \( id_i \) is instantiated with a name or constant \( id \). \( \hat{R}_t \{\text{id}/id, t/\tau\} \) denote an instance of the target user in which the target user instantiates the target data with \( t \) where \( t \) denotes any data which can be used to replace the target data. The unlinkability is modelled as strong secrecy [Bla04] of the target data: the adversary cannot distinguish an execution of \( R_i \) where \( \tau = t_1 \) from an execution where \( \tau = t_2 \), for \( t_1 \neq t_2 \).

**Definition 6.3 (priv)**. A well-formed protocol \( P_w \) satisfies data-privacy (priv) w.r.t. data \( \tau \) (\( \tau \in \text{bn}(R_i) \)), if

\[ \mathcal{C}_{P_w}[\hat{R}_i\{\text{id}/id_i, t_1/\tau\}] \approx_{\epsilon} \mathcal{C}_{P_w}[\hat{R}_i\{\text{id}/id_i, t_2/\tau\}]. \]

In the definition, \( \text{id} \) is a name or constant, \( t_1 \) and \( t_2 \) are free names. Since \( R_i := \nu \text{id} . \nu t . \hat{R}_t \), process \( \hat{R}_t \{\text{id}/id_i, t_1/\tau\} \) is an instance of role \( R_i \) where the identity is \( \text{id} \) and the target data is \( \tau \). The context \( \mathcal{C}_{P_w}[.] \) models honest third parties. Thus, \( \mathcal{C}_{P_w}[\hat{R}_i\{\text{id}/id_i, t_1/\tau\}] \) is an instance of the protocol \( P_w \), similarly for \( \mathcal{C}_{P_w}[\hat{R}_i\{\text{id}/id_i, t_2/\tau\}] \). The only difference between these two instances is the instantiation of the target data \( \tau \). Thus, this definition captures data-privacy by using the relation \( \approx_{\epsilon} \): the adversary cannot distinguish a user process with different target data.

**Example 6.4 (data-privacy)**. We study whether the system in Example 6.2 ensures request anonymity. Request anonymity is defined as the adversary cannot distinguish a user sending request \( \text{req}_1 \) from sending request \( \text{req}_2 \). This property can be formalised as data-privacy with respect to a request \( \text{req} \). In order to specify a request, we write the target user process in the following form:

\[ R_U := \nu \text{id} . \nu \text{req}.(\text{out}(c_p, (\text{id}, \text{req}))) || (\nu \text{req}.\text{out}(c_p, (\text{id}, \text{req}))). \]

Thus we have, \( \hat{R}_U \) defined as follows:

\[ \hat{R}_U := (\text{out}(c_p, (\text{id}, \text{req}))) || (\nu \text{req}.\text{out}(c_p, (\text{id}, \text{req}))). \]

Data-privacy w.r.t. \( \text{req} \) is formalised as

\[ \mathcal{C}_{ex}[\text{out}(c_p, (\text{id}, \text{req}))) || (\nu \text{req}.\text{out}(c_p, (\text{id}, \text{req})))] \approx_{\epsilon} \mathcal{C}_{ex}[\text{out}(c_p, (\text{id}, \text{req}))) || (\nu \text{req}.\text{out}(c_p, (\text{id}, \text{req})))], \]

where \( \mathcal{C}_{ex}[] := \nu c_p.(\hat{R}_U || R_{\text{TTP}} | \_). \)

The system does not satisfy request anonymity, because on the left hand side process of the equivalence, the adversary observes \( \text{req}_1 \) which does not appear on the right hand side process.
6.2.3 Modelling collaboration with the adversary

In order to define enforced privacy properties where the target user collaborates with the adversary and independency privacy properties where a set of third parties collaborate with the adversary, we need to model collaboration of users (a target user/third parties) with the adversary.

The process of a set of users is modelled as processes of each user in parallel. Since a user process is modelled as a role in a well-formed protocol and each user process can be any role, the set of users of a well-formed protocol $P_w$ is formally defined as a plain process $R_U := R_{u_1} | \cdots | R_{u_m}$, $\forall i \in \{1, \ldots, m\}$, $R_{u_i} \in \{R_1, \ldots, R_p\}$.

Inspired by the formal definition of coercion in [DKR09], the collaboration between a user and the adversary is formalised as a transformation of the user process. We extend it as a transformation of the process of a set of users. Note that a user need not always share all his information, e.g., a bribed user in a social network may reveal his relation with another user, but not his password. To be able to specify which information is shared, we formally define the set of information that a user has. Information of a user is expressed as a set of terms in the user process. Since the user processes are canonical in a well-formed protocol, bound names and variables are different in each user process. Thus, we can express information of a set of users as a set of terms appearing in the process of the set of users. Terms appearing in a plain process $R_U$ are given by $\text{Term}(R_U)$.

\begin{align*}
\text{Term}(0) &= \emptyset \\
\text{Term}(P \mid Q) &= \text{Term}(P) \cup \text{Term}(Q) \\
\text{Term}(\nu n.P) &= \{n\} \cup \text{Term}(P) \\
\text{Term}(\text{in}(v, x).P) &= \{x\} \cup \text{Term}(P) \\
\text{Term}(\text{out}(v, M).P) &= \{M\} \cup \text{Term}(P) \\
\text{Term}(\text{if } M = E N \text{ then } P \text{ else } Q) &= \text{Term}(P) \cup \text{Term}(Q)
\end{align*}

A collaboration specification then specifies which terms of a process are shared and how they are shared.

**Definition 6.5** (collaboration specification). A collaboration specification of a process $R_U$ is a tuple $\langle \Psi, \Phi, c_{\text{out}}, c_{\text{in}} \rangle$. $\Psi \subseteq \text{Term}(R_U)$ denotes the set of terms sent to the adversary each of which is of base type, $\Phi \subseteq \text{Term}(R_U)$ represents terms to be replaced by information provided by the adversary, $c_{\text{out}}$ is a fresh channel for sending information to the adversary, and $c_{\text{in}}$ is a fresh channel for reading information from the adversary, i.e., $c_{\text{out}}, c_{\text{in}} \notin \text{fn}(R_U) \cup \text{bn}(R_U)$.

Given a plain process $R_U$ and a collaboration specification $\langle \Psi, \Phi, c_{\text{out}}, c_{\text{in}} \rangle$ of the process, the transformation of $R_U$ is given by $R^t_{U(\Psi,\Phi,c_{\text{out}},c_{\text{in}})}$. Note that we use $c_{\text{in}} = \epsilon$ to denote that the adversary neither prepares information for the coerced users nor controls the conditional evaluations of the users.

**Definition 6.6** (collaboration behaviour). Let $R_U$ be a plain process, and $\langle \Psi, \Phi, c_{\text{out}}, c_{\text{in}} \rangle$ be a collaboration specification of $R_U$. Collaboration behaviour
of \( R_U \) according to \( (\Psi, \Phi, c_{out}, c_{in}) \) is defined as:

- \( 0^{(\Psi, \Phi, c_{out}, c_{in})} = 0 \)
- \( (P \mid Q)^{(\Psi, \Phi, c_{out}, c_{in})} = P^{(\Psi, \Phi, c_{out}, c_{in})} \mid Q^{(\Psi, \Phi, c_{out}, c_{in})} \)
- \( (!P)^{(\Psi, \Phi, c_{out}, c_{in})} = P^{(\Psi, \Phi, c_{out}, c_{in})} \)
- \( (\nu n.P)^{(\Psi, \Phi, c_{out}, c_{in})} = \begin{cases} \nu n.\text{out}(c_{out}, n).P^{(\Psi, \Phi, c_{out}, c_{in})} & \text{if } n \in \Psi, \\ \nu n.P^{(\Psi, \Phi, c_{out}, c_{in})} & \text{otherwise}, \end{cases} \)
- \( (\text{in}(v, x).P)^{(\Psi, \Phi, c_{out}, c_{in})} = \begin{cases} \text{in}(v, x).\text{out}(c_{out}, x).P^{(\Psi, \Phi, c_{out}, c_{in})} & \text{if } x \in \Psi, \\ \text{in}(v, x).P^{(\Psi, \Phi, c_{out}, c_{in})} & \text{otherwise}, \end{cases} \)
- \( (\text{out}(v, M).P)^{(\Psi, \Phi, c_{out}, c_{in})} = \begin{cases} \land c_{in} \neq \epsilon, \text{ where } x \text{ is a fresh variable,} \\ \text{out}(v, M).P^{(\Psi, \Phi, c_{out}, c_{in})} & \text{otherwise,} \end{cases} \)
- \( (\text{if } M =_E N \text{ then } P \text{ else } Q)^{(\Psi, \Phi, c_{out}, c_{in})} = \begin{cases} \text{in}(c_{in}, x).\text{if } x = \text{true then } P^{(\Psi, \Phi, c_{out}, c_{in})} \text{else } Q^{(\Psi, \Phi, c_{out}, c_{in})} & \text{if } c_{in} \neq \epsilon, \\ \text{where } x \text{ is a fresh variable and true is a constant,} \\ \text{if } M =_E N \text{ then } P^{(\Psi, \Phi, c_{out}, c_{in})} \text{else } Q^{(\Psi, \Phi, c_{out}, c_{in})} & \text{otherwise.} \end{cases} \)

Note that we only specify user behaviour in a collaboration with the adversary. The adversary’s behaviour may be omitted, as in the applied pi calculus the adversary is considered as the environment and does not need to be explicitly modelled. Our approach to reasoning about the adversary’s behaviour in a collaboration (e.g., enforcing a voter to cast a particular vote) follows the line of the definition of coercion-resistance in \[\text{DKR09}\]. Namely, a context \( C[] := \nu c_{out}.\nu c_{in}(\bot Q) \) models a specific way of collaboration of the adversary, where \( Q \) models the adversary’s behaviour in the context. In this way, we separate the adversary’s behaviour of distinguishing two processes, which is modelled by the environment, from the behaviour of collaborating with users which is modelled by the context.

Example 6.7 (collaboration). In the system in Example 6.2 suppose a user collaborates with the adversary in the following way. The user reveals his request \( \text{req}_i \) in a specific session with session identity \( i \), reads in an identity and a request from the adversary and then forwards the received message in session \( j \).

In order to specify the collaboration, we need to specify the session \( i \) and \( j \). The user process needs to be of the following form:

\[
R_U := \nu \text{id}.((\nu \text{req}.\text{out}(c_p,(\text{id}.\text{req}))) | (\nu \text{req}.\text{out}(c_p,(\text{id}.\text{req}))))
\]

The term sent to the adversary is \( \text{req}_i \), i.e., \( \Psi = \{\text{req}_i\} \). The replaced term is \( (\text{id}.\text{req}) \), i.e., \( \Phi = \{(\text{id}.\text{req})\} \). Therefore the collaboration is specified as \( (\Psi, \Phi, c_{out}, c_{in}) = \{(\text{req}_i),\{(\text{id}.\text{req})\}, c_{out}, c_{in}\} \).

Following Definition 6.6, the collaboration behaviour of the user is modelled as:

\[
R^{(\Psi, \Phi, c_{out}, c_{in})}_U := \nu \text{id}.((\nu \text{req}.\text{out}(c_p,\text{req}).\text{out}(c_p,(\text{id}.\text{req})))
\]

\[
| (\nu \text{req}.\text{in}(c_{in}, x).\text{out}(c_p,x))
\]

\[
|!(\nu \text{req}.\text{out}(c_p,(\text{id}.\text{req})))
\]
6.2.4 Modelling user coalitions

To define coalition privacy properties, we need to formally define a coalition between a target user and a set of defending third parties. The notion collaboration from the previous section cannot be adopted directly, as it does not specify the adversary’s behaviour, whereas a coalition must specify the behaviour of all involved users.

Given a set of users $R_U := R_{u_1} | \cdots | R_{u_m}$, a coalition of the users specifies communication between (potentially) each pair of users. For every communication, a coalition specification needs to make explicit who the sender and receiver are (unlike collaboration). Similar to the specification of collaboration, a coalition specification makes explicit which data is sent on which channel. To make the behaviour of both communicating parties explicit, we need to specify how the term in a communication is referred to in the receiver’s process. A communication in a coalition is specified as a tuple $(R_{u_i}, R_{u_j}, M, c, y)$ where $R_{u_i}, R_{u_j} \in \{R_{u_1}, \ldots, R_{u_m}\}$ $(R_{u_i} \neq R_{u_j})$ are the sender and receiver process, respectively; $M \in \text{Term}(R_{u_i})$ is the data sent in the communication; $c \notin \text{fn}(R_U) \cup \text{bn}(R_U)$ is a fresh channel used in the communication; $y \notin \text{fv}(R_U) \cup \text{bv}(R_U)$ is the variable used by the receiver to refer to the term $M$. A coalition specifies a set of communications of this type (denoted as $\Theta$). For the simplicity of modelling, we assume that for each communication, the coalition uses a distinct channel and distinct variable, i.e., $\forall (R_{u_i}, R_{u_j}, M, c, y) \in \Theta$ and $(R'_{u_i}, R'_{u_j}, M', c', y') \in \Theta$ we have $c \neq c' \land y \neq y'$.

A coalition specifies a set of terms which are communicated by the originating user process and are replaced in the coalition. In addition, a coalition needs to define how a term is replaced. In a collaboration, the adversary is assumed to be able to compute and prepare this, but in a coalition, no user can compute and prepare information for other users. Thus, this ability has to be explicitly specified in a coalition as a set of substitutions $\Delta = \{N/M \mid M \in \text{Term}(R_U)\}$. The new term $N$ are calculated from a set of terms $N_1, \ldots, N_n$ which are generated by the user, read in by the original process, or read in from coalition members. A successful coalition requires that there are no such situations where $N$ cannot be calculated in the user process when $M$ needs to be replaced.

Moreover, in a coalition, we allow the coalition to decide values of conditional evaluations (similar to collaboration, where the adversary decides this). Since no user in a coalition has the ability to specify the values of evaluations, these need to be assigned specifically. In addition, to add more flexibility, we allow a coalition to specify which evaluations are decided by the coalition and which are not. The evaluations of a plain user process $R_U$ is $\text{Eval}(R_U)$. The assignments of evaluations are specified as a set $\Pi \subseteq \{(e, b) \mid e \in \text{Eval}(R_U) \land b \in \{\text{true}, \text{false}\}\}$.

\[
\begin{align*}
\text{Eval}(0) &= \emptyset \\
\text{Eval}(P) &= \text{Eval}(P) \\
\text{Eval}(\forall P) &= \text{Eval}(P) \\
\text{Eval}(\exists P) &= \text{Eval}(P) \\
\text{Eval}(\text{in}(v, x), P) &= \text{Eval}(P) \\
\text{Eval}(\text{out}(v, M), P) &= \text{Eval}(P) \\
\text{Eval}(\text{if } M =_E N \text{ then } P \text{ else } Q) &= \{M =_E N\} \cup \text{Eval}(P) \cup \text{Eval}(Q)
\end{align*}
\]

**Definition 6.8** (coalition specification). A coalition of a set of users $R_U$ is specified as a tuple $(\Theta, \Delta, \Pi)$ where $\Theta$ is a set of communication, $\Delta$ is a set of substitutions and $\Pi$ is an assignment for a set of evaluations.
Note that this model does not include the coalition strategies in which the target users and defending third parties are able to generate new data, initiate new sessions, establishing new secrets, etc.

With the above setting, given a set of users \( R_U \) and a coalition specification \( \langle \Theta, \Delta, \Pi \rangle \) on users, the behaviour of a user in the coalition is modelled as a coalition transformation of the user’s original process.

**Definition 6.9 (coalition behaviour).** Let \( R_U := R_{u_1} | \cdots | R_{u_m} \) be a plain process of a set of users, \( \langle \Theta, \Delta, \Pi \rangle \) be a coalition specification of process \( R_U \), \( R \in \{ R_{u_1}, \cdots, R_{u_m} \} \) be a plain user process, the transformation of the process \( R \) in the coalition is given by \( R^{(\Theta, \Delta, \Pi)} \):

\[
R^{(\Theta, \Delta, \Pi)} = \nu\eta. (R^{(\Gamma, \Delta, \Pi)} | \text{in}(c_1, y'_1).!\text{out}(c'_1, y'_1) | \cdots | \text{in}(c_\ell, y'_\ell).!\text{out}(c'_\ell, y'_\ell))
\]

where \( \Gamma = \{ (R, R_{u_i}, M, c, y) | (R, R_{u_i}, M, c, y) \in \Theta \} \), \( \eta = \{ c'_1, \ldots, c'_\ell \} \), \( c'_1, \ldots, c'_\ell \) are fresh, \( \{ c_1, \ldots, c_\ell \} = \{ c | (R, R_{u_i}, M, c, y) \in \Theta \} \), \( y'_1, \ldots, y'_\ell \) are fresh variables, \( \xi = \{ (c_1, y'_1; c'_1), \ldots, (c_\ell, y'_\ell; c'_\ell) \} \) defines the association of channels and variables in process \( \text{in}(c_1, y'_1).!\text{out}(c'_1, y'_1) | \cdots | \text{in}(c_\ell, y'_\ell).!\text{out}(c'_\ell, y'_\ell) \), and \( R^{(\Gamma, \Delta, \Pi)} \) is given by:

- \( (\ell_F^{(\Gamma, \Delta, \Pi)}) \)  
- \( (P | Q)_{F}^{(\Gamma, \Delta, \Pi)} \)  
- \( (\langle P \rangle)_{F}^{(\Gamma, \Delta, \Pi)} \)  
- \( (\langle \nu \cdot \eta \rangle)_{F}^{(\Gamma, \Delta, \Pi)} \)  
- \( (\text{in}(v, x).P)_{F}^{(\Gamma, \Delta, \Pi)} \)  
- \( (\text{out}(v, M).P)_{F}^{(\Gamma, \Delta, \Pi)} \)  
- \( (\text{if} \ M = E \ N \text{ then} \ P \text{ else} \ Q)_{F}^{(\Gamma, \Delta, \Pi)} \)  

with \( F \) initially equals to \( \{ y_1, \ldots, y_\ell | (R_{u_i}, R, M, c, y) \in \Theta \} \).

Process \( \text{in}(c_1, y'_1).!\text{out}(c'_1, y'_1) | \cdots | \text{in}(c_\ell, y'_\ell).!\text{out}(c'_\ell, y'_\ell) \) models the receiving behaviour of process \( R \) in the coalition. The coalition specifies which channel is used to receive data. The received data on a channel are referred to as a distinct fresh variable. The received data is sent out over a distinct private channel. The association of channels and variables is modelled in \( \xi \). This sending behaviour is used for the process \( R^{(\Gamma, \Delta, \Pi)} \) to read the data when it is needed. Process \( R^{(\Gamma, \Delta, \Pi)} \)
models the sending behaviour, substitution of terms, assignments of evaluations. F captures the variables which are in \{y_1, \ldots, y_k\} and has not been read in yet.

Given a set of users \(R_U\) and a coalition specification \(\langle \Theta, \Delta, \Pi \rangle\) for them, the coalition is now modelled as \(R_U^{(\Theta, \Delta, \Pi)} = \nu \Omega.(R_u^{(\Theta, \Delta, \Pi)} | \cdots | R_m^{(\Theta, \Delta, \Pi)})\) where \(\Omega = \{c \mid \langle R_u, R_j, M, c, y \rangle \in \Theta\}\).

**Example 6.10 (coalition).** In the system in Example 6.12 suppose two users \(U_i\) and \(U_j\) cooperate in the following way: \(U_i\) sends his own request and the request from \(U_j\); \(U_j\) sends his own request and the request from \(U_i\).

\[
\begin{align*}
R_{U_i} & := \nu i.d_i.((\nu req, out(c_p, (id, req_i))) | (\nu req'_i, out(c_p, (id, req'_i)))) \\
& | (((\nu req, out(c_p, (id, req))))
R_{U_j} & := \nu i.d_j.((\nu req, out(c_p, (id, req_j))) | (\nu req'_j, out(c_p, (id, req'_j)))) \\
& | (((\nu req, out(c_p, (id, req))))
\end{align*}
\]

The coalition can be specified as \(\Theta = \{(R_u, R_i, req, c_j, y_j), (R_u, R_j, req_i, c_i, y_i)\}, \Delta = \{(i.d, y_i)/(id, req_i)\}, ((id, y_i)/(id, req_j)\}\) and \(\Pi = \emptyset\).

According to Definition 6.15, the process \((R_{U_i} \mid R_{U_j})^{(\Theta, \Delta, \Pi)}\) is as follows:

\[
\begin{align*}
\nu c'_i, \nu c'_j. \\
( (\nu i.d_i.((\nu req, out(c_i, req_i), out(c_p, (id, req_i))) & | (\nu req'_i, in(c'_i, y_j), out(c_p, (id, y_j)))) \\
& | (((\nu req, out(c_p, (id, req)))))) | \\
( (\nu i.d_j.((\nu req, out(c_j, req_j), out(c_p, (id, req_j))) & | (\nu req'_j, in(c'_j, y_i), out(c_p, (id, y_i)))) \\
& | (((\nu req, out(c_p, (id, req)))))) ) | \\
in(c_i, y'_i), !out(c'_i, y'_i) & | in(c_j, y'_j), !out(c'_j, y'_j)
\end{align*}
\]

### 6.3 Formalising the privacy notions

In our framework, the foundational property data-privacy, is formalised in a classical way as strong secrecy: equivalence of two processes where a variable is instantiated differently [Bla04]. Based on this property, we formalise enforced-privacy, independency-of-privacy and independency-of-enforced-privacy using the formalisation of collaboration. Using the formalisation of coalition, four corresponding coalition privacy properties are formalised.

#### 6.3.1 Enforced-privacy

Enforced-privacy is the unlinkability of a target user to his data even when the user collaborates with the adversary. Different collaborations impact privacy differently, so when we say a protocol satisfies enforced-privacy, it always refers to a specific collaboration specification.

Similar as in receipt-freeness and coercion-resistance in e-voting [DKR09], when a protocol \(P_w\) satisfies enforced-privacy w.r.t. a target data \(\tau\) (which belongs to role \(R_i\) and a collaboration specification \(\langle \Psi, \Phi, c_{out}, c_{in} \rangle\) defined on process \(\hat{R}_i\) (where \(R_i := \nu i.d, \nu \tau.R_i\)), there exists a process \(P_f\) for the target user to execute, such
that the adversary cannot distinguish between real collaboration with \( \tau = t_1 \) and fake collaboration (by means of process \( P_f \)) with \( \tau = t_2 \). In the epistemic notion of coercion-resistance, enforced-privacy can be defined as the existence of a *counter-strategy* for the target user to achieve his own goal, but the adversary cannot distinguish it from the target user following the adversary’s instructions [KT09].

**Definition 6.11 (epriv).** A well-formed protocol \( P_w \) satisfies enforced-privacy (epriv) w.r.t. target data \( \tau \) and collaboration specification \( \langle \Psi, \Phi, c_{\text{out}}, c_{\text{in}} \rangle \), if there exists a closed plain process \( P_f \), such that for any context \( C[.] := \nu c_{\text{out}}.\nu c_{\text{in}}.\langle \cdot | Q \rangle \) satisfying \( \text{bn}(P_w) \cap \text{fn}(C[.]) = \emptyset \) and \( C_{P_w}[C[\hat{R}_i^{(\Psi, \Phi, c_{\text{out}}, c_{\text{in}})}] \{\text{id}/i_{d_i}, t/\tau\}] \approx_{\ell} C_{P_w}[\hat{R}_i^{(\Psi, \Phi, c_{\text{out}}, c_{\text{in}})}] \{\text{id}/i_{d_i}, t/\tau\}] \), we have

1. \( C[P_f]^{\text{out}(c_{\text{out}})} \approx_{\ell} \hat{R}_i^{t}[\text{id}/i_{d_i}, t_2/\tau] \),
2. \( C_{P_w}[C[\hat{R}_i^{(\Psi, \Phi, c_{\text{out}}, c_{\text{in}})}] \{\text{id}/i_{d_i}, t/\tau\}] \approx_{\ell} C_{P_w}[C[P_f]] \),

where \( \tau \in \text{bn}(R_i) \), \( R_i := \nu \text{id}_i.\nu \tau.\hat{R}_i^{t} \), \( \langle \Psi, \Phi, c_{\text{out}}, c_{\text{in}} \rangle \) is defined on \( \hat{R}_i^{t} \), \( t \) is a free name representing a piece of data, and \( C[P_f]^{\text{out}(c_{\text{out}})} := \nu c_{\text{out}}.C[P_f] \langle \text{lin}(c_{\text{out}}, x) \rangle \).

The process \( \hat{R}_i^{(\Psi, \Phi, c_{\text{out}}, c_{\text{in}})} \{\text{id}/i_{d_i}, t/\tau\} \) models the behaviour of the collaborating target user. The behaviour of the adversary in the collaboration is implicitly modelled as \( Q \) in the context \( C[.] := \nu c_{\text{out}}.\nu c_{\text{in}}.\langle \cdot | Q \rangle \). Thus a specific collaboration is modelled as \( C[\hat{R}_i^{(\Psi, \Phi, c_{\text{out}}, c_{\text{in}})}] \{\text{id}/i_{d_i}, t/\tau\}] \). Note that sometimes the target data in the collaboration is not decided by \( \{t/\tau\} \), but by the context \( C[.] \). The target data is actually instantiated by \( C_{P_w}[C[\hat{R}_i^{(\Psi, \Phi, c_{\text{out}}, c_{\text{in}})}] \{\text{id}/i_{d_i}, t/\tau\}] \) \( \approx_{\ell} C_{P_w}[\hat{R}_i^{(\Psi, \Phi, c_{\text{out}}, c_{\text{in}})}] \{\text{id}/i_{d_i}, t_2/\tau\}] \). The first equivalence shows that even if the context \( C[.] \) is able to decide the target data, the target user can still actually instantiate the target data with \( t_2 \) by executing the process \( P_f \). The second equivalence shows that the adversary cannot distinguish the target user following the collaboration in process \( \hat{R}_i^{(\Psi, \Phi, c_{\text{out}}, c_{\text{in}})} \{\text{id}/i_{d_i}, t/\tau\} \) from executing the process \( P_f \), in the context of the adversary collaboration \( C[.] \).

### 6.3.2 Independency-of-privacy

Next, we account for attacking third parties. Based on data-privacy, we define independency-of-privacy to capture privacy when a set of third parties collaborate with the adversary. As different sets of third parties may differently influence the target user’s privacy, and since different collaboration amongst the same third parties leads to different privacy properties, independency-of-privacy is defined with respect to a set of third parties and a collaboration specification between them and the adversary.

**Definition 6.12 (third parties).** Given a well-formed protocol \( P_w \) and an instance of the target user \( \hat{R}_i \{\text{id}/i_{d_i}, t/\tau\} \), a set of third parties is defined as a set of users \( R_U := R_u_1 | \cdots | R_u_m \) where \( \forall i \in \{1, \cdots, m\}, R_u_i \neq \hat{R}_i \{\text{id}/i_{d_i}, t/\tau\} \). We use \( R_T \) to denote a set of attacking third parties and \( R_D \) to denote a set of defending third parties.

The collaboration between a set of attacking third parties \( R_T \) and the adversary is expressed as a collaboration specification \( \langle \Psi^t, \Phi^t, c^t_{\text{out}}, c^t_{\text{in}} \rangle \) defined on process \( R_T \).
The behaviour of the third parties in the collaboration is modelled as $R_T^{(\psi^t, \phi^t, c^t_{\text{out}}, c^t_{\text{in}})}$. Inspired by the domain-specific formal definitions, vote-independence [DLL11] in e-voting and independency-of-prescribing-privacy [ESORICS12] in e-health, the generic property independency-of-privacy is defined as follows: a well-formed protocol $P_w$ satisfies independency-of-privacy w.r.t. $(R_T, \langle \psi^t, \phi^t, c^t_{\text{out}}, c^t_{\text{in}} \rangle)$ and $\tau \in \text{bn}(R_t)$ if the adversary cannot distinguish the honest target user executing role $R_t$ with $\tau = t_1$ from the same user with $\tau = t_2$, even when the set of third parties $R_T$ collaborates with the adversary according to collaboration specification $\langle \psi^t, \phi^t, c^t_{\text{out}}, c^t_{\text{in}} \rangle$.

**Definition 6.13 (ipriv).** A well-formed protocol $P_w$ satisfies independency-of-privacy (ipriv) w.r.t. data $\tau$ and attacking third parties $(R_T, \langle \psi^t, \phi^t, c^t_{\text{out}}, c^t_{\text{in}} \rangle)$ if

$$C_{P_w}[\tilde{R}_t\{id/id_1, t_1/\tau\} | R_T^{(\psi^t, \phi^t, c^t_{\text{out}}, c^t_{\text{in}})}] \approx_{\ell} C_{P_w}[\tilde{R}_t\{id/id_1, t_2/\tau\} | R_T^{(\psi^t, \phi^t, c^t_{\text{out}}, c^t_{\text{in}})}],$$

where $\langle \psi^t, \phi^t, c^t_{\text{out}}, c^t_{\text{in}} \rangle$ is a collaboration specification of process $R_T$.

If the equivalence holds, then despite this collaboration, adversary cannot distinguish $\tilde{R}_t\{id/id_1, t_1/\tau\}$ in which the target user uses $\tau = t_1$ from $\tilde{R}_t\{id/id_1, t_2/\tau\}$ in which the target user uses $\tau = t_2$.

### 6.3.3 Independency-of-enforced-privacy

We define independency-of-enforced-privacy (iepriv) based on epriv in a similar fashion as ipriv. More precisely, iepriv of a protocol $P_w$ is defined w.r.t. target data $\tau \in \text{bn}(R_t)$, a collaboration specification $\langle \psi, \phi, c_{\text{out}}, c_{\text{in}} \rangle$ defined on process $\tilde{R}_t$ with $R_t := \nu d_1, \nu t_1, \tilde{R}_t$, and a set of attacking third parties together with a collaboration specification defined on the third parties processes $(R_T, \langle \psi^t, \phi^t, c^t_{\text{out}}, c^t_{\text{in}} \rangle)$. A well-formed protocol $P_w$ satisfies iepriv w.r.t. $\tau, \langle \psi, \phi, c_{\text{out}}, c_{\text{in}} \rangle$, $(R_T, \langle \psi^t, \phi^t, c^t_{\text{out}}, c^t_{\text{in}} \rangle)$, if there exists a closed plain process $P_f$ for the target user to execute, such that, despite the help of third parties $R_T$ according to $\langle \psi^t, \phi^t, c^t_{\text{out}}, c^t_{\text{in}} \rangle$, the adversary cannot distinguish between the target user collaborating with $\tau = t_1$, and him really using $\tau = t_2$ but faking collaboration for $\tau = t_1$ by $P_f$.

**Definition 6.14 (iepriv).** A well-formed protocol $P_w$ satisfies independency-of-enforced-privacy (iepriv) w.r.t. $\tau, \langle \psi, \phi, c_{\text{out}}, c_{\text{in}} \rangle$, and $(R_T, \langle \psi^t, \phi^t, c^t_{\text{out}}, c^t_{\text{in}} \rangle)$, if there exists a closed plain process $P_f$, s.t. for any $C[..] := \nu c_{\text{out}}, \nu c_{\text{in}}, (..Q)$ satisfying $\text{bn}(P_w) \cap \text{fn}(C[..]) = \emptyset$ and $C_{P_w}[C[\tilde{R}_t\{id/id_1, t_1/\tau\}] | R_T] \approx_{\ell} C_{P_w}[C[P_f]\{id/id_1, \tau/\} | R_T]$, we have

1. $C[P_f]\{id/id_1, t_2/\tau\} \approx_{\ell} \tilde{R}_t\{id/id_1, t_2/\tau\},$
2. $C_{P_w}[C[\tilde{R}_t\{id/id_1, t_1/\tau\}] | R_T^{(\psi^t, \phi^t, c^t_{\text{out}}, c^t_{\text{in}})}] \approx_{\ell} C_{P_w}[C[P_f] | R_T^{(\psi^t, \phi^t, c^t_{\text{out}}, c^t_{\text{in}})}],$

where $\langle \psi, \phi, c_{\text{out}}, c_{\text{in}} \rangle$ is a collaboration specification for $\tilde{R}_t$, and $\langle \psi^t, \phi^t, c^t_{\text{out}}, c^t_{\text{in}} \rangle$ is a collaboration specification of process $R_T$.

This formalisation adds the collaboration of third parties $R_T^{(\psi^t, \phi^t, c^t_{\text{out}}, c^t_{\text{in}})}$ to Definition 6.11.
6.3 Formalising the privacy notions

6.3.4 Coalition privacy properties

Corresponding to each privacy property defined above, we define coalition privacy properties which take into account defending third parties.

**Definition 6.15** (defensive coalition). Given an instance of the target user \( \hat{R}_i \{ \text{id/id, t/} \} \), a set of defending third parties \( R_D \), and a coalition specification \( \langle \Theta, \Delta, \Pi \rangle \) defined on \( \hat{R}_i \{ \text{id/id, t/} \} | R_D \), the coalition is modelled as \( \nu \Omega. (\hat{R}_i \{ \text{id/id, t/} \} | R_D)^{(\Theta, \Delta, \Pi)} \), where \( \Omega = \{ c | \langle R_{u_1}, R_{u_2}, M, c, y \rangle \in \Theta \} \).

The target user’s behaviour in the coalition is \( \hat{R}_i \{ \text{id/id, t/} \}^{(\Theta, \Delta, \Pi)} = \nu \eta. ((\hat{R}_i \{ \text{id/id, t/} \})^{(\Gamma, \Delta, \Pi)} | P_\gamma) \), where \( \eta \) is a set of fresh channels \( \{ c'_1, \ldots, c'_\nu \} \), \( \Gamma = \{ \{ \hat{R}_i \{ \text{id/id, t/} \}, R_{u_1}, M, c, y \} | \{ \hat{R}_i \{ \text{id/id, t/} \}, R_{u_1}, M, c, y \} \in \Theta \} \), and \( P_\gamma := \text{in}(c_1, y'_1) : \text{out}(c'_1, y'_1) \mid \cdots \mid \text{in}(c_\nu, y'_\nu) : \text{out}(c'_\nu, y'_\nu) \) with \( \{ y'_1, \ldots, y'_\nu \} \) being fresh variables, \( \{ c_1, \ldots, c_\nu \} = \{ c | \langle R_{u_1}, R_i \{ \text{id/id, t/} \}, M, c, y \rangle \in \Theta \} \). The third parties’ behaviour in the coalition is \( R_D^{(\Theta, \Delta, \Pi)} \).

**Coalition-privacy** Intuitively, coalition-privacy means that a target user’s privacy is preserved due to the cooperation of a set of defending third parties. A well-formed protocol \( P_w \) satisfies coalition-privacy w.r.t. data \( \tau \in \text{bn}(R_i) \) and \((R_D, \langle \Theta, \Delta, \Pi \rangle)\), where \( \langle \Theta, \Delta, \Pi \rangle \) is defined on \( \hat{R}_i \) | \( R_D \) and \( R_i := \nu \text{id/} \), if the adversary cannot distinguish an honest user in role \( \hat{R}_i \) using \( \tau \) from the user actually using \( \tau = t_1 \) while helped by a set of defending third parties.

**Definition 6.16** (cpriv). A well-formed protocol \( P_w \) satisfies coalition-privacy (cpriv) w.r.t. data \( \tau \) and coalition \((R_D, \langle \Theta, \Delta, \Pi \rangle)\) if

\[
C_{P_w} [\hat{R}_i \{ \text{id/id}, t_1/\tau \} | R_D] \approx \nu \Omega. (\hat{R}_i \{ \text{id/id}, t_2/\tau \} | R_D)^{(\Theta, \Delta, \Pi)}
\]

where \( \langle \Theta, \Delta, \Pi \rangle \) is a coalition specification defined on \( \hat{R}_i \{ \text{id/id}, t_2/\tau \} | R_D \).

In the definition, the coalition is modelled as \( \nu \Omega. (\hat{R}_i \{ \text{id/id}, t_2/\tau \} | R_D)^{(\Theta, \Delta, \Pi)} \), where the target user instantiates the target data with \( t_2 \). The equivalence shows that the adversary cannot distinguish the target user instantiating the target data with \( t_2 \) in the coalition from the target user instantiating the target data with \( t_1 \).

Thus, coalition-privacy captures privacy when there exists a set of third parties cooperating with the target user following a pre-defined coalition specification.

**Coalition-enforced-privacy** Taking into account defending third parties, we define coalition-enforced-privacy based on enforced-privacy. As before, coalition-enforced-privacy specifies a target data \( \tau \) and a collaboration specification of the target user \( \langle \Psi, \Phi, c_{\text{out}}, c_{\text{in}} \rangle \). Similar as in coalition-privacy, coalition-enforced-privacy specifies a set of defending third parties \( R_D \) and a coalition specification \( \langle \Theta, \Delta, \Pi \rangle \) as well. In coalition-enforced-privacy, the target user both cooperates with the adversary and defending third parties. Similar to enforced-privacy, we assume that the target user lies to the adversary if it is possible. We do not assume that the target user lies to the defending third parties, as they help the target user maintain privacy.

Intuitively, coalition-enforced-privacy means that a target user is able to lie to the adversary about his target data when helped by defending third parties – the adversary cannot tell whether the user lied. This property is modelled as
the combination of coalition-privacy and enforced-privacy: a protocol $P_w$ satisfies coalition-enforced-privacy w.r.t. $\tau \in \text{bn}(R_i)$, $\langle \Psi, \Phi, c_{\text{out}}, c_{\text{in}} \rangle$ and $(R_D, \langle \Theta, \Delta, \Pi \rangle)$, for $\langle \Psi, \Phi, c_{\text{out}}, c_{\text{in}} \rangle$ a collaboration specification defined on $\hat{R}_i$ ($\hat{R}_i := \nu \text{id}_i, \nu \tau, \hat{R}_i$), and $\langle \Theta, \Delta, \Pi \rangle$ a coalition specification defined on the target user and $R_D$, if there exists a process $P_f$, such that the adversary cannot distinguish between genuine collaboration with $\tau = t_1$ and faking collaboration using $P_f$ with the help of the coalition for $\tau = t_2$.

**Definition 6.17 (cepriv).** A well-formed protocol $P_w$ satisfies coalition-enforced-privacy (cepriv) w.r.t. data $\tau$, $\langle \Psi, \Phi, c_{\text{out}}, c_{\text{in}} \rangle$ and $(R_D, \langle \Theta, \Delta, \Pi \rangle)$, if there exists a closed plain process $P_f$, such that for any $C[\_] := \nu c_{\text{out}}, \nu c_{\text{in}}, (\_ \mid Q)$ satisfying $\text{bn}(P_w) \cap \text{fn}(C[\_]) = \emptyset$ and $C_{P_w}[C[R_i^{\langle \Psi, \Phi, c_{\text{out}}, c_{\text{in}} \rangle, \{\text{id}/id_i, t/\tau\} \mid R_D] \approx_{\ell} C_{P_w}[R_i^{\langle \Psi, \Phi, c_{\text{out}}, c_{\text{in}} \rangle, \{\text{id}/id_i, t_1/\tau\} \mid R_D)$, we have

1. $\nu \Omega.(\nu \eta.(C[P_f]^{\text{out}(c_{\text{out}})}) | P_f) \mid R_D^{\langle \Theta, \Delta, \Pi \rangle} = \approx_{\ell} \nu \Omega.(\hat{R}_i^{\text{id}/id_i, t_2/\tau} | P_f) \mid R_D^{\langle \Theta, \Delta, \Pi \rangle}$,
2. $C_{P_w}[C[R_i^{\langle \Psi, \Phi, c_{\text{out}}, c_{\text{in}} \rangle, \{\text{id}/id_i, t_1/\tau\} \mid R_D] \approx_{\ell} C_{P_w}[\nu \Omega.(C[P_f] | P_f) \mid R_D^{\langle \Theta, \Delta, \Pi \rangle}]$,

where $\Omega, \eta, P_f$ are defined in Definition 6.13 $\langle \Psi, \Phi, c_{\text{out}}, c_{\text{in}} \rangle$ is defined on $\hat{R}_i$, $\langle \Theta, \Delta, \Pi \rangle$ is a coalition specification defined on $\hat{R}_i^{\text{id}/id_i, t_2/\tau} \mid R_D$.

The collaboration between the target user and the adversary instantiating the target data with $t_1$ is modelled by the equivalence $C_{P_w}[C[R_i^{\langle \Psi, \Phi, c_{\text{out}}, c_{\text{in}} \rangle, \{\text{id}/id_i, t/\tau\} \mid R_D] \approx_{\ell} C_{P_w}[R_i^{\langle \Psi, \Phi, c_{\text{out}}, c_{\text{in}} \rangle, \{\text{id}/id_i, t_1/\tau\} \mid R_D]$. The target user’s actual behaviour of instantiating the target data with $t_2$ in process $P_f$ is modelled as the first equivalence. The second equivalence shows that the adversary cannot distinguish the process in which the target user follows the collaboration with the adversary from the process in which the target user lies to the adversary with the help of defending third parties.

**Coalition-independency-of-privacy** Similarly, we define coalition-independency-of-privacy with respect to a target data $\tau$, a set of attacking third parties with a collaboration specification $(R_T, \langle \Psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t \rangle)$, and a set of defending third parties $R_D$ with a coalition specification $(\Theta, \Delta, \Pi)$. Note that we require that there is no intersection between attacking third parties and defending third parties, i.e., $R_T \cap R_D = \emptyset$, as we assume a third party cannot be both attacking and defending at the same time. A well-formed protocol $P_w$ satisfies coalition-independency-of-privacy w.r.t. $\tau$, $(R_T, \langle \Psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t \rangle)$ and $(R_D, \langle \Theta, \Delta, \Pi \rangle)$, if the adversary, even with the collaboration of a set of attacking third parties, cannot distinguish the target user instantiating $\tau = t_1$ from the target user actually instantiating $\tau = t_2$ in the coalition with the help of defending third parties.

**Definition 6.18 (cipriv).** A well-formed protocol $P_w$ satisfies coalition-independency-of-privacy (cipriv) w.r.t. data $\tau$, $(R_T, \langle \Psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t \rangle)$, and $(R_D, \langle \Theta, \Delta, \Pi \rangle)$, if

$$C_{P_w}[R_i^{\text{id}/id_i, t_1/\tau} \mid R_D] \approx_{\ell} C_{P_w}[R_T^{\langle \Psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t \rangle}] \
\approx_{\ell} C_{P_w}[\nu \Omega.(\hat{R}_i^{\text{id}/id_i, t_2/\tau} | R_D^{\langle \Theta, \Delta, \Pi \rangle}) \mid R_T^{\langle \Psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t \rangle}]$$

where $\langle \Psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t \rangle$ is a collaboration specification of process $R_T$, $\langle \Theta, \Delta, \Pi \rangle$ is a coalition specification defined on $\hat{R}_i^{\text{id}/id_i, t_2/\tau} \mid R_D$. 

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Coalition-independency-of-enforced-privacy

Finally, we consider the case combining all situations together: the target user collaborates with the adversary following \( \langle \Psi', \Phi', c_{out}', c_{in}' \rangle \), a set of attacking third parties \( R_T \) collaborate with the adversary following \( \langle \Psi^t, \Phi^t, c_{out}^t, c_{in}^t \rangle \), and a set of defending third parties \( R_D \) and a coalition \( \langle \Theta, \Delta, II \rangle \). We formally define coalition-independency-of-enforced-privacy below.

**Definition 6.19** (ciepriv). A well-formed protocol \( P_w \) satisfies coalition-independency-of-enforced-privacy (ciepriv) w.r.t. \( \tau \), \( \langle \Psi, \Phi, c_{out}, c_{in} \rangle \), \( (R_T, \langle \Psi^t, \Phi^t, c_{out}^t, c_{in}^t \rangle) \) and \( (R_D, \langle \Theta, \Delta, II \rangle) \), if there exists a closed plain process \( P_f \) such that for any context \( C[. ] \) := \( \nu c_{out}. \nu c_{in}. (\tau \mid Q) \) satisfying \( bn(P_w) \cap fn(C[. ]) = \emptyset \) and \( C_{Pw} [C[R_T^t, \Psi^t, c_{out}^t] \{\nu \mid id_1, t_2 \tau\}] \mid R_T \mid R_D] \approx_t C_{Pw} [R_T^t, \Psi^t, c_{out}^t] \{\nu \mid id_1, t_1 \tau\}] \mid R_T \mid R_D] \), we have

1. \( \nu \Omega. (\nu \eta. C[P_f]^{out(c'_{out'})} \mid P_f \gamma) \mid R_D^{\langle \Theta, \Delta, II \rangle}) \approx_t \nu \Omega. ((R_T^{\langle \nu \mid id_1, t_2 \tau\} \mid R_D^{\langle \Theta, \Delta, II \rangle}),
2. \( \nu \Omega. (C[P_f]^{\langle \Theta, \Delta, II \rangle \mid \nu \eta. (C[P_f] | P_f \gamma) \mid R_D^{\langle \Theta, \Delta, II \rangle}), \)

where \( \Omega, \gamma, P_f \) are defined in Definition 6.13. \( \langle \Psi, \Phi, c_{out}, c_{in} \rangle \) is a collaboration specification defined on \( R_T \), \( \langle \Psi^t, \Phi^t, c_{out}^t, c_{in}^t \rangle \) is a collaboration specification defined on \( R_T \), \( \langle \Theta, \Delta, II \rangle \) is a collaboration specification defined on \( R_D \).

As certain coalitions may fail to maintain privacy, the coalition privacy properties can be generalised by requiring the existence of a successful coalition. The general version of coalition privacy properties allow us to reason about the existence of a coalition (a strategy) such that a user’s privacy is preserved. How to find such a coalition is an interesting topic for studying coalition privacy properties. Each property defined in the above can be instantiated in many different forms by specifying the parameters of the property (such as target data, collaboration, coalition). Furthermore, only the target user is allowed to lie to the adversary – we do not consider lying third parties. Properties, ipriv, iepriv, cpriv and ciepriv, can be extended by allowing third parties to lie.

### 6.3.5 An example of coalition privacy

We modelled a system in Example 6.2 and defined a coalition \( \langle \Psi, \Phi, c_{out}, c_{in} \rangle \) of two users in the system in Example 6.10. According to Definition 6.16, the system does not satisfy cpriv w.r.t. target data req and coalition \( \langle \Theta, \Delta, II \rangle \). The reason is that without coalition, all the requests the adversary observed are different; with the coalition, the adversary can detect that two requests are sent twice.

We show that the system in Example 6.2 may satisfy cpriv with respect to a different coalition. For instance, the following coalition specification in which the two users swap their requests.

\[ \langle \Theta', \Delta', II' \rangle = (\emptyset, \{(id_1, req_1)/\text{id}_2, \text{req}_1\}, \{(id_2, req_1)/\text{id}_1, \text{req}_1\}, \emptyset) \]
According to Definition 6.15, the coalition process is as follows.
\[ \nu \Omega.(R_{U_i} \mid R_{U_j})^{(\Theta, \Delta, \Pi)} \doteq \left( \nu \text{id}_i.(\nu \text{req}_i.\text{out}(c_p, (\text{id}_i, \text{req}_i))) \mid \neg(\nu \text{req}_i.\text{out}(c_p, (\text{id}_i, \text{req}_i))) \right) \mid \left( \nu \text{id}_j.(\nu \text{req}_j.\text{out}(c_p, (\text{id}_j, \text{req}_j))) \mid \neg(\nu \text{req}_j.\text{out}(c_p, (\text{id}_j, \text{req}_j))) \right). \]

Since the adversary cannot detect who sent which request, a user’s request is anonymous. This coalition prevents the situation, in which one only user is involved and thus his request is known by the adversary.

Now consider the following collaboration specification in which the user \( U_i \) needs to forward the private message to the adversary.

\[ (\Psi, \Phi, c_{out}, c_{in}) = \{\{(\text{id}_i, \text{req}_i)\}, \emptyset, c_{out}, \ell\}. \]

The collaboration behaviour of \( U_i \) is modelled as follows:
\[ R_{U_i}^{(\Psi, \Phi, c_{out}, c_{in})} \doteq \nu \text{id}_i.(\nu \text{req}_i.\text{out}(c_p, (\text{id}_i, \text{req}_i))).\text{out}(c_p, (\text{id}_i, \text{req}_i)) \mid \neg(\nu \text{req}_i.\text{out}(c_p, (\text{id}_i, \text{req}_i))) \]

The system does not satisfy \( c_{priv} \) w.r.t. \( \text{req} \), collaboration \( (\Psi, \Phi, c_{out}, c_{in}) \) and coalition \( (R_{U_j}, (\Theta, \Delta, \Pi)) \), since the system does not satisfies \( c_{priv} \) w.r.t. \( \text{req} \) and coalition \( (R_{U_j}, (\Theta, \Delta, \Pi)) \). However, the system satisfies \( c_{priv} \) w.r.t. \( \text{req} \), collaboration \( (\Psi, \Phi, c_{out}, c_{in}) \) and coalition \( (R_{U_i}, (\Theta', \Delta', \Pi')) \). Because there exists a process
\[ P_f := (\nu \text{id}_i.(\nu \text{req}_i.\text{out}(c_p, (\text{id}_i, \text{req}_i))).\text{out}(c_p, (\text{id}_i, \text{req}_i)) \mid \neg(\nu \text{req}_i.\text{out}(c_p, (\text{id}_i, \text{req}_i))) \]

such that the two equivalence in Definition 6.17 are satisfied. That is, context \( C[\_] \) satisfies
\[ C_{ex}[C[R_{U_i}^{(\Psi, \Phi, c_{out}, c_{in})}] \mid R_{U_j}] \approx_{\ell} C_{ex}[R_{U_i}^{(\Psi, \Phi, c_{out}', c_{in})}] \mid R_{U_j}], \]

Since \( U_i \) does not read information from the adversary, we have the context being empty and
\[ R_{U_i}^{(\Psi, \Phi, c_{out}', c_{in})} := R_{U_i}^{(\Psi, \Phi, c_{out}, c_{in})} \]

with \( c_{out} = c_{out}' \).

Accordingly, \( \nu \Omega.(\nu \eta.(C[P_f]^{\text{out}(c_{out}')}) \mid P_\gamma) \mid R_{D}^{(\Theta, \Delta, \Pi)} \doteq (\nu \text{id}_i.(\nu \text{req}_i.\text{out}(c_p, (\text{id}_i, \text{req}_i))).\text{out}(c_p, (\text{id}_i, \text{req}_i)) \mid \neg(\nu \text{req}_i.\text{out}(c_p, (\text{id}_i, \text{req}_i))) \mid (\nu \text{id}_j.(\nu \text{req}_j.\text{out}(c_p, (\text{id}_j, \text{req}_j)))).\text{out}(c_p, (\text{id}_j, \text{req}_j)) \mid \neg(\nu \text{req}_j.\text{out}(c_p, (\text{id}_j, \text{req}_j))) \)

Since \( (\nu \text{req}_i.\text{out}(c_p, (\text{id}_i, \text{req}_i))).\text{out}(c_p, (\text{id}_i, \text{req}_i)) \mid \neg(\nu \text{req}_i.\text{out}(c_p, (\text{id}_i, \text{req}_i))) \) works the same as \( (\nu \text{req}_i.\text{out}(c_p, (\text{id}_i, \text{req}_i))).\text{out}(c_p, (\text{id}_i, \text{req}_i)) \), we have the first equivalence,
\[ \nu \Omega.(\nu \eta.(C[P_f]^{\text{out}(c_{out}')}) \mid P_\gamma) \mid R_{D}^{(\Theta, \Delta, \Pi)} \approx_{\ell} \nu \Omega.(R_{U_i} \mid R_{U_j})^{(\Theta, \Delta, \Pi)}. \]

Similarly, since
\[ R_{U_i}^{(\Psi, \Phi, c_{out}, c_{in})} \mid R_{U_j} \approx_{\ell} P_f \mid R_{U_i}^{(\Theta, \Delta, \Pi)} \]

we have the second equivalence,
\[ C_{ex}[R_{U_i}^{(\Psi, \Phi, c_{out}, c_{in})}] \mid R_{U_j} \approx_{\ell} C_{ex}[\nu \Omega.(\nu \eta.(C[P_f] \mid P_\gamma) \mid R_{U_j}^{(\Theta, \Delta, \Pi)}]. \]
6.4 Relations between the privacy notions

We show the relations between the privacy properties in Figure 6.1: we use $\rho$ to denote the specification of a target user’s collaboration with the adversary $\langle \Psi, \Phi, c_{out}, c_{in} \rangle$, $\theta$ to denote the specification of a set of attacking third parties and their collaboration with the adversary $\langle R_T, \langle \Psi^t, \Phi^t, c_{out}^t, c_{in}^t \rangle \rangle$, and $\delta$ to denote the specification of a set of defending third parties and their coalition with the target user $\langle R_D, \langle \Theta, \Delta, \Pi \rangle \rangle$.

The left diamond in Figure 6.1 shows the relations between privacy properties which do not consider defending third parties while the right diamond shows the relations between privacy properties which consider defending third parties. In the left diamond, $epriv_\rho$ and $ipriv_\theta$ are stronger than priv, meaning that if a protocol satisfies $epriv_\rho$ or $ipriv_\theta$, then the protocol satisfies priv. Intuitively, if the adversary cannot break privacy with the help from the target user (in $epriv_\rho$) or from a set of attacking third parties (in $ipriv_\theta$), the adversary cannot break privacy without any help (in priv). Similarly, if the adversary cannot break privacy with the help from both target user and attacking third parties (in $iepriv_\rho, \theta$), the adversary cannot break privacy with the help from only one of them (in $epriv_\rho$ and $ipriv_\theta$). Thus, $iepriv_\rho, \theta$ is stronger than both enforced-privacy $\rho$ and $ipriv_\theta$. This is described as Theorem 6.20.

Theorem 6.20. (1) $\forall \theta$, $iepriv_\rho, \theta \implies epriv_\rho$, (2) $\forall \rho$, $iepriv_\rho, \theta \implies ipriv_\theta$, (3) $\forall \rho$, $epriv_\rho \implies priv$, and (4) $\forall \theta$, $ipriv_\theta \implies priv$.

Proof sketch: The proof of $\forall \rho$, $iepriv_\rho, \theta \implies ipriv_\theta$ and $\forall \rho$, $epriv_\rho \implies priv$ follows the strategy of how to prove coercion-resistance $\implies$ receipt-freeness $\implies$ vote-privacy given by Delaune et al. [DKR09]. For all $\rho$, when a protocol satisfies $epriv_\rho$, for an adversary context $C[.]$, three equivalences in Definition 6.11 hold. From the equivalences, we can deduce that $C_{P_{\rho}}[\hat{R}_i(\hat{\Psi}, \hat{\Phi}, c_{out}^i) \{id/id^i, t_1/\tau \}] \approx \ell C_{P_{\rho}}[C[P]]$. By applying the evaluation context $\nu c_{out}^i(\cdot, \cdot)(c_{out}^i, x)$ on both side of the equivalence, we prove that $C_{P_{\rho}}[\hat{R}_i(id/id^i, t_1/\tau)] \approx \ell C_{P_{\rho}}[C[P]]^{out(c_{out}^i)}$. Because of the first equivalence in Definition 6.11 $C[P]$ \^{out(c_{out}^i)} $\approx \ell \hat{R}_i(id/id^i, t_2/\tau)$, we deduce
the equivalence $C_{P_a}[\hat{R}, \{id/\text{id}_a, t_1/\tau\}] \approx \epsilon C_{P_a}[\hat{R}, \{id/\text{id}_a, t_2/\tau\}]$. This coincides with the equivalence in Definition 6.3. Thus we prove that $\forall \rho$, $\epsilon \text{priv}_{\rho, \theta} \Rightarrow \text{priv}$.

Similarly we prove $\forall \rho$, $\epsilon \text{ipriv}_{\rho, \theta} \Rightarrow \text{ipriv}_\theta$. $\forall \theta$, $\epsilon \text{ipriv}_{\rho, \theta} \Rightarrow \text{priv}$ can be proved as follows: for an adversary context $C[\_] := \nu c_{\text{out}} \cdot \nu c_{\text{in}} \cdot (\_ | Q)$ satisfying $\text{bn}(P_w) \cap \text{fn}(C[\_]) = \emptyset \land C_{P_a}[R_T^{(\Psi', \Phi^{\theta'}, \epsilon_{\text{out}}, \epsilon_{\text{in}}^{'})}] \approx \epsilon C_{P_a}[R_T^{(\Psi', \Phi^{\theta'}, \epsilon_{\text{out}}, \epsilon_{\text{in}}^{'})}]$, we show that $\epsilon \text{ipriv}_{\theta} \Rightarrow \text{priv}$. By applying $C[\_]$ and the evaluation context $\nu c_{\text{out}} \cdot (\_ | \text{lin}(c_{\text{out}} \cdot x))$ on both side of the equivalence in Definition 6.13 we have $C_{P_a}[\hat{R}, \{id/\text{id}_a, t_1/\tau\} | R_T] \approx \epsilon C_{P_a}[\hat{R}, \{id/\text{id}_a, t_2/\tau\} | R_T]$. By applying rule $P = P | P$, the third parties’ behaviour $R_T$ is absorbed by the environment. Thus, the equivalence in Definition 6.3 is satisfied. Similarly reasoning holds for proving $\forall \theta$, $\epsilon \text{priv}_{\rho, \theta} \Rightarrow \epsilon \text{priv}_\rho$. Precise proofs are available in Appendix [B.2].

Moreover, the implication relations in Theorem 6.20 are uni-directional, in the sense that we can disprove the opposite directions by presenting counter-examples (see details in Appendix [B.2]). We can apply the same technique to prove the relations in the right diamond. Thus we have the following theorem. Precise proofs are available in Appendix [B.3].

**Theorem 6.21.** (1) $\forall \theta$, $\epsilon \text{ciepriv}_{\rho, \theta, \delta} \Rightarrow \epsilon \text{cepriv}_{\rho, \delta}$, (2) $\forall \rho$, $\epsilon \text{ciepriv}_{\rho, \theta, \delta} \Rightarrow \epsilon \text{ciepriv}_{\theta, \delta}$, (3) $\forall \rho$, $\epsilon \text{cepriv}_{\rho, \theta} \Rightarrow \epsilon \text{cpriv}_\delta$, and (4) $\forall \theta$, $\epsilon \text{cpriv}_{\theta, \delta} \Rightarrow \epsilon \text{cpriv}_\delta$.

Each privacy property in the left diamond has a weaker corresponding property in the right diamond, meaning that if a protocol satisfies a privacy property in the left diamond, there exists a coalition such that the property satisfies the corresponding coalition privacy property in the right diamond. Intuitively, if a protocol preserves privacy of a target user without any help from third parties, the protocol can still preserve his privacy with the help from others.

**Theorem 6.22.** (1) $\epsilon \text{priv}_{\rho, \theta} \Rightarrow \exists \delta$, $\epsilon \text{ciepriv}_{\rho, \theta, \delta}$, (2) $\epsilon \text{priv}_{\rho} \Rightarrow \exists \delta, \epsilon \text{cepriv}_{\rho, \delta}$, (3) $\epsilon \text{ipriv}_\theta \Rightarrow \exists \delta$, $\epsilon \text{cpriv}_{\theta, \delta}$, and (4) $\epsilon \text{priv} \Rightarrow \exists \delta$, $\epsilon \text{cpriv}_\delta$.

**Proof sketch:** When a protocol satisfies $\epsilon \text{priv}$, the equivalence in Definition 6.3 holds. It is easy to see that the equivalence in Definition 6.3 coincides with the one in Definition 6.16 when the coalition is set empty. The same reasoning holds for proving other relations in the theorem. Precise proofs are available in Appendix [B.4].

Generally, given a set of defending third parties $R_D$, when a protocol satisfies $\epsilon \text{priv}$, the requirement that the protocol also satisfies $\epsilon \text{cpriv}_\delta$ is the following equivalence: $\nu \Omega.(\hat{R}, \{id/\text{id}_a, t_2/\tau\} | R_D) \approx \epsilon (\hat{R}, \{id/\text{id}_a, t_2/\tau\} | R_D)$. When the coalition is of the form $\langle \theta, \emptyset, \emptyset \rangle$, this requirement is satisfied. However, not all coalition specifications defined on $R_D$ can satisfy the requirement. Therefore, even when a protocol satisfies $\epsilon \text{priv}$, some coalition specification may fail to satisfy $\epsilon \text{cpriv}_\delta$. The observation holds for other relations in Theorem 6.22 as well. Note that the requirement ‘$\exists \delta$’ makes the coalition privacy properties in Theorem 6.22 coincide with their general extensions as discussed previously in Section 6.3.4.
6.5 Application

Privacy notions modelled as strong secrecy can be captured by data-privacy. For instance, doctor anonymity (see Definition 5.5) is data-privacy where the target data is a user’s identity. Various domain-specific properties, which capture privacy in domains where data-privacy is too strong to be satisfied, can be instantiated by coalition-privacy. For instance,

- strong bidding-price-secrecy for non-winning bidders (see Definition 4.1) in sealed-bid e-auctions is defined as the adversary cannot determine a bidder’s bidding-price, assuming the existence of a winning bid. This can be instantiated as coalition-privacy where the target data is a bid \( p_b \), the defending third party is the winning bidder \( P_B \{ d/Id \} \) and the coalition specification is \( \langle \emptyset, \emptyset, \emptyset \rangle \).

- Prescribing-privacy (see Definition 5.1) is defined as the adversary cannot determine a doctor’s prescription with the existence of a counter-balancing doctor. This can be instantiated as coalition-privacy where the target data is a prescription \( \text{presc} \), the defending third party is the counter-balancing doctor \( \{ d/Id \} \) and the coalition specification is \( \langle \emptyset, \Delta, \emptyset \rangle \) where the substitution \( \Delta \) specifies how to replace the counter-balancing doctor’s prescription \( \{ p_A / p_B \} \).

- Vote-privacy (see Definition 3.14) is defined as the adversary cannot determine a voter’s vote with the existence of a counter-balancing voter. This can be instantiated as coalition-privacy where the target data is a vote \( \text{vote} \), the defending third party is the counter-balancing voter \( \{ c/vote \} \) and the coalition specification is \( \langle \emptyset, \Delta, \emptyset \rangle \) where the substitution \( \Delta \) specifies how to replace the counter-balancing voter’s vote \( \{ a/c \} \).

Enforced privacy notions like receipt-freeness or coercion-resistance in e-voting can be captured by either enforced-privacy or coalition-enforced-privacy.

- Receipt-freeness in e-voting (see Definition 3.17) can be instantiated by coalition-enforced-privacy w.r.t. vote and \( \langle P_v B \{ c/vote \}, \emptyset, \emptyset, c \rangle \) (the target data and the coalition are the same as in vote-privacy), and the collaboration specification is \( \langle \Psi, \emptyset, c_{out}, c \rangle \) where \( \Psi \) contains all private terms generated and read-in in the target voter process. \( \Psi \) in a process \( R \) is given by \( \text{OutTerm}(R) \).

\[
\begin{align*}
\text{OutTerm}(0) &= \emptyset \\
\text{OutTerm}(P \mid Q) &= \text{OutTerm}(P) \cup \text{OutTerm}(Q) \\
\text{OutTerm}(\parallel P) &= \text{OutTerm}(P) \\
\text{OutTerm}(\forall n.P) &= \{ n \} \cup \text{OutTerm}(P) \quad \text{when } n \text{ is name of base type,} \\
\text{OutTerm}(\forall n.P) &= \text{OutTerm}(P) \quad \text{otherwise} \\
\text{OutTerm}(\text{in}(v, x).P) &= \{ x \} \cup \text{OutTerm}(P) \quad \text{when } n \text{ is name of base type,} \\
\text{OutTerm}(\text{in}(v, x).P) &= \text{OutTerm}(P) \quad \text{otherwise} \\
\text{OutTerm}(\text{out}(v, M).P) &= \text{OutTerm}(P) \\
\text{OutTerm}(\text{if } M =_E N \text{ then } P \text{ else } Q) &= \text{OutTerm}(P) \cup \text{OutTerm}(Q)
\end{align*}
\]
In a similar way, coercion-resistance in e-voting (see Definition 3.19) is an instance of coalition-enforced-privacy w.r.t. vote and coalition specification \((P_{vB}\{c/vote\}, \langle \emptyset, \{\{a/c\}, \emptyset\}\rangle)\) (the target data and the coalition are the same as in vote-privacy), and the cooperation specification is \(\langle \Psi, \Phi, c_{out}, c_{in} \rangle\) where \(\Psi\) contains all private terms generated and read-in in the target voter process and \(\Phi\) contains all the send out terms. \(\Phi\) in a process \(R\) is given by \(\text{ReplaceTerm}(R)\).

\[
\begin{align*}
\text{ReplaceTerm}(0) & = \emptyset \\
\text{ReplaceTerm}(P | Q) & = \text{ReplaceTerm}(P) \cup \text{ReplaceTerm}(Q) \\
\text{ReplaceTerm}(v_{\text{n}}.P) & = \text{ReplaceTerm}(P) \\
\text{ReplaceTerm}(\text{in}(v, x).P) & = \text{ReplaceTerm}(P) \\
\text{ReplaceTerm}(\text{out}(v, M).P) & = \{M\} \cup \text{ReplaceTerm}(P) \\
\text{ReplaceTerm}(\text{if } M = E N \text{ then } P \text{ else } Q) & = \text{ReplaceTerm}(P) \cup \text{ReplaceTerm}(Q)
\end{align*}
\]

The two independency of privacy properties, i.e., independency-of-prescribing-privacy and independence-vote-privacy are instances of coalition-independency-of-privacy.

\begin{itemize}
\item Independency of prescribing-privacy (see Definition 5.3) can be instantiated as coalition-independency-of-privacy w.r.t. target data \(\text{presc}\), defending third parties \(\langle \text{init}_{dr}\{d_{B}/Id_{dr}\}, \langle P_{dr}\{d_{B}/Id_{dr}\} | \text{main}_{dr}\{d_{B}/Id_{dr}, p_{B}/\text{presc}\}\rangle \rangle\), coalition specification \(\langle \emptyset, \{\{p_{A}/p_{B}\}\}, \emptyset \rangle\) with the attacking third parties \(R_{i}\) and third party collaboration \(\langle \Psi, \emptyset, c_{out}, \epsilon \rangle\) where \(\Psi\) contains all private terms generated and read-in in \(R_{i}\), i.e., \(\Psi = \text{OutTerm}(R_{i})\).
\item Vote-independence [DLL11]: A voting process respects vote-independence if for all votes \(a\) and \(c\)
\[
\begin{align*}
C_{v}[P_{vA}\{a/vote\} | P_{vB}\{c/vote\} | P_{vC}^{c_{out}, c_{in}}] \\
\approx_{\epsilon} C_{v}[P_{vA}\{a/vote\} | P_{vB}\{c/vote\} | P_{vC}^{c_{out}, c_{in}}].
\end{align*}
\]
\end{itemize}

This property can also be considered as an instance of coalition-independency-of-privacy, where the target data is and the coalition are the same as in vote-privacy, the set of attacking third parties is a third voter \(P_{vC}\), and the collaboration specification of the third voter is \(\langle \Psi, \Phi, c_{out}, c_{in} \rangle\) where \(\Psi\) are all generated \(\text{OutTerm}(P_{vC})\) and \(\Phi\) are all read-in terms \(\text{ReplaceTerm}(P_{vC})\) in the third voter process.

6.6 Conclusions

In this chapter, we considered both negative and positive influences of third parties to a target user’s privacy. Furthermore, we identified a new privacy notion: coalition privacy, where third parties help defend the target user’s privacy. In addition, we presented a formal framework which allows us to give domain-independent formalisations of the privacy notions accounting for third parties. We defined a standard form of protocols in which any protocol may be expressed.
To formally define enforced privacy properties and independency of privacy properties, we modelled collaboration between users and the adversary. A collaboration specifies precisely what information is shared and how it is shared. As such, this modelling provides the necessary flexibility for expressing various types of collaboration. To model coalition privacy properties, we proposed the notion of coalition in our framework. This notion formally expresses the behaviour of and information shared between a target user and a set of third parties.

In our framework, the basic privacy property data-privacy was formalised in a classical way as strong secrecy: equivalence of two processes where a variable is instantiated differently [Bla04]. Based on this property, we formalised enforced-privacy, independency-of-privacy and independency-of-enforced-privacy using the formalisation of collaboration. Using the formalisation of coalition, four corresponding coalition privacy properties were formalised.

Finally, we discussed the relations between the defined properties. We proved that these properties are hierarchically related, and we showed that various privacy definitions from literature are instances of properties in our hierarchy.
Conclusions and future work

As motivated in the introduction (Chapter 1), privacy is a desirable requirement in services based on the Internet. To ensure privacy against adversaries controlling the network, cryptography is widely used in protocols. However, the design and verification of cryptographic protocols are well known to be error-prone. Formal approaches have shown to be effective in addressing this problem. Therefore we argue that formalising privacy notions is a necessary step to verify privacy claims of a protocol. In particular, a strong privacy notion – enforced privacy was proposed recently to ensure privacy against bribery and coercion. In this thesis, we studied enforced privacy formally using a process algebra – the applied π calculus.

We first studied existing requirements and formalisations of enforced privacy (Chapter 2). We found that enforced privacy is required in domains such as e-voting, e-auctions and e-health. However, formalisations of enforced privacy focused on the e-voting domain. We formalised enforced privacy in other domains: e-auctions and e-health. We formalised enforced privacy in the e-auction domain in a similar way as the formalisations in the e-voting domain (Chapter 4). We performed a case study to validate our formalisations. E-auction systems are similar to e-voting systems in the sense that roles can be naturally divided into two types: participants and authorities. In contrast, e-health systems involve a far more complex constellation of roles, some of which may not be trustworthy. Hence, protecting a user’s privacy when third parties cooperating with the adversary was required in e-health. Therefore, in addition to enforced privacy, we formalised an independency of privacy property of e-health systems to capture such third parties’ cooperating behaviour. Furthermore, we verified the formalised properties of a Belgian e-health protocol which was proposed for practical use. Using the experience of formalising domain-specific enforced privacy, we proposed a general formalisation of enforced privacy in a formal framework. In addition, we took third parties’ influence into account. On the one hand, a third party can influence a target user’s privacy negatively by cooperating with the adversary to break the user’s privacy. On the other hand, a third party can influence a target user’s privacy positively by cooperating with the target user to maintain the user’s privacy. We generalised the formalisations of privacy properties taking into account negative third parties in the framework. In addition, we proposed privacy properties taking into account positive third parties and formalised the properties in the framework as well. Finally, we proved the relations between the formalised privacy properties in the framework.
Chapter 7 Conclusions and future work

7.1 Summary of contributions

In this thesis, we studied enforced privacy: privacy with respect to an adversary who can bribe or coerce users in addition to control the network. More specifically, we

1. studied enforced privacy in e-auctions (Chapter 4),
2. studied enforced privacy in e-health (Chapter 5), and
3. developed a formal verification framework in which we generalised enforced privacy and studied enforced privacy in the presence of others (Chapter 6).

Chapter 4: Enforced privacy in e-auctions

In the study of enforced privacy in e-auctions, our main contribution is that we proposed formalisations of two privacy properties of auction protocols: bidding-price-secrecy and receipt-freeness, following definitions of vote-privacy and receipt-freeness in e-voting [DKR09]. We have modelled the AS02 protocol in the applied pi calculus, verified bidding-price-secrecy of the protocol automatically, using ProVerif, and receipt-freeness of the protocol manually.

Chapter 5: Enforced privacy in e-health

The main contribution in the study of enforced privacy in e-health is that we identified three enforced privacy properties of e-health systems: enforced prescribing-privacy, independency of prescribing-privacy, independency of enforced prescribing-privacy. In addition, we are the first to provide formal definitions for them. Furthermore, we developed an in-depth applied pi model of the DLV08 e-health protocol [dDLVV08] which is rather complicated and aims for practical use in Belgium. Furthermore, we formally analysed privacy and enforced privacy properties of the protocol, as well as regular security and privacy properties. We have found ambiguities in the protocol which potentially lead to flaws on privacy, and proposed suggestions for fixing them. The formal analysis of the DLV08 protocol, together with the analysis of the AS02 protocol, provide insights on the design of protocols preserving enforced privacy.

Chapter 6: Enforced privacy in the presence of others

We generalised domain-specific enforced privacy properties of the e-voting, e-auction and e-health systems. Inspired by the requirement of independency of privacy in e-health systems, we took into account privacy properties where third parties cooperate with the adversary and generalised them as independency of privacy properties. In addition, we considered privacy properties where third parties cooperate with the target user, and proposed coalition privacy. We also formalised the privacy properties in a new formal framework and formally prove their relations. In the framework, we defined a standard form of protocols which is able to represent any protocol. To formally define enforced privacy properties and independency of privacy properties, we model collaboration between users and the adversary. The collaboration allows us to precisely specify which information is shared and how it is shared, thereby providing the necessary flexibility for modelling various types of collaboration. To model coalition privacy properties, we proposed the property of coalition in our framework to formally capture the behaviour and shared information among a target user and a set of third parties. The formal framework allows...
us to give domain-independent formalisations of the privacy properties. We formalised data-privacy, enforced-privacy, independency-of-privacy and independency-of-enforced-privacy using the formalisation of collaboration. Using the formalisation of coalition, four corresponding coalition privacy properties are formalised. Finally, we formally discussed how the formalised privacy properties are related in a privacy hierarchy. In addition, we showed that many existing formalisations are instances of properties in our hierarchy. It appears that the formalisations of enforced privacy in other domains can benefit from our framework.

7.2 Future work

Enforced privacy is still rather new compared to classical privacy properties such as anonymity and untraceability. Enforced privacy requirements have only been formally studied in a few domains, e-voting, e-auctions and e-health. The adversary's ability to bribe or coerce users does not depend on domains. As such, enforced privacy needs to be studied in other domains. For instance, in online social networks, normally a user maintains the link between the identities and pseudonyms of his friends. The following scenario that a user may be coerced to reveal the link has been identified in social networks [BMP11]. Thus, the following requirement has been identified: a user should be able to give fake associations between pseudonyms and identities of his friends to a coercer [BMP11]. Once the requirements are identified, designing systems enforcing user's privacy is needed. Development of systems providing enforced privacy will benefit from privacy-enforcing techniques used to guarantee receipt-freeness and coercion-resistance in e-voting, e-auctions and e-health, for example chameleon bit commitments, untappable channels and zero-knowledge proofs, as used in the AS02 e-auction protocol. As design of cryptographic protocols is well-known to be error-prone, a claimed enforced privacy property needs to be formally verified. To do so, formalising such a property is a necessary step. The formalisations of a claimed enforced privacy property will benefit from our formal framework.

In the two domains where enforced privacy has been studied in this thesis, e-auctions and e-health, there is still interesting research that needs to be carried out. For instance, in e-auctions, we only formalised receipt-freeness in sealed-bid e-auctions as it is required in the literature. Privacy notions due to different types of cooperation between a bribed or coerced user and the adversary have not been formalised. Thus, an interesting research direction is to formalise such privacy notions, for example, coercion-resistance, in e-auctions. In addition, the privacy notions we defined in e-auctions (bidding-price-secrecy and receipt-freeness) aim to protect privacy for non-winning bidders. Chen et al. proposed an auction protocol which can ensure the winner's privacy as well [CLK03]. It is interesting to formalise and verify privacy and enforced privacy for this protocol.

Compared to the study of enforced privacy in the e-auction and e-voting domains, where many systems providing enforced privacy have been proposed, the study of enforced privacy in the e-health domain is in its infancy. Especially, e-health systems require a strong privacy, the combination of enforced privacy and independency of privacy, for instance, the DLV08 e-health protocol requires doctor privacy even if both pharmacists and doctors cooperate with the adversary. How to en-
sure such strong privacy properties in e-health remains a challenge, for instance, improving the DLV08 protocol to satisfy the independency of enforced prescribing-privacy.

Moreover, as we have already mentioned in Chapter 6, there are some interesting research directions based on the formal framework. A coalition privacy notion formalised in the framework is always with respect to a specific coalition. A coalition between a target user and third parties may fail to maintain privacy of the target user. How to find a coalition and synthesise a strategy for the coalition to satisfy some coalition privacy properties for a protocol is an interesting research direction. In addition, in the formal framework, we did not consider that the third parties cooperating with the adversary may lie to the adversary. Taking this third parties’ lying into account will lead to new privacy notions. Thus, how to extend our privacy hierarchy to capture situations where a third party is coerced but has a strategy to lie to the adversary needs to be addressed.

Our work focuses on formalising enforced privacy notions. The verification of enforced privacy properties in our case studies benefit from verification techniques provided by the tool ProVerif. However, this tool may report false attacks, or not terminate. As we can see, equivalences in the enforced privacy formalisations are often complex. Developing verification techniques which are efficient for verifying such equivalences is an interesting direction. Furthermore, ProVerif can only help prove whether an equivalence hold, it cannot, for example, prove whether there exists a process in which a bribed or coerced user can successfully cheat the adversary. How to find such processes automatically remains a challenge.
A

Full proofs in Chapter 4

A.1 Full proof of receipt-freeness of AS02

We show the detailed proof of the equivalence eq1 in Section 4.6.2

\[
(\text{let } untapch = untapch_{\text{bA}} \text{ in let } privch = privch_{\text{bA}} \text{ in}
\]
\[
\approx_{\ell} (\text{let } p_b = c \text{ inlet } untapch = untapch_{\text{bA}} \text{ in}
\]
\[
\text{let } privch = privch_{\text{bA}} \text{ in let } ch = ch_{\text{bA}} \text{ in } P_b),
\]
where \( P_f^{\text{out(chc.)}} \equiv \nu \text{chc.}(P_f || \text{lin(chc, } x)) \), \( P_f \) is defined in Figure 4.12 and \( P_b \) is defined as in Figure 4.16.

In order to prove the equivalence, we can prove that \( P_f^{\text{out(chc.)}} \approx_{\ell} P_b\{c/p_b\} \), i.e.,

\( (\nu \text{chc.}(P_f || \text{lin(chc, } x)) \approx_{\ell} P_b\{c/p_b\} \).

Let \( P' := (\nu \text{chc.}(P_f || \text{lin(chc, } x)) \) and \( Q' := P_b\{c/p_b\} \). Process \( P' \) is shown in Figure A.1 and process \( Q' \) is shown in Figure A.3

\[
P' := \nu \text{chc.}(\]
\[
p1. \quad \text{in(privch, } ssk_b)\).
\[
p2. \quad \text{out(chc, } ssk_b)\).
\[
p3. \quad \nu \text{ sk}_b. \text{ out(chc, sk}_b)\).
\[
p4. \quad \text{out(chc, sign(pk(sk}_b), sksk_b))}\).
\[
p5. \quad \nu \text{ r}_1, \ldots, \nu \text{ r}_a, \ldots, \nu \text{ r}_c, \ldots, \nu \text{ r}_m\).
\[
p6. \quad \text{out(chc, (r}_1, \ldots, f(r}_a), \ldots, f(r}_c), \ldots, r_m))\).
\[
p7. \quad \text{let } \text{cnt}_{p1} = \text{commit}(r_1, pk(sk}_b), m_{\text{no}}) \text{ in}
\]
\[
p8. \quad \ldots
\]
\[
p9. \quad \text{let } \text{cnt}_{ps} = \text{commit}(r_1, pk(sk}_b), m_{\text{no}}) \text{ in}
\]
\[
p10. \quad \ldots
\]
\[
p11. \quad \text{let } \text{cnt}_{pc} = \text{commit}(r_1, pk(sk}_b), m_{\text{yes}}) \text{ in}
\]
\[
p12. \quad \ldots
\]
\[
p13. \quad \text{let } \text{cnt}_{pm} = \text{commit}(r_1, pk(sk}_b), m_{\text{no}}) \text{ in}
\]
\[
p14. \quad \text{out(chc, sign((cnt}_{p1}, \ldots, cnt}_{pm}), sksk_b))\).
\[
p15. \quad \text{out(untapch, sign((r}_1, \ldots, r_a), \ldots, r_c), \ldots, r_m), sksk_b))\)
\[
p16. \quad ||\text{lin(chc, } x))
\]

Figure A.1: The process \( \nu \text{chc.}(P_f || \text{lin(chc, } x)) \).

The transitions of process \( P' \) are shown in Figure A.2.

In Figure A.2 \( P'_1, \ldots, P'_7 \) are sub-processes in Figure A.1
\( P' \xrightarrow{\text{in(privch,}\, y)} \nu \text{ch}. P'_{1}\{y/s\text{sk}_b\} \)

\[ \xrightarrow{\text{(COMM)}} \nu \text{ch}. P'_{2}\{y/s\text{sk}_b\} \]

\[ \xrightarrow{\nu x_1. \text{out(ch,}\, x_1)} \nu \text{ch} \cdot \nu \text{sk}_b. P'_{3}\{y/s\text{sk}_b\} \]

\[ \xrightarrow{\text{(COMM)}} \nu \text{ch} \cdot \nu \text{sk}_b. (P'_4 \mid \{\text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b)/x_1\}\{y/s\text{sk}_b\}) \]

\[ \xrightarrow{\nu x_2. \text{out(ch,}\, x_2)} \nu \text{ch} \cdot \nu \text{sk}_b. \nu r_1. \cdots \nu r_m. (P'_5 \mid \{\text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b)/x_1\}) \]

\[ \cdots \]

\[ \xrightarrow{\nu x_3. \text{out(ch,}\, x_3)} \nu \text{ch} \cdot \nu \text{sk}_b. \nu r_1. \cdots \nu r_m. (P'_6 \mid \{\text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b)/x_1\}) \]

\[ \cdots \]

Figure A.2: Transitions of \( P' \)

- \( P'_1 \) is the sub-process \( p_2 \) to \( p_{16} \).
- \( P'_2 \) is the sub-process \( p_3 \) to \( p_{16} \).
- \( P'_3 \) is the sub-process \( p_4 \) to \( p_{16} \).
- \( P'_4 \) is the sub-process \( p_5 \) to \( p_{16} \).
- \( P'_5 \) is the sub-process \( p_7 \) to \( p_{16} \).
- \( P'_6 \) is the sub-process \( p_7 \) to \( p_{16} \).
- \( P'_7 \) is the sub-process \( p_{15} \) to \( p_{16} \).

\[ Q' := \]

\[ q_1. \quad \text{in(chPriv, ssk}_b). \]

\[ q_2. \quad \nu \, ssk_b. \text{out(ch, sign(pk(sk}_b), ssk}_b)). \]

\[ q_3. \quad \nu \, r_1. \cdots \nu \, r_m. \]

\[ q_4. \quad \text{let } cmtp_1 = \text{commit}(r_1, \text{pk}(sk}_b), M_{no}) \, \text{in} \]

\[ q_5. \quad \text{...} \]

\[ q_6. \quad \text{let } cmtp_n = \text{commit}(r_n, \text{pk}(sk}_b), M_{no}) \, \text{in} \]

\[ q_7. \quad \text{...} \]

\[ q_8. \quad \text{let } cmtp_c = \text{commit}(r_c, \text{pk}(sk}_b), M_{yes}) \, \text{in} \]

\[ q_9. \quad \text{...} \]

\[ q_{10}. \quad \text{let } cmtp_m = \text{commit}(r_m, \text{pk}(sk}_b), M_{no}) \, \text{in} \]

\[ q_{11}. \quad \text{out(ch, sign((cmtp_1, \cdots, cmtp_m), ssk}_b)). \]

\[ q_{12}. \quad \text{out(untapch, sign((r_1, \cdots, r_m), ssk}_b)) \]

Figure A.3: The bidder process \( P_{b}\{c/p_{b}\} \).

The transitions of process \( Q' \) are shown in Figure A.4.

In Figure A.4, \( Q'_1, \ldots, Q'_4 \) are sub-processes in Figure A.3.

- \( Q'_1 \) is the sub-process \( q_2 \) to \( q_{12} \).
A.1 Full proof of receipt-freeness of AS02

\[ Q' \xrightarrow{\text{in(privech, y)}} Q_1' \{ y / \text{ssk}_b \} \]
\[ \nu x_1. \text{out(ch, } x_1) \rightarrow \nu \text{sk}_b. (Q_2' \mid \{ \text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b) / x_1 \}) \{ y / \text{ssk}_b \} \]
\[ \nu x_2. \text{out(ch, } x_2) \rightarrow \nu \text{sk}_b. \nu r_1. \ldots . \nu r_m. (Q_3' \mid \{ \text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b) / x_1 \} \mid \{ \text{sign}(\text{cmt}^{p_1}, \ldots , \text{cmt}^{p_m}), \text{ssk}_b / x_2 \}) \{ y / \text{ssk}_b \} \]
\[ \nu x_3. \text{out(ch, } x_3) \rightarrow \nu \text{sk}_b. \nu r_1. \ldots . \nu r_m. (Q_4' \mid \{ \text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b) / x_1 \} \mid \{ \text{sign}(r_1, \ldots , r_m, \text{ssk}_b / x_3) \}) \{ y / \text{ssk}_b \} \]

Figure A.4: Transitions of \( Q' \).

- \( Q_2' \) is the sub-process \( q_3 \) to \( q_{12} \).
- \( Q_3' \) is the sub-process \( q_{12} \).
- \( Q_4' \) is the sub-process 0.

We build a relation \( \mathcal{R} \) as follows: \( P' \mathcal{R} Q', P' \mathcal{R} Q_1', P' \mathcal{R} Q_1, P' \mathcal{R} Q_1', P' \mathcal{R} Q_2, P' \mathcal{R} Q_2', P' \mathcal{R} Q_3', \) where

\[
\begin{align*}
P_1' &:= (\nu \text{ch}. P'_1 \{ y / \text{ssk}_b \}) \\
P_2' &:= (\nu \text{ch}. P'_2 \{ y / \text{ssk}_b \}) \\
P_3' &:= (\nu \text{ch}. \nu \text{sk}_b. P'_3 \{ y / \text{ssk}_b \}) \\
P_4' &:= (\nu \text{ch}. \nu \text{sk}_b. (P'_4 \mid \{ \text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b) / x_1 \}) \{ y / \text{ssk}_b \}) \\
P_5' &:= (\nu \text{ch}. \nu \text{sk}_b. \nu r_1. \ldots . \nu r_m. (P'_5 \mid \{ \text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b) / x_1 \}) \{ y / \text{ssk}_b \}) \\
P_6' &:= (\nu \text{ch}. \nu \text{sk}_b. \nu r_1. \ldots . \nu r_m. (P'_6 \mid \{ \text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b) / x_1 \} \mid \{ \text{sign}(\text{cmt}^{p_1}, \ldots , \text{cmt}^{p_m}), \text{ssk}_b / x_2 \}) \{ y / \text{ssk}_b \}) \\
Q_1' &:= Q_1' \{ y / \text{ssk}_b \} \\
Q_2' &:= (\nu \text{sk}_b. (Q_2' \mid \{ \text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b) / x_1 \}) \{ y / \text{ssk}_b \}) \\
Q_3' &:= (\nu \text{sk}_b. \nu r_1. \ldots . \nu r_m. (Q_3' \mid \{ \text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b) / x_1 \} \mid \{ \text{sign}(\text{cmt}^{p_1}, \ldots , \text{cmt}^{p_m}), \text{ssk}_b / x_2 \}) \{ y / \text{ssk}_b \}).
\end{align*}
\]

We prove that \( P' \approx_s Q' \). Since \( \text{frame}(P') := \nu \text{sk}_b. \nu r_1. \ldots . \nu r_m \) and \( \text{frame}(Q') := \nu \text{ch}. \nu \text{sk}_b. \nu r_1. \ldots . \nu r_m \), we have that \( \text{domain}(P') = \text{domain}(Q') \) and \( M = E N \) in \( \text{frame}(P') \) iff \( M = E N \) in \( \text{frame}(Q') \), thus, we have \( \text{frame}(P') \approx_s \text{frame}(Q') \). Therefore, \( P' \approx_s Q' \).

Similarly, from the above we can prove that \( P_1 \approx_s Q_1', P_2 \approx_s Q_1', P_3 \approx_s Q_1', P_4 \approx_s Q_2', P_5 \approx_s Q_2', P_6 \approx_s Q_3' \).

(1) \( \text{frame}(P_1') := (\nu \text{ch}.0) \) and \( \text{frame}(Q_1') := 0 \). We can see that \( \text{domain}(P_1') = \text{domain}(Q_1') \). According to rule NEW \( - 0 \), we have that \( \text{frame}(P_1') \approx_s \text{frame}(Q_1') \), and thus, \( M = E N \) in \( \text{frame}(P_1') \) iff \( M = E N \) in \( \text{frame}(Q_1') \). Therefore, \( P_1 \approx_s Q_1' \).

(2) Since \( \text{frame}(P_2') := (\nu \text{ch}.0) \) as well, from (1) we have that \( P_2 \approx_s Q_1' \).

(3) Since \( \text{frame}(P_3') := \nu \text{ch}. \nu \text{sk}_b.0 \), we can see that \( \text{domain}(P_1') = \text{domain}(Q_1') \). According to rule NEW \( - 0 \), we have that \( \text{frame}(P_3') \approx_s \text{frame}(Q_1') \) as well. Therefore, \( M = E N \) in \( \text{frame}(P_3') \) iff \( M = E N \) in \( \text{frame}(Q_1') \), and thus, \( P_3 \approx_s Q_1' \).

(4) \( \text{frame}(P_4') := \nu \text{ch}. \nu \text{sk}_b. \{ \{ \text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b) / x_1 \} \{ y / \text{ssk}_b \} \} \) and \( \text{frame}(Q_2') := \nu \text{sk}_b. \{ \{ \text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b) / x_1 \} \{ y / \text{ssk}_b \} \} \). We can see that \( \text{domain}(P_4') = \text{domain}(Q_2') \).
According to rule PAR – 0, we have that

$$\text{frame}(P^4) \equiv \nu \text{ch}.\nu \text{sk}_b. (\{\text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b)/x_1 \mid 0\}\{y/\text{ssk}_b\}).$$

According to rule NEW – PAR, we have that

$$\nu \text{ch}.\nu \text{sk}_b. (\{\text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b)/x_1 \mid 0\}\{y/\text{ssk}_b\})$$

$$\equiv \nu \text{sk}_b. (\{\text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b)/x_1 \mid \nu \text{ch}.0\}\{y/\text{ssk}_b\}).$$

According to rule NEW – 0, we have that $\nu \text{ch}.0 \equiv 0$ and thus

$$\nu \text{sk}_b. (\{\text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b)/x_1 \mid \nu \text{ch}.0\}\{y/\text{ssk}_b\})$$

$$\equiv \nu \text{sk}_b. (\{\text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b)/x_1 \}\{y/\text{ssk}_b\}).$$

Therefore, $\text{frame}(P^4) \equiv \nu \text{sk}_b. (\{\text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b)/x_1 \}\{y/\text{ssk}_b\}).$

That is $\text{frame}(P^4) \equiv \text{frame}(Q^2)$. Therefore, we have $\text{frame}(P^4) \approx_s \text{frame}(Q^2)$, and thus, $M =_E N$ in $\text{frame}(P^4)$ iff $M =_E N$ in $\text{frame}(Q^2)$. Therefore, $P^1 \approx_s Q^2$.

(5) $\text{frame}(P^5) := \nu \text{ch}.\nu \text{sk}_b.\nu r_1.\cdots.\nu r_m. (\{\text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b)/x_1 \}\{y/\text{ssk}_b\})$. We can see that $\text{domain}(P^5) = \text{domain}(Q^2)$. Similar as in (3), we can prove that $\text{frame}(P^5) \equiv \text{frame}(P^4)$. Therefore, $\text{frame}(P^5) \approx_s \text{frame}(Q^2)$ as well. Thus, $M =_E N$ in $\text{frame}(P^5)$ iff $M =_E N$ in $\text{frame}(Q^2)$. Hence, we have $P^5 \approx_s Q^2$.

(6) We have the two frames, $\text{frame}(P^6)$ and $\text{frame}(Q^3)$, defined as follows:

$$\text{frame}(P^6) := \nu \text{ch}.\nu \text{sk}_b.\nu r_1.\cdots.\nu r_m. (\{\text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b)/x_1 \} | \{\text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b)/x_2 \})\{y/\text{ssk}_b\}).$$

$$\text{frame}(Q^3) := \nu \text{sk}_b.\nu r_1.\cdots.\nu r_m. (\{\text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b)/x_1 \} | \{\text{sign}(\text{pk}(\text{sk}_b), \text{ssk}_b)/x_2 \})\{y/\text{ssk}_b\}).$$

We can see that $\text{domain}(P^6) = \text{domain}(Q^3)$. Similar as in (1), we prove that $\text{frame}(P^6) \approx_s \text{frame}(Q^3)$. Thus, $M =_E N$ in $\text{frame}(P^6)$ iff $M =_E N$ in $\text{frame}(Q^3)$. Hence, we have $P^6 \approx_s Q^3$.

We can see that the relation $\mathcal{R}$ satisfies the definition of labelled bisimilarity (see Definition 3.11). If $P'$ does a internal reduction to $P^1$, then we have $P^1 \mathcal{R} Q'$. If $Q$ does a labelled transition to $Q'$, then $P$ can do three internal reductions and a labelled transition to $P^4$, and $Q^2 \mathcal{R} P^4$.

$$\square$$

Proving the second equivalence holds is similar. Below, we explain how the proof runs.

Recall the proof obligation:

$${}_C \text{AS02}[\{ \text{let } p_b = \mathbf{a} \text{ in let } \text{untapch} = \text{untapch}_{bA} \text{ in} \}$$

$$\text{let } privch = \text{privch}_{bA} \text{ in let } ch = \text{ch}_{bA} \text{ in } P_b)^{\text{chc}} |$$

$${}_C \text{AS02}[\{ \text{let } p_b = \mathbf{d} \text{ in let } \text{untapch} = \text{untapch}_{bB} \text{ in} \}$$

$$\text{let } privch = \text{privch}_{bB} \text{ in let } ch = \text{ch}_{bB} \text{ in } P_b]$$

$${}_C \text{AS02}[\{ \text{let } privch = \text{privch}_{bA} \text{ in let } ch = \text{ch}_{bA} \text{ in } P_f \}$$

$$|$$

$${}_C \text{AS02}[\{ \text{let } p_b = \mathbf{d} \text{ in let } \text{untapch} = \text{untapch}_{bB} \text{ in} \}$$

$$\text{let } privch = \text{privch}_{bB} \text{ in let } ch = \text{ch}_{bB} \text{ in } P_b].$$
Since the processes on each side of the equivalence consists of several concurrent processes, including the key distribution process, \( n \) bidder processes and an auctioneer process. In addition, the key distribution process and the auctioneer process contain sub-processes which are in parallel. Thus, the transitions of the process on each side are too complicated to be showed here. The proof of this equivalence is similar to the proof of the first equivalence as shown above. The difference is that for proving the second equivalence, the states and transitions are more complicated.

Generally speaking, since the contexts are the same on both sides, if the process on the left side can do a labelled transition by one of the parallel sub-processes in the context, or an internal reduction between sub-processes in the context, the corresponding sub-processes on the right hand side context can do the same transition. And Vice versa.

The transitions caused by the two bidder processes, \( P_{bA} \{a/p_b\}^{chc} \) and \( P_{bB} \{d/p_b\} \) on the left side, \( P_f \) and \( P_{bB} \{d/p_b\} \) on the right side, are showed in Figure 4.13. From Figure 4.13, we can see that if the left side can do a transition by the two bidder processes, the right side can do a corresponding transition by the two bidder processes as well. Figure 4.13 also shows the change of adversary knowledge caused by transitions of the two bidder processes. We can see, form Figure 4.13, that the change of adversary knowledge is the same on both sides for most transitions. The only exception is \( x_5 \). In the proof sketch in Section 4.6.2, we showed that the adversary cannot distinguish the difference of \( x_5 \) between left and right side.

Therefore, for a state on the left side, there is a corresponding state on the right side. We build a relation between the corresponding states. The relations satisfies the second and third items in the definition of labelled bisimilarity (see Definition 3.11). Then we prove that the frames of the corresponding states are static equivalence, in a similarly way as we prove the first equivalence. Generally speaking, most frames are the same, due to the similarity of the processes on both sides. In such a way, the second equivalence is proved.

\[ \square \]
B

Full proofs in Chapter 6

B.1 Auxiliary lemmas

Theorem 1. If $A \approx_{\ell} B$ and $B \approx_{\ell} C$, then $A \approx_{\ell} C$.

Proof. 1. Since $A \approx_{\ell} B$, according to Definition 3.11 we have $A \approx_s B$. Similarly, since $B \approx_{\ell} C$, we have $B \approx_s C$.

According to Definition 3.8, since $A \approx_s B$, we have $\text{frame}(A) \approx_s \text{frame}(B)$. That is, 1) $\text{domain}(\text{frame}(A)) = \text{domain}(\text{frame}(B))$ and 2) $\forall$ terms $M, N$: $(M =_{E} N)$ in frame $\text{frame}(A)$ iff $(M =_{E} N)$ in frame $\text{frame}(B)$. Similarly, since $B \approx_s C$, we have $\text{frame}(B) \approx_s \text{frame}(C)$. Thus, 3) $\text{domain}(\text{frame}(B)) = \text{domain}(\text{frame}(C))$ and 4) $\forall$ terms $M, N$: $(M =_{E} N)$ in frame $\text{frame}(B)$ iff $(M =_{E} N)$ in frame $\text{frame}(C)$.

Because of 1) and 3), we have $\text{domain}(\text{frame}(A)) = \text{domain}(\text{frame}(C))$. Because of 2) and 4), we have $\forall$ terms $M, N$: $(M =_{E} N)$ in frame $\text{frame}(A)$ iff $(M =_{E} N)$ in frame $\text{frame}(C)$. Therefore, $\text{frame}(A) \approx_s \text{frame}(C)$, and thus, $A \approx_s C$.

2. We build a relation $\mathcal{R}$: $A \mathcal{R} C$ if $A \approx_{\ell} P$ and $P \approx_{\ell} C$ for some $P$.

2.1 Since $A \approx_{\ell} B$, according to Definition 3.11 we have that if $A \rightarrow A'$ then $B \rightarrow B'$ and $A' \approx_{\ell} B'$ for some $B'$. Similarly, since $B \approx_{\ell} C$, we have that if $B \rightarrow B'$ then $C \rightarrow C'$ and $B' \approx_{\ell} C'$ for some $C'$.

Therefore, if $B \rightarrow B'$, then $C \rightarrow C'$ and $B' \approx_{\ell} C'$ for some $C'$. From the above, we can see, if $A \rightarrow A'$ then $C \rightarrow C'$ and $A' \approx_{\ell} B'$ and $B' \approx_{\ell} C'$ for some $B'$ and $C'$. Thus, we have if $A \rightarrow A'$ then $C \rightarrow C'$ and $A' \mathcal{R} C'$ for some $C'$. Vice versa, we prove that if $C \rightarrow C'$ then $A \rightarrow_{\ell} A'$ and $A' \mathcal{R} C'$ for some $A'$, in a similar way.

2.2 Since $A \approx_{\ell} B$, according to Definition 3.11 we have that if $A \rightarrow_{\ell} A'$ and $fv(\alpha) \subseteq \text{domain}(A)$ and $\text{bn}(\alpha) \cap \text{fn}(B) = \emptyset$; then $B \rightarrow_{\ell} B_1 \rightarrow_{\ell} B'$ and $A' \approx_{\ell} B'$ for some $B'$.

5) Since $A \approx_{\ell} B$, we have $\text{domain}(A) = \text{domain}(B)$ (see second paragraph). Therefore, from the condition $fv(\alpha) \subseteq \text{domain}(A)$, we have $fv(\alpha) \subseteq \text{domain}(B)$.

6) Since $B \approx_{\ell} C$, we have that if $B \rightarrow_{\ell} B_1$ then $C \rightarrow_{\ell} C_1$ and $B_1 \approx_{\ell} C_1$ for some $C_1$ (see fourth paragraph).

Since internal reduction does not introduce active substitutions, thus does not change the domain, i.e., $\text{domain}(B) = \text{domain}(B_1)$. Form 5), we get $fv(\alpha) \subseteq \text{domain}(B_1)$.

Assume $\text{bn}(\alpha) \cap \text{fn}(C) = \emptyset$, since internal reduction does not introduce free names, we have $\text{bn}(\alpha) \cap \text{fn}(C_1) = \emptyset$.

Since $B_1 \approx_{\ell} C_1$, we have that if $B_1 \rightarrow_{\ell} B'$ and $fv(\alpha) \subseteq \text{domain}(B_1)$ and $\text{bn}(\alpha) \cap$
From the above, we prove that 
\[ A \sim C \quad \text{for some } C'. \]
Combining 5) and 6), we have that if \( A \overset{\alpha}{\rightarrow} A' \) and \( \text{fv}(\alpha) \subseteq \text{domain}(A) \) and \( \text{bn}(\alpha) \cap \text{fn}(C) = \emptyset \); then \( C \overset{\alpha}{\rightarrow} C' \) and \( A' \sim B' \) and \( B' \sim C' \) for some \( C' \).

Therefore, if \( A \overset{\alpha}{\rightarrow} A' \) and \( \text{fv}(\alpha) \subseteq \text{domain}(A) \) and \( \text{bn}(\alpha) \cap \text{fn}(C) = \emptyset \); then \( C \overset{\alpha}{\rightarrow} C' \) and \( A' \sim C' \) for some \( C' \).

In a similar way, we can prove that if \( C \overset{\alpha}{\rightarrow} C' \) and \( \text{fv}(\alpha) \subseteq \text{domain}(C) \) and \( \text{bn}(\alpha) \cap \text{fn}(A) = \emptyset \); then \( A \overset{\alpha}{\rightarrow} A' \) and \( A' \overset{\alpha}{\rightarrow} C' \) for some \( A' \).

From the above step 1, 2.1, and 2.2, we prove that \( A \sim C \).

**Theorem 2.** If \( A \equiv B \) and \( C \equiv D \), and \( A \approx C \) then \( B \approx D \).

**Proof.** From the following lemma in [AF01]: Static equivalence is closed by structural equivalence, we have that if \( A \equiv B \), then \( A \approx B \).

That is, accoding to Definition 3.8 if \( A \approx B \); then
1) \( \text{domain(frame}(A)) = \text{domain(frame}(B)) \) and
2) \( \forall \text{ terms } M, N: (M =_E N) \text{ in frame } \text{frame}(A) \text{ iff } (M =_E N) \text{ in frame } \text{frame}(B). \)

According to Definition 3.11 since \( A \approx C \), we have, \( A \approx C \), that is,
3) \( \text{domain(frame}(A)) = \text{domain(frame}(C)) \) and
4) \( \forall \text{ terms } M, N: (M =_E N) \text{ in frame } \text{frame}(A) \text{ iff } (M =_E N) \text{ in frame } \text{frame}(C). \)

Combining 1) and 3), we have \( \text{domain(frame}(B)) = \text{domain(frame}(C)). \)

Combining 2) and 4), we have \( \forall \text{ terms } M, N: (M =_E N) \text{ in frame } \text{frame}(B) \text{ iff } (M =_E N) \text{ in frame } \text{frame}(C). \) Thus, \( B \approx C. \)

We build a relation \( R \) as \( B \sim C \) if \( B \approx C \).

Since \( A \approx C \), we have that if \( C \rightarrow C' \) then \( A \rightarrow^{*} A' \) and \( A' \approx C' \) for some \( A' \).

Since internal reduction is closed under structural equivalence, we have that if \( A \equiv B \) and \( A \rightarrow^{*} A' \), then \( B \rightarrow^{*} B' \) and \( B' = A' \).

Therefore, if \( C \rightarrow C' \) then \( B \rightarrow^{*} B' \) and \( B' \approx C' \) for some \( B' \). That is, if \( C \rightarrow C' \) then \( B \rightarrow^{*} B' \) and \( B' \sim C' \) for some \( B' \).

In a similar way, we can prove that if \( B \rightarrow B' \) then \( C \rightarrow^{*} C' \) and \( B' \sim C' \) for some \( C' \).

5) Assume \( B \overset{\alpha}{\rightarrow} B' \). According to rule STRUCT, we have that if \( B \overset{\alpha}{\rightarrow} B' \) and \( A \equiv B \) then \( A \overset{\alpha}{\rightarrow} B' \).

6) Since \( A \approx C \), we have that if \( A \overset{\alpha}{\rightarrow} B' \) and \( \text{fv}(\alpha) \subseteq \text{domain}(A) \) and \( \text{bn}(\alpha) \cap \text{fn}(C) = \emptyset \); then \( C \overset{\alpha}{\rightarrow} C' \) and \( B' \approx C' \) for some \( C' \).

7) Since we have \( \text{domain}(A) = \text{domain}(B) \) (see first paragraph), we have if \( \text{fv}(\alpha) \subseteq \text{domain}(A) \) then \( \text{fv}(\alpha) \subseteq \text{domain}(B) \).

Combining 5) 6) and 7), we have if \( B \overset{\alpha}{\rightarrow} B' \) and \( \text{fv}(\alpha) \subseteq \text{domain}(B) \) and \( \text{bn}(\alpha) \cap \text{fn}(C) = \emptyset \); then \( C \overset{\alpha}{\rightarrow} C' \) and \( B' \approx C' \) for some \( C' \).

In a similar way, we can prove that if \( C \overset{\alpha}{\rightarrow} C' \) and \( \text{fv}(\alpha) \subseteq \text{domain}(C) \) and \( \text{bn}(\alpha) \cap \text{fn}(B) = \emptyset \); then \( B \overset{\alpha}{\rightarrow} B' \) and \( B' \approx C' \) for some \( B' \).

From the above, we prove that \( B \approx C \). Similary, we can prove that \( A \approx B \). By lemma 1] we have if \( A \approx C \), \( B \approx C \), then \( A \approx B \). Similarly, if \( A \approx B \) and \( A \approx D \) then \( B \approx D \).

**Theorem 3.** Let \( Q \) be a closed plain process and \( c'_{out} \) be a channel name such that \( c'_{out} \not\in \text{fn}(Q) \cup \text{bn}(Q) \). Let \( C_h[.] := \nu c'_{out} .(\bot \mid \text{lin}(c'_{out}, x)) \). We have \( Q(\psi, b, c'_{out}, t) \) out\( (c'_{out}) \)
\(\varepsilon \nu c_{\text{out}}'. (Q^{(\Psi, \Phi, c_{\text{out}}, \epsilon)}) \mid \text{lin}(c_{\text{out}}, x)) \approx_{\ell} Q\) [DKR09]

**Theorem 4.** Let \(C_1[\_] = \nu u_1, (\_ | B_1)\) and \(C_2[\_] = \nu u_2, (\_ | B_2)\) be two evaluation contexts such that \(\hat{u}_1 \cap (\text{fv}(B_2) \cup \text{fv}(B_2)) = \emptyset\) and \(\hat{u}_2 \cap (\text{fv}(B_1) \cup \text{fv}(B_1)) = \emptyset\). We have that \(C_1[C_2[A]] \equiv C_2[C_1[A]]\) for any extended process \(A\) [DKR09].

**Theorem 5.** Let \(A | B\) be a process, \(ch\) be a channel name in \(A\), \(ch\) never appears in \(B\). \((A | B)^{\text{out}(ch)} \equiv A^{\text{out}(ch)} | B\).

**Proof.**

\((A | B)^{\text{out}(ch)} := \nu \text{ch}.((A | B) \mid \text{lin}(ch, x))\)

\(A^{\text{out}(ch)} | B := (\nu \text{ch}.(A \mid \text{lin}(ch, x))) | B\)

Since \(ch\) never appears in \(B\), we have (rule NEW-PAR)

\((\nu \text{ch}.(A \mid \text{lin}(ch, x))) | B \equiv \nu \text{ch}.((A \mid \text{lin}(ch, x)) | B)\),

Because of rule PAR-C and rule PAR-A, we have

\((A | B) \mid \text{lin}(ch, x) \equiv A \mid \text{lin}(ch, x) | B\),

Thus,

\(\nu \text{ch}.((A | B) \mid \text{lin}(ch, x)) \equiv \nu \text{ch}.((A \mid \text{lin}(ch, x)) | B)\).

By transitivity of structural equivalence, we have

\((A | B)^{\text{out}(ch)} \equiv A^{\text{out}(ch)} | B\).

\[
\square
\]

**B.2 Theorem 6.20**

(3) \(\forall \rho, \text{epriv}_\rho \rightarrow \text{priv}\)

We prove the statement in the following two directions: 1. \(\forall \rho, \text{epriv}_\rho \rightarrow \text{priv}\) 2. \(\exists \rho, \text{priv} \not\rightarrow \text{epriv}_\rho\)

1. \(\forall \rho\), when a protocol satisfies \(\text{epriv}_\rho\), we prove that the protocol also satisfies \(\text{priv}\).

For a collaboration \(\rho = (\Psi, \Phi, c_{\text{out}}, c_{\text{in}})\), when a well-formed protocol \(P_w\) satisfies \(\text{epriv}\) w.r.t. \(\tau\) and \(\Psi, \Phi, c_{\text{out}}, c_{\text{in}}\), there exists a closed plain process \(P_f\) such that for any context \(C[\_] := \nu c_{\text{out}}.\nu c_{\text{in}}. (\_ | Q)\) satisfying \(\text{bn}(P_w) \cap \text{fn}(C[\_]) = \emptyset\) and

**eq1:**

\[C_{P_w}[R_{1}^{(\Psi, \Phi, c_{\text{out}}, c_{\text{in}})}\{\text{id}/id_1, t/\tau\}] \approx_{\ell} C_{P_w}[R_{1}^{(\Psi, \Phi, c_{\text{out}}, c_{\text{in}})}\{\text{id}/id_1, t_1/\tau\}],\]

we have

**eq2:**

\[C[P_f]^{\text{out}(c_{\text{out}}')} \approx_{\ell} R_{1}\{\text{id}/id_1, t_2/\tau\},\]
and

**eq3:**
\[ C_{P_w}[C[\hat{R}_i^{\langle \Psi, \Phi, c_{out}, c_{in} \rangle} \{id/\mathit{id}_i, t/\tau \}]] \approx_{\ell} C_{P_w}[C[P_f]]. \]

1) According to Lemma 1 (transitivity of \( \approx_{\ell} \)), combining (eq1) and (eq3), we have

**eq4:**
\[ C_{P_w}[\hat{R}_i^{\langle \Psi, \Phi, c_{out}, c_{in} \rangle} \{id/\mathit{id}_i, t_1/\tau \}] \approx_{\ell} C_{P_w}[C[P_f]]. \]

2) By applying the evaluation context \( C[\_] := \nu c_{out}.([\text{lin}(c_{out}, x)) \ (x \text{ is a fresh variable}) \) on both sides of (eq4), we have

**eq5:**
\[ C_h[C_{P_w}[\hat{R}_i^{\langle \Psi, \Phi, c_{out}, c_{in} \rangle} \{id/\mathit{id}_i, t_1/\tau \}]] \approx_{\ell} C_h[C_{P_w}[C[P_f]]]. \]

3) According to Lemma 3 by swapping position of context \( C_h[\_] \) and \( C_{P_w}[\_] \), the left side of (eq5) is structural equivalent to

\[ C_{P_w}[C_h[\hat{R}_i^{\langle \Psi, \Phi, c_{out}, c_{in} \rangle} \{id/\mathit{id}_i, t_1/\tau \}]], \]

and the right side of (eq5) is structural equivalent to \( C_{P_w}[C_h[C[P_f]]] \). According to Lemma 2 the above two processes are bisimilar, that is

**eq6:**
\[ C_{P_w}[C_h[\hat{R}_i^{\langle \Psi, \Phi, c_{out}, c_{in} \rangle} \{id/\mathit{id}_i, t_1/\tau \}]] \approx_{\ell} \hat{R}_i[\mathit{id}/%id_i, t_1/\tau]. \]

4) By Lemma 3 we have the following equivalence

\[ C_h[\hat{R}_i^{\langle \Psi, \Phi, c_{out}, c_{in} \rangle} \{id/\mathit{id}_i, t_1/\tau \}] \approx_{\ell} \hat{R}_i[\mathit{id}/%id_i, t_1/\tau]. \]

By applying the context \( C_{P_w}[\_] \) on both sides of the above equivalence, we have

**eq7:**
\[ C_{P_w}[C_h[\hat{R}_i^{\langle \Psi, \Phi, c_{out}, c_{in} \rangle} \{id/\mathit{id}_i, t_1/\tau \}]] \approx_{\ell} C_{P_w}[\hat{R}_i[\mathit{id}/%id_i, t_1/\tau]]. \]

That is, the left side of (eq6) is equivalent to \( C_{P_w}[\hat{R}_i[\mathit{id}/%id_i, t_1/\tau]]. \)

5) By Lemma 3 we have \( C[P_f]^{\text{out}(c_{out}) := C_h[C[P_f]]. \) Thus, we can replace the process \( C[P_f]^{\text{out}(c_{out})} \) in (eq2) with \( C_h[C[P_f]]. \) That is, \( C_h[C[P_f]] \approx_{\ell} \hat{R}_i[\mathit{id}/%id_i, t_2/\tau]. \) By applying context \( C_{P_w}[\_] \) on both sides of the above equivalence, we have

**eq8:**
\[ C_{P_w}[C_h[C[P_f]]] \approx_{\ell} C_{P_w}[\hat{R}_i[\mathit{id}/%id_i, t_2/\tau]]. \]

That is, the right side of (eq6) is equivalent to \( C_{P_w}[\hat{R}_i[\mathit{id}/%id_i, t_2/\tau]]. \)

6) According to Lemma 2 combining (eq6), (eq7) and (eq8), we have

**eq9:**
\[ C_{P_w}[\hat{R}_i[\mathit{id}/%id_i, t_1/\tau]] \approx_{\ell} C_{P_w}[\hat{R}_i[\mathit{id}/%id_i, t_2/\tau]]. \]

The equivalence (eq9) coincides with the equivalence in Def. 6.3. Thus, the protocol \( P_w \) satisfies priv.

\[
\Box
\]

2. There exists \( \rho \) such that \( \text{priv} \Rightarrow \text{epriv}_\rho \).

We prove the statement by showing an example in which a protocol satisfies priv but not epriv\(_\rho \) for some \( \rho \) as in Ex. 6.3.
Example B.1. Protocol $Q := \nu_{\tau.\nu.s.out}(ch,\text{enc}(s,r))$ where $ch$ is a public channel, satisfies $\text{priv}$ w.r.t. $s$, but not $\text{epriv}$ w.r.t. $s$ and $\langle \{r\},0,c_{out},\epsilon \rangle$. The adversary cannot distinguish $\text{enc}(s_1,r)$ and $\text{enc}(s_2,r)$, thus the protocol satisfies $\text{priv}$ w.r.t. $s$. However, when $Q$ is coerced to reveal $r$, there is no way for $Q$ to cheat the adversary. Because of the perfect encryption assumption, any other nonce cannot be used to decrypt $\text{enc}(s,r)$, thus, the adversary will find out whether the user lied.

(4) $\forall \theta$, $\text{ipriv}_\theta \implies \text{priv}$

Note that in $\text{ipriv}_\theta$, we assume the existence of a set of attacking third parties $R_T$. Thus, when we consider $\text{priv}$, we have the same assumption that there exists the same set of third parties $R_D$.

We prove the statement in the following two directions: 1. $\forall \theta$, $\text{ipriv}_\theta \implies \text{priv}$ 2. $\exists \theta$, $\text{priv} \iff \text{ipriv}_\theta$

1. $\forall \theta = (R_T, (\Psi^t, \Phi^t, c^t_{out}, c^t_{in}))$, when a protocol satisfies $\text{ipriv}_\theta$, we prove that the protocol also satisfies $\text{priv}$ with the existence of $R_T$.

For a collaboration of third parties $\theta = (R_T, (\Psi^t, \Phi^t, c^t_{out}, c^t_{in}))$, when a well-formed protocol $P_w$ satisfies $\text{ipriv}$ w.r.t. $\tau$ and $(R_T, (\Psi^t, \Phi^t, c^t_{out}, c^t_{in}))$, the following equivalence holds.

eqi1: $C_{i,[]} := \nu c_{out} \nu c_{in}. (- | Q)

satisfying $\text{bn}(P_w) \cap \text{fn}(C_{i,[]}) = \emptyset$ and

eqi2: $C_{i,[R_T(\Psi^t, \Phi^t, c^t_{out}, c^t_{in})]} \approx_{\ell} R_T(\Psi^t, \Phi^t, c^t_{out}, c^t_{in})$.

2) By applying the context $C_{i,[]}$ on both sides of (eqi1), we have

eqi3: $C_{i,C_P}[R_T(\Psi^t, \Phi^t, c^t_{out}, c^t_{in})] \approx_{\ell} C_{i,C_P}[R_T(\Psi^t, \Phi^t, c^t_{out}, c^t_{in})]$.

3) By applying the evaluation context $C_{i,[]} := \nu c_{out} \cdot (- | \text{ln}(c_{out}, x)) \ (x \text{ is a fresh variable})$, on both sides of (eqi3), we have

eqi4: $C_{i,}[C_P[R_T(\Psi^t, \Phi^t, c^t_{out}, c^t_{in})]] \approx_{\ell} C_{i,}[C_P[R_T(\Psi^t, \Phi^t, c^t_{out}, c^t_{in})]]$. 
4) According to Lemma 4 by swapping contexts $C^i_h[\_]$ and $C_{P_w}[\_],$ the left side of (eqi4) is structural equivalent to

$$C_{P_w}[C^i_h[C_t[\hat{R}_i\{\text{id}/id_i, t_1/\tau\} | R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}]]]$$

That is,

**eqi5:**

$$C^i_h[C_t^i[C_P_{w}[\hat{R}_i\{\text{id}/id_i, t_1/\tau\} | R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}]]]$$

$$\equiv C_{P_w}[C^i_h[C_t[\hat{R}_i\{\text{id}/id_i, t_1/\tau\} | R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}]]]$$

Since $c_{out}^i$ and $c_{in}^i$ are fresh channel names, they do not appear in $\hat{R}_i\{\text{id}/id_i, t_1/\tau\}$. According to Lemma 5, we have are able to move the position of the context $C^i_h[\_]$, thus have

**eqi6:**

$$C_{P_w}[C^i_h[C_t^i[\hat{R}_i\{\text{id}/id_i, t_1/\tau\} | R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}]]]$$

$$\equiv C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_1/\tau\} | C^i_h[C_t[R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}]]]$$

Thus, combining (eqi5) and (eqi6), we have

**eqi7:**

$$C^i_h[C_t^i[C_P_{w}[\hat{R}_i\{\text{id}/id_i, t_1/\tau\} | R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}]]]$$

$$\equiv C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_1/\tau\} | C^i_h[C_t[R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}]]]$$

5) Similarly, the right side of (eqi4) satisfies the following equivalence,

**eqi8:**

$$C^i_h[C_t^i[C_P_{w}[\hat{R}_i\{\text{id}/id_i, t_2/\tau\} | R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}]]]$$

$$\equiv C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_2/\tau\} | C^i_h[C_t[R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}]]]$$

6) According to Lemma 2 combining (eqi7), (eqi8) and (eqi4), we have

**eqi9:**

$$C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_1/\tau\} | C^i_h[C_t[R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}]]]$$

$$\approx\eta C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_2/\tau\} | C^i_h[C_t[R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}]]]$$

7) By applying the context $C^i_h[\_]$ on both sides of (eqi2), we obtain

$$C^i_h[C_t[R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}]] \approx\eta C^i_h[R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}].$$

According to Lemma 3 from the above equivalence, we have

$$C^i_h[R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}] \approx\eta R_T.$$  

By Lemma 1 (transitivity of the above two equivalences), we have

**eqi10:**

$$C^i_h[C_t[R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}]] \approx\eta R_T.$$  

8) Thus, the left side of (eqi9) satisfies the following equivalence (by applying context $C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_1/\tau\} | \_]$ on both sides of (eqi10))

$$C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_1/\tau\} | C^i_h[C_t[R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}]]] \approx\eta C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_1/\tau\} | R_T].$$

The right side of (eqi9) satisfies the following equivalence (by applying context $C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_2/\tau\} | \_]$ on both sides of (eqi10))

$$C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_2/\tau\} | C^i_h[C_t[R_T^{(\psi, \phi, c_{out}^i, c_{in}^i)}]]] \approx\eta C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_2/\tau\} | R_T].$$
According to Lemma 11 (transitivity), from (eqi9), we have 
\[ C_{P_w}[\tilde{R}_t\{id/\text{id}_1, t_1/\tau\} | R_T] \approx_{\ell} C_{P_w}[\tilde{R}_t\{id/\text{id}_1, t_2/\tau\} | R_T] \]

9) According to the definition of third parties – third parties are third party processes running in parallel. The context \( C_{P_w}[\_] \) has the following form
\[ C_{P_w}[\_] := \nu \cdot \text{genkey} \mid R_1 \mid \ldots \mid R_p \mid \_ \]
Thus, according to rule 
\[ !P \equiv P || P, \]
\( R_T \) can be absorbed by the context. Thus, \( C_{P_w}[\_ | R_T] \) is a type of context where there requires \( R_T \) to be present. We define \( C'_{P_w}[\_] := C_{P_w}[\_ | R_T] \), where \( R_T \) has to be present in the context, we have
\[ \text{eqi12:} \]
\[ C'_{P_w}[\tilde{R}_t\{id/\text{id}_1, t_1/\tau\}] \approx_{\ell} C'_{P_w}[\tilde{R}_t\{id/\text{id}_1, t_2/\tau\}] \]
Therefore, the protocol satisfies \( \text{priv} \) w.r.t. \( \tau \) with the existence of \( R_T \). \( \square \)

2. There exists \( \theta \) such that \( \text{priv} \iff \text{ipriv}_\theta \).
We prove the statement by showing an example in which a protocol satisfies \( \text{priv} \) but not \( \text{ipriv}_\theta \) for some \( \theta \) as in Eq. 132.

**Example B.2.** The following protocol

\[ P := \nu \cdot \text{untapch}.(Q \mid Q') \]
\[ Q := \nu \cdot \text{s.out}(\text{untapch}, s) \]
\[ Q' := \text{in}(\text{untapch}, x) \]
where \( \text{untapch} \) is an untappable channel, satisfies \( \text{priv} \) w.r.t. \( s \), but not \( \text{ipriv}_\theta \) w.r.t. \( s \) and \( (Q', \{\{x\}, \emptyset, \text{out}_c, \epsilon\}) \). Since the communication is untappable, the adversary cannot distinguish \( s_1 \) from \( s_2 \), thus the protocol satisfies \( \text{priv} \) w.r.t. \( s \). However, when the communication partner \( Q' \) reveals the secret information he reads in on the untappable channel, \( s \) is revealed.

(2) \( \forall \rho, \text{ipriv}_{\rho,\theta} \implies \text{ipriv}_\theta \)

Similar as proving \( \forall \rho, \text{epriv}_\rho \implies \text{priv} \), we prove the statement in the following two directions: 1. \( \forall \rho, \text{ipriv}_{\rho,\theta} \implies \text{ipriv}_\theta \)

2. \( \exists \rho, \theta, \text{ipriv}_\theta \iff \text{iepriv}_{\rho,\theta} \)

1. \( \forall \rho \), when a protocol satisfies \( \text{iepriv}_{\rho,\theta} \) for some \( \theta \), we prove that the protocol also satisfies \( \text{ipriv}_\theta \).

For a collaboration \( \rho = (\Psi, \Phi, c_{\text{out}}, c_{\text{in}}) \), when a well-formed protocol \( P_w \) satisfies \( \text{iepriv} \) w.r.t. \( \tau \), \( (\Psi, \Phi, c_{\text{out}}, c_{\text{in}}) \) and \( (R_T, (\Psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t)) \) there exists a closed plain process \( P_f \), such that for any context \( C[\_] := \nu c_{\text{out}}.\nu c_{\text{in}}.(\_ | Q) \) satisfying \( \text{bn}(P_w) \cap \text{fn}(C[\_]) = \emptyset \) and
\[ \text{eqi1:} \]
\[ C_{P_w}[C[\tilde{R}_t(\Psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t)] \{id/\text{id}_1, t/\tau\}] | R_T] \approx_{\ell} C_{P_w}[\tilde{R}_t(\Psi^t, c_{\text{out}}^t, c_{\text{in}}^t) \{id/\text{id}_1, t_1/\tau\}] | R_T], \]
we have
\[
\mathcal{C}[P]^{\langle \text{id}/\text{id}_i, \text{t}_2/\tau \rangle} \approx_\ell \tilde{R}_i \{ \text{id}/\text{id}_i, \text{t}_2/\tau \},
\]
and
\[
\text{eqie3}:
\]
\[
\mathcal{C}_{P_w}[\mathcal{C}[\tilde{R}_i^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)} \{ \text{id}/\text{id}_i, \text{t}/\tau \}] \mid R_T^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)}]
\approx_\ell \mathcal{C}_{P_w}[\mathcal{C}[P] \mid R_T^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)}].
\]

We first prove the following statement: If a context which provides information for the collaborating target user \( \mathcal{C}^t[\cdot] := \nu_{c_{\text{out}} \cdot \nu_{c_{\text{in}}}}(\cdot | Q) \) satisfies \( \text{bn}(P_w) \cap \text{fn}(\mathcal{C}[\cdot]) = \emptyset \) and
\[
\text{eqie4}:
\]
\[
\mathcal{C}_{P_w}[\mathcal{C}[\tilde{R}_i^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)} \{ \text{id}/\text{id}_i, \text{t}/\tau \}] \mid R_T^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)}]
\approx_\ell \mathcal{C}_{P_w}[\mathcal{C}[P] \mid R_T^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)}],
\]
then this context satisfies (eqie1) when \( R_T \) exists.

\textbf{Proof.} Since (eqie4) holds for any context of the adversary which provides information for the collaborating third parties, for a specific context \( \mathcal{C}_t[\cdot] \) of the adversary providing information for the collaborating third parties, (eqie4) should hold.
\[
\mathcal{C}_t[\cdot] := \nu_{c_{\text{out}} \cdot \nu_{c_{\text{in}}}}(\cdot | Q)
\]
satisfying \( \text{bn}(P_w) \cap \text{fn}(\mathcal{C}_t[\cdot]) = \emptyset \) and
\[
\text{eqie41}:
\]
\[
\mathcal{C}_t[R_T^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)}] \approx_\ell R_T^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)}.
\]
Since (eqie4) holds in context \( \mathcal{C}_t[\cdot] \), we apply context \( \mathcal{C}_t[\cdot] \) and evaluation context \( \mathcal{C}_t^e[\cdot] := \nu_{c_{\text{out}} \cdot (\cdot | \text{fn}(\mathcal{C}_t^e[\cdot]))} (x) \) is a fresh variable) on both sides of (eqie4), we have
\[
\text{eqie42}:
\]
\[
\mathcal{C}_t^e[\mathcal{C}_t[R_T^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)}] \mid \mathcal{C}_t[R_T^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)}]]
\approx_\ell \mathcal{C}_t^e[\mathcal{C}_t[R_T^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)}] \mid \mathcal{C}_t[R_T^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)}]]].
\]
Similar as proving \( \forall \theta, \text{ipriv}_\theta \Rightarrow \text{priv} \), by Lemma 5, we move the position of the contexts \( \mathcal{C}_t[\cdot] \) and \( \mathcal{C}_t^e[\cdot] \), and have
\[
\text{eqie43}:
\]
\[
\mathcal{C}_{P_w}[\mathcal{C}[\tilde{R}_i^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)} \{ \text{id}/\text{id}_i, \text{t}/\tau \}] \mid \mathcal{C}_t[R_T^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)}]]
\approx_\ell \mathcal{C}_{P_w}[\mathcal{C}[\tilde{R}_i^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)} \{ \text{id}/\text{id}_i, \text{t}/\tau \}] \mid \mathcal{C}_t[R_T^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)}]]].
\]
By applying context \( \mathcal{C}_t[\cdot] \) on both sides of (eqie41) we have
\[
\text{eqie44}:
\]
\[
\mathcal{C}_t[R_T^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)}] \approx_\ell \mathcal{C}_t[R_T^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)}].
\]
According to Lemma 3 we have
\[
\mathcal{C}_t[R_T^{(\Phi, \varepsilon, \text{id}_i, \text{c}_m)}] \approx_\ell R_T
Thus, by transitivity, combining the above equivalence and (eqie44), we have **eqie45:**
\[
C_h[C_{\ell}[R_T^{(\psi^t,\phi^t,c_{\text{out}}^t,c_{\text{in}}^t)}]] \approx_{\ell} R_T
\]
By applying context \(C_p\) on both sides of (eqie45), we have
\[
C_p[C'[\hat{R}_i^{(\psi,\phi,c_{\text{out}}^t,c_{\text{in}}^t)}] \{id/id_i, t/\tau\}] = \approx_{\ell} C_p[C_{\ell}[R_T^{(\psi^t,\phi^t,c_{\text{out}}^t,c_{\text{in}}^t)}]]
\]
By applying context \(C_p\) on both sides of (eqie45), we have
\[
C_p[C'_{\hat{R}_i^{(\psi,\phi,c_{\text{out}}^t,c_{\text{in}}^t)}] \{id/id_i, t_1/\tau\}] = \approx_{\ell} C_p[C_{\ell}[R_T^{(\psi^t,\phi^t,c_{\text{out}}^t,c_{\text{in}}^t)}]]
\]
Because of (eqie43), combining the above two equivalences, we have
\[
C_p[C'_{\hat{R}_i^{(\psi,\phi,c_{\text{out}}^t,c_{\text{in}}^t)}] \{id/id_i, t/\tau\}] \approx_{\ell} C_p[C'_{\hat{R}_i^{(\psi,\phi,c_{\text{out}}^t,c_{\text{in}}^t)}] \{id/id_i, t_1/\tau\}] \approx_{\ell} R_T
\]
Thus, the statement is proved.

1) Since the context \(C'\) satisfies \(bn(P_w) \cap fn(C) = \emptyset\) and **eqie51:** (replacing \(C\) with \(C'\) in (eqie1))
\[
C_p[C'_{\hat{R}_i^{(\psi,\phi,c_{\text{out}}^t,c_{\text{in}}^t)}] \{id/id_i, t/\tau\}] \approx_{\ell} C_p[C_{\ell}[\hat{R}_i^{(\psi,\phi,c_{\text{out}}^t,c_{\text{in}}^t)}] \{id/id_i, t_1/\tau\}] \approx_{\ell} R_T
\]
for \(C'\), (eqie2) and (eqie3) should hold by replacing \(C\) with \(C'\).

**eqie52:**
\[
C'[P]_{\hat{R}_i^{(\psi,\phi,c_{\text{out}}^t,c_{\text{in}}^t)}} \approx_{\ell} \hat{R}_i{id/id_i, t_2/\tau},
\]
**eqie53:**
\[
C_p[C'_{\hat{R}_i^{(\psi,\phi,c_{\text{out}}^t,c_{\text{in}}^t)}] \{id/id_i, t/\tau\}] \approx_{\ell} C_p[C'_{\hat{R}_i^{(\psi^t,\phi^t,c_{\text{out}}^t,c_{\text{in}}^t)}] \{id/id_i, t_1/\tau\}] \approx_{\ell} R_T^{(\psi^t,\phi^t,c_{\text{out}}^t,c_{\text{in}}^t)}
\]
2) Combining (eqie4) and (eqie53), we have **eqie6:**
\[
C_p[C'_{\hat{R}_i^{(\psi,\phi,c_{\text{out}}^t,c_{\text{in}}^t)}] \{id/id_i, t_1/\tau\}] \approx_{\ell} C_p[C'_{\hat{R}_i^{(\psi^t,\phi^t,c_{\text{out}}^t,c_{\text{in}}^t)}] \{id/id_i, t_1/\tau\}] \approx_{\ell} R_T^{(\psi^t,\phi^t,c_{\text{out}}^t,c_{\text{in}}^t)}
\]
3) By applying evaluation context \(C_h\) := \(\nu c'_{\text{out}}(\_ \mid \text{lin}(c_{\text{out}}, x))\) \((x\text{ is a fresh variable})\) on both sides of (eqie6), we have **eqie7:**
\[
C_h[C_p[C'_{\hat{R}_i^{(\psi,\phi,c_{\text{out}}^t,c_{\text{in}}^t)}] \{id/id_i, t_1/\tau\}] \approx_{\ell} C_h[C_p[C'_{\hat{R}_i^{(\psi^t,\phi^t,c_{\text{out}}^t,c_{\text{in}}^t)}] \{id/id_i, t_1/\tau\}] \approx_{\ell} R_T^{(\psi^t,\phi^t,c_{\text{out}}^t,c_{\text{in}}^t)}
\]
4) By Lemma 4 and Lemma 5 we move the position of context \( C_h[\_] \) and have

\[
\text{eqie8: } \quad C_{P_w}[\hat{R}_i[\hat{R}_i[\hat{R}_i\{\id/\id_i, t_1/\tau\} \mid R_T^{(\psi^i, c_{\text{out}}^i, c_{\text{in}}^i)}] \approx_\ell C_{P_w}[C[P]] \mid R_T^{(\psi^i, c_{\text{out}}^i, c_{\text{in}}^i)}].
\]

6) Because of Lemma 3

\[
C_h[\hat{R}_i[\hat{R}_i\{\id/\id_i, t_1/\tau\}] \approx_\ell \hat{R}_i[\id/\id_i, t_1/\tau],
\]

thus we have that the left side of (eqie8) is equivalent to

\[
C_{P_w}[\hat{R}_i[\id/\id_i, t_1/\tau] \mid R_T^{(\psi^i, c_{\text{out}}^i, c_{\text{in}}^i)}]
\]

Because of (eqie52), we have

\[
C_h[C[P]] := C[P]^{\text{out}(\psi^i, \epsilon)} \approx_\ell \hat{R}_i[\id/\id_i, t_2/\tau].
\]

Thus, by applying context \( C_{P_w}[\_ \mid R_T^{(\psi^i, c_{\text{out}}^i, c_{\text{in}}^i)}] \) on both sides of the equivalence, we have that the right side of (eqie8) is equivalent to

\[
C_{P_w}[\hat{R}_i[\id/\id_i, t_2/\tau] \mid R_T^{(\psi^i, c_{\text{out}}^i, c_{\text{in}}^i)}]
\]

By Lemma 1 (transitivity), we have

\[
C_{P_w}[\hat{R}_i[\id/\id_i, t_1/\tau] \mid R_T^{(\psi^i, c_{\text{out}}^i, c_{\text{in}}^i)}] \approx_\ell C_{P_w}[\hat{R}_i[\id/\id_i, t_2/\tau] \mid R_T^{(\psi^i, c_{\text{out}}^i, c_{\text{in}}^i)}]
\]

The above equivalence coincides with the equivalence in ipriv (Def: [6.13]). Thus, the protocol satisfies ipriv_\( \theta \).

There exists \( \rho, \theta \) such that ipriv_\( \theta \) \( \iff \) iepriv_\( \rho, \theta \).

We prove the statement by showing an example in which a protocol satisfies ipriv_\( \theta \) but not iepriv_\( \rho, \theta \) for some \( \rho \) as in Ex. B.3.

**Example B.3. Protocol**

\[
P := Q | Q'
\]

\[
Q := \nu r . \nu s . \text{out}(\text{ch}, \text{enc}(s, r))
\]

\[
Q' := \text{in}(\text{ch}, x)
\]

where ch is a public channel, satisfies ipriv w.r.t. s and \( (Q', \{x\}, \emptyset, c_{\text{out}}, \epsilon) \), but not iepriv w.r.t. s, \( \{r\}, \emptyset, c_{\text{out}}, \epsilon \) and \( (Q', \{x\}, \emptyset, c_{\text{out}}, \epsilon) \).

When the third party Q' is coerced for the information he read. The revealing of information from Q' does not help increase the adversary’s knowledge. Therefore, the adversary still cannot distinguish \( \text{enc}(s_1, r) \) and \( \text{enc}(s_2, r) \), even when Q' reveals information, thus the protocol satisfies ipriv w.r.t. s and \( (Q', \{x\}, \emptyset, c_{\text{out}}, \epsilon) \).

However, when Q is also coerced to reveal r, the adversary can decrypt the message \( \text{out}(\text{ch}, \text{enc}(s, r)) \) and find s. In addition, there is no way for Q to cheat the adversary. Due to the perfect encryption assumption, any other nonce cannot be used to decrypt \( \text{enc}(s, r) \), thus, the adversary will find out whether the user lied. Therefore, the protocol does not satisfy iepriv w.r.t. s, \( \{r\}, \emptyset, c_{\text{out}}, \epsilon \) and \( (Q', \{x\}, \emptyset, c_{\text{out}}, \epsilon) \).
∀θ, ieprivρ,θ =⇒ eprivρ

We prove the statement in the following two directions: 1. ∀θ, ieprivρ,θ =⇒ eprivρ
2. ∃ρ, θ, eprivρ =⇒ ieprivρ,θ

1. ∀θ = (R_T, (Ψ, Φ, c_{out}, c_{in})), when a protocol satisfies ieprivρ,θ for some ρ, we prove that the protocol also satisfies eprivρ with the existence of R_T.

For a collaboration of third parties θ = (R_T, (Ψ, Φ, c_{out}, c_{in})), when a well-formed protocol P_w satisfies iepriv w.r.t. target data τ, (Ψ, Φ, c_{out}, c_{in}) and (R_T, (Ψ, Φ, c_{out}, c_{in})), there exists a closed plain process P_f such that for any context C[.] := νc_{out}.νc_{in}.(- | Q) satisfying bn(P_w) ∩ fn(C[.]) = ∅ and

eqiee1:

C_{P_w}[C[R_i(Ψ, Φ, c_{out}, c_{in}) \{id/\nu id_i, τ/τ\}] | R_T] ≅_ℓ C_{P_w}[\hat{R}_i(Ψ, Φ, c_{out}, c_{in}) \{id/\nu id_i, τ/τ\}] | R_T],

we have
eqiee2:

C[P_f]^\nu c_{out}(\nu c_{out}^1) ≅_ℓ \hat{R}_\nu \nu id_i, τ/τ],

eqiee3:

C_{P_w}[C[\hat{R}_i(Ψ, Φ, c_{out}, c_{in}) \{id/\nu id_i, τ/τ\}] | R_T(Ψ, Φ, c_{out}, c_{in})] ≅_ℓ C_{P_w}[C[P_f] | R_T(Ψ, Φ, c_{out}, c_{in})].

1) Since for any context of the adversary which provides information for the collaborating third parties, the equivalence (eqiee3) holds. Thus, for the following context C_t[.] of the adversary, the equivalence still holds. C_t[.] := νc_{out}\nu c_{in}(\nu Q) satisfying bn(P_w) ∩ fn(C_t[.]) = ∅ and

eqiee4:

C_t[R_T(Ψ, Φ, c_{out}, c_{in})] ≅_ℓ R_T(Ψ, Φ, c_{out}, c_{in}).

That is, by applying the context C_t[.] on both sides of (eqiee3), we have,
eqiee5:

C_t[C_{P_w}[C[\hat{R}_i(Ψ, Φ, c_{out}, c_{in}) \{id/\nu id_i, τ/τ\}] | R_T(Ψ, Φ, c_{out}, c_{in})]] ≅_ℓ C_t[C_{P_w}[C[P_f] | R_T(Ψ, Φ, c_{out}, c_{in})]]

2) By applying the evaluation context C_h[.] := νc_{out}(\nu ln(c_{out}, x)) (x is a fresh variable), on both sides of (eqiee5), we have

eqiee6:

C_h[C_t[C_{P_w}[C[\hat{R}_i(Ψ, Φ, c_{out}, c_{in}) \{id/\nu id_i, τ/τ\}] | R_T(Ψ, Φ, c_{out}, c_{in})]]] ≅_ℓ C_h[C_t[C_{P_w}[C[P_f] | R_T(Ψ, Φ, c_{out}, c_{in})]].

3) By Lemma 4 and Lemma 5, we move the position of the contexts C_h[.] and C_t[.] in (eqiee6) and have

eqiee7:

C_{P_w}[C[\hat{R}_i(Ψ, Φ, c_{out}, c_{in}) \{id/\nu id_i, τ/τ\}] | C_h[C_t[R_T(Ψ, Φ, c_{out}, c_{in})]]] ≅_ℓ C_{P_w}[C[P_f] | C_h[C_t[R_T(Ψ, Φ, c_{out}, c_{in})]].
4) By applying context \( C'_h[C_P[w]C[C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}]\{\text{id}/id_i,t/\tau]\}] \) on both sides of (eqiee4), we have

\[
C'_h[C_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}]\{\text{id}/id_i,t/\tau]\} \approx_c C[P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}]\{\text{id}/id_i,t/\tau\}].
\]

5) By Lemma 5, we move the position of context \( C'_h[\ ] \) and have (eqiee9):

\[
C_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}]\{\text{id}/id_i,t/\tau\} \approx_c C_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}]\{\text{id}/id_i,t/\tau\}.
\]

6) By Lemma 1 (transitivity), combining (eqiee7) and (eqiee9), we have (eqiee10):

\[
C_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}]\{\text{id}/id_i,t/\tau\} \approx_c C_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}]\{\text{id}/id_i,t/\tau\}.
\]

7) According to Lemma 5 (hide on channel), we have

\[
C'_h[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}] \approx_c R_T.
\]

8) By Lemma 1 (transitivity), combining the above equivalence and (eqiee4), we have

\[
C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}]\approx_c R_T.
\]

9) Thus, by applying context \( C_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}] \) on both sides of the above equivalence, the left side of (eqiee10) is bisimilar to

\[
C_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}] \approx_c R_T.
\]

and by applying context \( C_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}] \) on both sides of the above equivalence, the right side of (eqiee10) is bisimilar to

\[
C_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}] \approx_c R_T.
\]

Thus, (eqiee11):

\[
C_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}] \approx_c R_T.
\]

10) Because of rule

\[
!P \equiv P || P,
\]

\( R_T \) can be absorbed by the context. That is, \( C_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}] \) is a type of context where there requires \( R_T \) to be present. We define \( C'_P[w] := C_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}] \), where \( R_T \) has to be present in the context. Thus, we have (eqiee12):

\[
C_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}] \approx_c C_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}]\{\text{id}/id_i, t/\tau\}.
\]

From (eqiee1), by replacing the context \( C_P[w] \) with \( C'_P[w] \), we have (eqiee13):

\[
C'_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}]\{\text{id}/id_i, t/\tau\}] \approx_c C'_P[w]C[\hat{R}_{\psi,\Phi,c_{\text{out}},c_{\text{in}}}]\{\text{id}/id_i, t_1/\tau\}.
\]

Therefore, for any context \( C[\ ] \) satisfying (eqiee13), (eqiee2) and (eqiee12) hold. Thus, the protocol satisfies epriv_p. 

\( \square \)
B.3 Theorem. [6.21]

2. There exists \( \theta, \rho \) such that \( epriv_\rho \iff i epriv_{\rho, \theta} \).

We prove the statement by showing an example in which a protocol satisfies \( epriv_\rho \) but not \( i epriv_{\rho, \theta} \) for some \( \theta \) as in Ex. B.4.

Example B.4. The following protocol

\[
\begin{align*}
P & := \nu \text{untapch}(Q \mid Q') \\
Q & := \nu s.\text{out}(\text{untapch}, s) \\
Q' & := \text{in}(\text{untapch}, x)
\end{align*}
\]

where \( \text{untapch} \) is an untappable channel, satisfies \( epriv \) w.r.t. \( s \) and collaboration \( \langle \{s\}, \emptyset, c_{out}, e \rangle \), but not \( i epriv \) w.r.t. \( s \), collaboration \( \langle \{s\}, \emptyset, c_{out}, e \rangle \) and third party collaboration \( \langle Q', \{x\}, \emptyset, c_{out}, e \rangle \).

Since the communication is untappable, \( Q \) can lie about \( s \) to be \( s' \), the adversary cannot detect whether \( Q \) lied, thus the protocol satisfies \( epriv \) w.r.t. \( s \) and \( \langle \{s\}, \emptyset, c_{out}, e \rangle \). However, when the communication partner \( Q' \) reveals the secret information that he reads in on the untappable channel, \( s \) is revealed. Thus, the protocol does not satisfies \( i epriv \) w.r.t. \( s \), \( \langle \{s\}, \emptyset, c_{out}, e \rangle \) and \( \langle Q', \{x\}, \emptyset, c_{out}, e \rangle \).

B.3 Theorem. [6.21]

(3) \( \forall \rho, \ cpriv_{\rho, \delta} \implies cpriv_\delta \)

With the above assumption, we prove the statement in the following two directions:

1. \( \forall \rho, \ cpriv_{\rho, \delta} \implies cpriv_\delta \)

2. \( \exists \rho, \delta, \ cpriv_{\rho, \delta} \iff cepriv_{\rho, \delta} \)

1. \( \forall \rho \), when a protocol satisfies \( cepriv_{\rho, \delta} \) for some \( \delta \), we prove that the protocol also satisfies \( cpriv_\delta \).

For a collaboration \( \rho = \langle \Psi, \Phi, c_{out}, c_{in} \rangle \), when a well-formed protocol \( P_w \) satisfies \( cepriv \) w.r.t. \( \tau, \langle \Psi, \Phi, c_{out}, c_{in} \rangle \) and \( \langle R_D, \langle \Theta, \Delta, \Pi \rangle \rangle \), there exists a closed plain process \( P_f \), such that for any context \( C[\_] := \nu c_{out}.\nu c_{in}.(\_ \mid Q) \) satisfying \( bn(P_w) \cap fn(C[\_]) = \emptyset \) and

**eqc1:**

\[
C_{P_w}[C[\hat{R}_1\{id/\hat{id}_1, t/\tau\}^{(\Psi, \Phi, c_{out}, c_{in})}] \approx_\ell C_{P_w}[\hat{R}_1\{id/\hat{id}_1, t_1/\tau\}^{(\Psi, \Phi, c_{out}, c_{in})}],
\]

we have

**eqc2:**

\[
\nu \Omega.(\nu \eta.(C[P_f])^{\text{out}(c_{out}^\ast)} \mid P_\gamma) \mid R_D^{(\Theta, \Delta, \Pi)}) \approx_\ell \nu \Omega.(\hat{R}_1\{id/\hat{id}_1, t_2/\tau\} \mid R_D^{(\Theta, \Delta, \Pi)}),
\]

**eqc3:**

\[
C_{P_w}[C[\hat{R}_1\{id/\hat{id}_1, t/\tau\}^{(\Psi, \Phi, c_{out}, c_{in})}] \mid R_D] \approx_\ell C_{P_w}[\nu \Omega.(\nu \eta.(C[P_f]) \mid P_\gamma) \mid R_D^{(\Theta, \Delta, \Pi)}].
\]

1) By applying context \( C_h[\_] \) on both side of (eqc3), we have

**eqc4:**

\[
C_h[C_{P_w}[C[\hat{R}_1\{id/\hat{id}_1, t/\tau\}^{(\Psi, \Phi, c_{out}, c_{in})}] \mid R_D]] \approx_\ell C_h[C_{P_w}[\nu \Omega.(\nu \eta.(C[P_f]) \mid P_\gamma) \mid R_D^{(\Theta, \Delta, \Pi)}]].
\]
2) By Lemma 5, we move the position of $C_h[\_\_\_\_\_\_]$, and have
\[
C_{P_w}[C_h[C[\ell_1{id/\ell_1}, t/\ell_1]\{\psi, \phi, c_{\text{out}}, c_{\text{in}}\}] | R_D]
\approx \ell C_{P_w}[\nu \Omega.((\nu \eta.(C_h[C[P_j]] | P_\gamma)) | R_D^{(\Theta, \Delta, \Pi)})].
\]

3) The context $C_{P_w}[\_\_\_\_\_]$ has the following form:
\[
C_{P_w}[\_\_\_\_\_] := \nu \tilde{c}.(\text{genkey} \ | \! R_1 | \ldots | \! R_p | \_).
\]
Because of (eqc1) and rule $!P \equiv P \ | \! P$, we have
\[
C_{P_w}[C[C[\ell_1{id/\ell_1}, t/\ell_1]\{\psi, \phi, c_{\text{out}}, c_{\text{in}}\}] | R_D]
\approx \ell C_{P_w}[\nu \Omega.((\nu \eta.(C_h[C[P_j]] | P_\gamma)) | R_D^{(\Theta, \Delta, \Pi)})].
\]

4) By applying $C_h[\_\_\_\_\_]$ on both side of (eqc6), we have
\[
C_h[C_{P_w}[C[C[\ell_1{id/\ell_1}, t/\ell_1]\{\psi, \phi, c_{\text{out}}, c_{\text{in}}\}] | R_D]]
\approx \ell C_h[C_{P_w}[\nu \Omega.((\nu \eta.(C_h[C[P_j]] | P_\gamma)) | R_D^{(\Theta, \Delta, \Pi)})]].
\]

5) By Lemma 5, we move the position of $C_h[\_\_\_\_\_]$ and have
\[
C_{P_w}[C_h[C[C[\ell_1{id/\ell_1}, t/\ell_1]\{\psi, \phi, c_{\text{out}}, c_{\text{in}}\}] | R_D]]
\approx \ell C_{P_w}[C_h[C[\ell_1{id/\ell_1}, t_1/\ell_1]\{\psi, \phi, c_{\text{out}}, c_{\text{in}}\}] | R_D]].
\]

6) By Lemma 5, combining (eqc5) and (eqc8), we have
\[
C_{P_w}[C_h[C[C[\ell_1{id/\ell_1}, t_1/\ell_1]\{\psi, \phi, c_{\text{out}}, c_{\text{in}}\}] | R_D]]
\approx \ell C_{P_w}[C[C[\ell_1{id/\ell_1}, t_1/\ell_1]\{\psi, \phi, c_{\text{out}}, c_{\text{in}}\}] | R_D]].
\]

7) By Lemma 5, we have
\[
C_{P_w}[C[C[\ell_1{id/\ell_1}, t_1/\ell_1]\{\psi, \phi, c_{\text{out}}, c_{\text{in}}\}] | R_D]]
\approx \ell C_{P_w}[\nu \Omega.((\nu \eta.(C_h[C[P_j]] | P_\gamma)) | R_D^{(\Theta, \Delta, \Pi)})].
\]
Thus, we have (by applying context $C_{P_w}[\_\_\_\_\_\_]$ on the above equivalence)
\[
C_{P_w}[\nu \Omega.((\nu \eta.(C_h[C[P_j]] | P_\gamma)) | R_D^{(\Theta, \Delta, \Pi)})]
\approx \ell C_{P_w}[\nu \Omega.((\nu \eta.(C_h[C[P_j]] | P_\gamma)) | R_D^{(\Theta, \Delta, \Pi)})]
\]
That is, the left side of (eqc9) is equivalent to
\[
C_{P_w}[\nu \Omega.((\nu \eta.(C_h[C[P_j]] | P_\gamma)) | R_D^{(\Theta, \Delta, \Pi)})]
\]

8) According to Lemma 5, we have
\[
\nu \Omega.((\nu \eta.(C_h[C[P_j]] | P_\gamma)) | R_D^{(\Theta, \Delta, \Pi)}) \approx \ell \nu \Omega.((\nu \eta.(C_h[C[P_j]] | P_\gamma)) | R_D^{(\Theta, \Delta, \Pi)})
\]
Because of (eqc2), we have
\[
\nu \Omega.((\nu \eta.(C_h[C[P_j]] | P_\gamma)) | R_D^{(\Theta, \Delta, \Pi)}) \approx \ell \nu \Omega.((\nu \eta.(C_h[C[P_j]] | P_\gamma)) | R_D^{(\Theta, \Delta, \Pi)})
\]

9) By applying context $C_{P_w}[\_\_\_\_\_\_]$ on both sides of (eqc11), we have
\[
C_{P_w}[\nu \Omega.((\nu \eta.(C_h[C[P_j]] | P_\gamma)) | R_D^{(\Theta, \Delta, \Pi)})]
\approx \ell C_{P_w}[\nu \Omega.((\nu \eta.(C_h[C[P_j]] | P_\gamma)) | R_D^{(\Theta, \Delta, \Pi)})]
\]
That is, the right side of (eqc9) is equivalent to
\[ C_{P_w}[\nu\Omega.((\hat{R}_t\{\text{id}/\text{id}_1,\ t_2/\tau}\{R_D\})^{(\theta,\Delta,\Pi)}]. \]

10) Combining (eqc10) and (eqc12), we have
\[ C_{P_w}[\hat{R}_t\{\text{id}/\text{id}_1,\ t_1/\tau\} \ | \ R_D] \approx' C_{P_w}[\nu\Omega.((\hat{R}_t\{\text{id}/\text{id}_1,\ t_2/\tau\})|R_D)^{(\theta,\Delta,\Pi)}]. \]

Therefore, the protocol satisfies cpriv. \(\square\)

2. There exists \(\rho, \delta\) such that cpriv\(_{\rho,\delta}\) \(\iff\) cepriv\(_{\rho,\delta}\).

We prove the statement by showing an example in which a protocol satisfies cpriv\(_{\rho,\delta}\) but not cepriv\(_{\rho,\delta}\) for some \(\rho, \delta\). As shown in Section 6.5, vote-privacy is an instance of cpriv where the defending third party is the counter-balancing voter, and the coalition is the counter-balancing voter replaces his vote to counter balance to target voter’s vote, and receipt-freeness is an instance of cepriv with the same defending third party and coalition. The protocol FOO02 [FOO02] is shown to satisfy vote-privacy but not receipt-freeness [DKR09].

(4) \(\forall \theta, \) cpriv\(_{\theta,\delta}\) \(\implies\) cpriv\(_{\delta}\)

We prove the statement in the following two directions: 1. \(\forall \theta, \) cpriv\(_{\theta,\delta}\) \(\implies\) cpriv\(_{\delta}\)

2. \(\exists \theta, \delta, \) cpriv\(_{\delta}\) \(\iff\) cpriv\(_{\theta,\delta}\)

1. \(\forall \theta = (R_T, (\Psi^t, \Phi^t, c_{\text{out}}^t, c_m^t))\), when a protocol satisfies ipriv\(_{\theta,\delta}\) for some \(\delta\), we prove that the protocol also satisfies cpriv\(_{\delta}\) with the existence of \(R_T\).

For a collaboration of third parties \(\theta = (R_T, (\Psi^t, \Phi^t, c_{\text{out}}^t, c_m^t))\), when a well-formed protocol \(P_w\) satisfies cpriv w.r.t. \(\tau, (R_T, (\Psi^t, \Phi^t, c_{\text{out}}^t, c_m^t))\) and \((R_D, (\theta, \Delta, \Pi))\) the following equivalence holds.

eqc1:
\[
C_{P_w}[\hat{R}_t\{\text{id}/\text{id}_1,\ t_1/\tau\} \ | \ R_D] \approx' C_{P_w}[\nu\Omega.((\hat{R}_t\{\text{id}/\text{id}_1,\ t_2/\tau\})|R_D)^{(\theta,\Delta,\Pi)}] \approx' C_{P_w}[\nu\Omega.((\hat{R}_t\{\text{id}/\text{id}_1,\ t_2/\tau\})|R_D)^{(\theta,\Delta,\Pi)}].
\]

Since for all context of the adversary which supplies information needed by the collaborating third parties the protocol satisfies cpriv\(_{\theta,\delta}\), thus, for the following context which provides information for collaborating third parties, \(C_{i}[] := \nu c_{\text{out}}^i, \nu c_m^i, (\_ | Q)\) satisfying bn\((P_w) \cap fn(C_{i}[]) = \emptyset\) and

eqc2:
\[
C_i[R_t^{(\Psi^t, \Phi^t, c_{\text{out}}^t, c_m^t)}] \approx C_t[R_t^{(\Psi^t, \Phi^t, c_{\text{out}}^t, c_m^t)}],
\]

the protocol satisfies cpriv\(_{\theta,\delta}\).

1) By applying context \(C_{i}[]\) on both sides of (eqc1), we have

eqc3:
\[
C_{i}[C_{P_w}[\hat{R}_t\{\text{id}/\text{id}_1,\ t_1/\tau\} \ | \ R_D] \approx C_{i}[C_{P_w}[\nu\Omega.((\hat{R}_t\{\text{id}/\text{id}_1,\ t_2/\tau\})|R_D)^{(\theta,\Delta,\Pi)}] \approx C_{i}[C_{P_w}[\nu\Omega.((\hat{R}_t\{\text{id}/\text{id}_1,\ t_2/\tau\})|R_D)^{(\theta,\Delta,\Pi)}].
\]

B.3 Theorem [6.21]
2) By applying the evaluation context $\hat{C}_1^t[\cdot] := \nu C_{out}^{t}(\nu \log(c_{out}^{t}, x))$ ($x$ is a fresh variable), on both sides of (eqci3), we have
\[
\text{eqci4:}
\]
\[
C_1^t[C_{P,w}[\hat{R}_t\{id/id_i, t_1/\tau\} \mid R_D \mid R_T^{(\psi_i,\phi_i,c_{out}^{t},c_{in}^{t})}]] \\
\\approx \ell C_1^t[C_{P,w}[\nu \Omega.((\hat{R}_t\{id/id_i, t_2/\tau\} \mid R_D)^{\Theta, \Delta, H}) \mid R_T^{(\psi_i,\phi_i,c_{out}^{t},c_{in}^{t})}]].
\]
3) According to Lemma 4 and Lemma 5, we move the position of contexts $C_1^t[\cdot]$ and $C_1[\cdot]$ and have
\[
\text{eqci5:}
\]
\[
C_{P,w}[\hat{R}_t\{id/id_i, t_1/\tau\} \mid R_D \mid C_1^t[C_{P,w}[\nu \Omega.((\hat{R}_t\{id/id_i, t_2/\tau\} \mid R_D)^{\Theta, \Delta, H}) \mid C_1^t[R_T^{(\psi_i,\phi_i,c_{out}^{t},c_{in}^{t})}]]].
\]
4) By applying context $C_1^t[\cdot]$ on both sides of (eqci2), we have
\[
C_1^t[C_{P,w}[\nu \Omega.((\hat{R}_t\{id/id_i, t_2/\tau\} \mid R_D)^{\Theta, \Delta, H}) \mid C_1^t[R_T^{(\psi_i,\phi_i,c_{out}^{t},c_{in}^{t})}]] \\
\approx \ell C_1^t[R_T^{(\psi_i,\phi_i,c_{out}^{t},c_{in}^{t})}].
\]
Because of Lemma 3, we have
\[
C_1^t[R_T^{(\psi_i,\phi_i,c_{out}^{t},c_{in}^{t})}] \approx \ell R_T.
\]
Thus, by transitivity, combining the above two equivalences, we have
\[
C_1^t[C_{P,w}[\nu \Omega.((\hat{R}_t\{id/id_i, t_2/\tau\} \mid R_D)^{\Theta, \Delta, H}) \mid C_1^t[R_T^{(\psi_i,\phi_i,c_{out}^{t},c_{in}^{t})}]] \approx \ell R_T.
\]
Thus, by applying context $C_{P,w}[\hat{R}_t\{id/id_i, t_1/\tau\} \mid R_D \mid \_]$ and context $C_{P,w}[\nu \Omega.((\hat{R}_t\{id/id_i, t_2/\tau\} \mid R_D)^{\Theta, \Delta, H}) \mid \_]$ on both sides of the above equivalence, because of transitivity via (eqci5), we have
\[
\text{eqci6:}
\]
\[
C_{P,w}[\hat{R}_t\{id/id_i, t_1/\tau\} \mid R_D \mid R_T] \\
\approx \ell C_{P,w}[\nu \Omega.((\hat{R}_t\{id/id_i, t_2/\tau\} \mid R_D)^{\Theta, \Delta, H}) \mid R_T].
\]
5) Since $R_T$ can be absorbed by the context $C_{P,w}[\_]$, we have
\[
\text{eqci7:}
\]
\[
C_{P,w}[\hat{R}_t\{id/id_i, t_1/\tau\} \mid R_D] \approx \ell C_{P,w}[\nu \Omega.((\hat{R}_t\{id/id_i, t_2/\tau\} \mid R_D)^{\Theta, \Delta, H})].
\]
Thus, the protocol satisfies $\text{cpriv}$. \hfill $\square$

2. There exists $\theta$ such that $\text{cpriv}_\delta \implies \text{cpriv}_{\theta, \delta}$.

We prove the statement by showing an example in which a protocol satisfies $\text{cpriv}_\delta$ for some $\delta$ but not $\text{cpriv}_{\theta, \delta}$ for some $\theta$. For instance, Dreier et al. proved that the protocol by Lee et al. [LBD+03] satisfies vote-privacy – an instance of $\text{cpriv}$ where coalition is the counter-balancing voter votes differently from the target voter, but not vote-independence – an instance of $\text{cpriv}$ where the coalition is the same as in $\text{cpriv}$ and the attacking third party is the third voter [DLLT].

(2) $\forall \rho, \text{ciepriv}_{\rho, \theta, \delta} \implies \text{ciepriv}_{\theta, \delta}$.

We prove the statement in the following two directions: 1. $\forall \rho, \text{ciepriv}_{\rho, \theta, \delta} \implies \text{ciepriv}_{\theta, \delta}$ 2. $\exists \rho, \theta, \delta, \text{ciepriv}_{\theta, \delta} \implies \text{ciepriv}_{\rho, \theta, \delta}$.
1. ∀\(\rho\), when a protocol satisfies ciepriv\(_{\rho,\theta,\delta}\) for some \(\theta, \delta\), we prove that the protocol also satisfies cipriv\(_{\theta,\delta}\).

For a collaboration \(\rho = (\Psi, \Phi, c_{out}, c_{in})\), when a well-formed protocol \(P_w\) satisfies ciepriv w.r.t. \(\tau, (\Psi, \Phi, c_{out}, c_{in})\), \((R_T, (\Psi^t, \Phi^t, c_{out}^t, c_{in}^t))\) and \(R_D, (\Theta, \Delta, \Pi)\), there exists a closed plain process \(P_f\), such that for any context \(C_{[\cdot]} := \nu c_{out}. \nu c_{in} (\cdot \mid Q)\) satisfying \(bn(P_w) \cap fn(C_{[\cdot]}) = \emptyset\)

\[\text{eqiei1:}\]
\[C_{P_w}[C_{\hat{R}_i} \{\text{id}/id_i, \text{t}/\tau\} (\Psi, \Phi, c_{out}, c_{in})] \mid R_T \mid R_D\]
\[\approx \_ \_ \] \[C_{P_w}[\hat{R}_i \{\text{id}/id_i, \text{t}_1/\tau\} (\Psi, \Phi, c_{out}, c_{in})] \mid R_T \mid R_D,\]

we have

\[\text{eqiei2:}\]
\[\nu \Omega. (\nu \eta. (C_{[\cdot]}^\text{out}(\cdot)) \mid P_f) \mid R_D^{(\Theta, \Delta, \Pi)} \approx \_ \_ \] \[\nu \Omega. (\nu \eta. (C_{[\cdot]} \mid P_f) \mid R_D^{(\Theta, \Delta, \Pi)},\]

\[\text{eqiei3:}\]
\[C_{P_w}[C_{\hat{R}_i} \{\text{id}/id_i, \text{t}/\tau\} (\Psi, \Phi, c_{out}, c_{in})] \mid R_D \mid R_T^{(\Psi^t, \Phi^t, c_{out}^t, c_{in}^t)}\]
\[\approx \_ \_ \] \[C_{P_w}[\hat{R}_i \{\text{id}/id_i, \text{t}_1/\tau\} (\Psi, \Phi, c_{out}, c_{in})] \mid R_D \mid R_T^{(\Psi^t, \Phi^t, c_{out}^t, c_{in}^t)},\]

then this context satisfies the following equivalence (replacing \(C_{[\cdot]}\) with \(C_{[\cdot]}\) in (eqiei1)) when \(R_T\) exists.

\[C_{P_w}[C_{\hat{R}_i} \{\text{id}/id_i, \text{t}/\tau\} (\Psi, \Phi, c_{out}, c_{in})] \mid R_T \mid R_D\]
\[\approx \_ \_ \] \[C_{P_w}[\hat{R}_i \{\text{id}/id_i, \text{t}_1/\tau\} (\Psi, \Phi, c_{out}, c_{in})] \mid R_T \mid R_D,\]

2) Thus, for \(C_{[\cdot]}\), the following equivalence holds (replacing \(C_{[\cdot]}\) with \(C_{[\cdot]}\) in (eqiei2) and (eqiei3)).

\[\text{eqiei5:}\]
\[\nu \Omega. (\nu \eta. (C_{[\cdot]}^\text{out}(\cdot)) \mid P_f) \mid R_D^{(\Theta, \Delta, \Pi)} \approx \_ \_ \] \[\nu \Omega. (\nu \eta. (C_{[\cdot]} \mid P_f) \mid R_D^{(\Theta, \Delta, \Pi)},\]

\[\text{eqiei6:}\]
\[C_{P_w}[C_{\hat{R}_i} \{\text{id}/id_i, \text{t}/\tau\} (\Psi, \Phi, c_{out}, c_{in})] \mid R_D \mid R_T^{(\Psi^t, \Phi^t, c_{out}^t, c_{in}^t)}\]
\[\approx \_ \_ \] \[C_{P_w}[\hat{R}_i \{\text{id}/id_i, \text{t}_1/\tau\} (\Psi, \Phi, c_{out}, c_{in})] \mid R_D \mid R_T^{(\Psi^t, \Phi^t, c_{out}^t, c_{in}^t)},\]

3) Combining (eqiei4) and (eqiei6), we have

\[\text{eqiei7:}\]
\[C_{P_w}[\nu \Omega. (\nu \eta. (C_{[\cdot]} \mid P_f) \mid R_D^{(\Theta, \Delta, \Pi)} \mid R_T^{(\Psi^t, \Phi^t, c_{out}^t, c_{in}^t)},\]
\[\approx \_ \_ \] \[C_{P_w}[\hat{R}_i \{\text{id}/id_i, \text{t}_1/\tau\} (\Psi, \Phi, c_{out}, c_{in})] \mid R_D \mid R_T^{(\Psi^t, \Phi^t, c_{out}^t, c_{in}^t)}\]
4) By applying evaluation context \( C_h[.] := \nu c_{out} \cdot (\_ | \in(c_{out}, x)) \) (\( x \) is a fresh variable) on both sides of (eqiei7), we have (eqiei8):

\[
C_h[C_{\text{P}_w}[\nu \Omega \cdot (\nu \eta \cdot (C'[P_f] | P_{\gamma}) | R_{D}^{(\theta, \Delta, II)} | R_{T}^{(\Psi, \Phi, c_{out}', c_{m}')} \]) \\
\approx_{\ell} C_h[C_{\text{R}_i}[\text{id} / \text{id}_i, t_1 / \tau]^{(\Psi, \Phi, c_{out}', c_{m}')} | R_{D} | R_{T}^{(\Psi, \Phi, c_{out}', c_{m}')}]
\]

5) By Lemma 4 and Lemma 5 we move the position of context \( C_h[.] \) and have (eqiei9):

\[
C_{\text{P}_w}[\nu \Omega \cdot (\nu \eta \cdot (C_h[C'[P_f] | P_{\gamma}) | R_{D}^{(\theta, \Delta, II)} | R_{T}^{(\Psi, \Phi, c_{out}', c_{m}')} \]) \\
\approx_{\ell} C_{\text{P}_w}[C_h[\text{id} / \text{id}_i, t_1 / \tau]^{(\Psi, \Phi, c_{out}', c_{m}')} | R_{D} | R_{T}^{(\Psi, \Phi, c_{out}', c_{m}')}]
\]

6) Because of Lemma 3 we have

\[
C_h[\hat{R}_i[\text{id} / \text{id}_i, t_1 / \tau]^{(\Psi, \Phi, c_{out}', c_{m}')} \approx_{\ell} \hat{R}_i[\text{id} / \text{id}_i, t_1 / \tau],
\]

thus, we have that the right side of (eqiei9) is equivalent to

\[
C_{\text{P}_w}[\hat{R}_i[\text{id} / \text{id}_i, t_1 / \tau] | R_{D} | R_{T}^{(\Psi, \Phi, c_{out}', c_{m}')}}.
\]

7) By applying context \( C_{\text{P}_w}[.] | R_{T}^{(\Psi, \Phi, c_{out}', c_{m}')} \) on both sides of (eqiei2), we have

\[
C_{\text{P}_w}[\nu \Omega \cdot (\nu \eta \cdot (C'[P_f])^{\text{out}(c_{out}') | P_{\gamma}) | R_{D}^{(\theta, \Delta, II)} | R_{T}^{(\Psi, \Phi, c_{out}', c_{m}')} \]) \\
\approx_{\ell} C_{\text{P}_w}[\nu \Omega \cdot (\hat{R}_i[\text{id} / \text{id}_i, t_2 / \tau]) | R_{D})^{(\theta, \Delta, II)} | R_{T}^{(\Psi, \Phi, c_{out}', c_{m}')}}
\]

That is, the left side of (eqiei9) is equivalent to

\[
C_{\text{P}_w}[\nu \Omega \cdot (\hat{R}_i[\text{id} / \text{id}_i, t_2 / \tau]) | R_{D})^{(\theta, \Delta, II)} | R_{T}^{(\Psi, \Phi, c_{out}', c_{m}')}}
\]

Therefore, by transitivity, we have

\[
C_{\text{P}_w}[\hat{R}_i[\text{id} / \text{id}_i, t_1 / \tau] | R_{D} | R_{T}^{(\Psi, \Phi, c_{out}', c_{m}')}} \\
\approx_{\ell} C_{\text{P}_w}[\nu \Omega \cdot (\hat{R}_i[\text{id} / \text{id}_i, t_2 / \tau]) | R_{D})^{(\theta, \Delta, II)} | R_{T}^{(\Psi, \Phi, c_{out}', c_{m}')}}
\]

Therefore, the protocol satisfies cipriv_{\theta, \delta}.

\[
\square
\]

2. There exists \( \rho, \theta, \delta \) such that cipriv_{\theta, \delta} \neq ciepriv_{\rho, \theta, \delta}.

We prove the statement by showing an example in which a protocol satisfies cipriv_{\theta, \delta} but not ciepriv_{\rho, \theta, \delta} for some \( \rho, \theta, \delta \). For instance, Dreier et al. prove that the voting protocol FOO92 [FOO92] satisfies vote-independence – an instance of cipriv where the coalition is the counter-balancing voter votes differently from the target voter and the attacking third party is the third voter, but not vote-independence with passive collaboration – an instance of ciepriv where the coalition and attacking third party are the same as in cipriv and the collaboration is forwarding private information to the adversary [DLL11].
(1) \( \forall \theta, \text{ciepriv}_{\rho,\theta,\delta} \implies \text{cepriv}_{\rho,\delta} \)

We prove the statement in the following two directions: 1. \( \forall \theta, \text{ciepriv}_{\rho,\theta,\delta} \implies \text{cepriv}_{\rho,\delta} \) 2. \( \exists \rho, \theta, \delta, \text{cepriv}_{\rho,\delta} \not\implies \text{ciepriv}_{\rho,\theta,\delta} \)

1. \( \forall \theta = (R_T, (\Psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t)), \) when a protocol satisfies \( \text{ciepriv}_{\rho,\theta,\delta} \) for some \( \rho, \delta, \) we prove that the protocol also satisfies \( \text{cepriv}_{\rho,\delta} \) with the existence of \( R_T. \)

For a collaboration of third parties \( \theta = (R_T, (\Psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t)), \) when a well-formed protocol \( P_w \) satisfies \( \text{ciepriv} \) w.r.t. \( \tau, (\Psi, \Phi, c_{\text{out}}, c_{\text{in}}), (R_D, (\Theta, \Delta, \Pi)) \) and \( (R_T, (\Psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t)), \) there exists a closed plain process \( P_f, \) such that for any context \( C[,]:=\nu c_{\text{out}},\nu c_{\text{in}},(-)Q \) satisfying \( \text{bn}(P_w) \cap \text{fn}(C[,]) = \emptyset \) and \( \text{eqciee1:} \)

\[
C_{P_w}[C[R_i^{(\Psi,\Phi,c_{\text{out}},c_{\text{in}})}\{\text{id}\}/\text{id}_i, t/\tau)] \mid R_T \mid R_D] \\
\approx \ell C_{P_w}[\nu \Omega.(\nu\eta.(C[P_f] \mid P_\gamma) \mid R_D^{(\Theta,\Delta,\Pi)})] \mid R_T^{(\Psi,\Phi,c_{\text{out}},c_{\text{in}})}]
\]

we have \( \text{eqciee2:} \)

\[
\nu \Omega.(\nu\eta.(C[P_f] \mid P_\gamma) \mid R_D^{(\Theta,\Delta,\Pi)}) \approx \ell \nu \Omega.(\nu\eta.(C[P_f] \mid P_\gamma) \mid R_D^{(\Theta,\Delta,\Pi)}) \mid R_T^{(\Psi,\Phi,c_{\text{out}},c_{\text{in}})}
\]

1) Since for any context of the adversary which provides information for the collaborating third parties, the equivalence (eqciee3) holds. Thus, for the following context \( C[,]:=\nu c_{\text{out}},\nu c_{\text{in}},(-)Q \) satisfying \( \text{bn}(P_w) \cap \text{fn}(C[,]) = \emptyset \) and \( \text{eqciee4:} \)

\[
C_{\theta}[R_T^{(\Psi,\Phi,c_{\text{out}},c_{\text{in}})}] \approx \ell R_T^{(\Psi,\Phi,c_{\text{out}},c_{\text{in}})}
\]

Therefore, by applying the context \( C[,]:=\nu c_{\text{out}},\nu c_{\text{in}},(-)Q \) satisfying \( \text{bn}(P_w) \cap \text{fn}(C[,]) = \emptyset \) and \( \text{eqciee5:} \)

\[
C_{\ell}[C_{P_w}[C[R_i^{(\Psi,\Phi,c_{\text{out}},c_{\text{in}})}\{\text{id}\}/\text{id}_i, t/\tau]] \mid R_D \mid R_T^{(\Psi,\Phi,c_{\text{out}},c_{\text{in}})}]
\approx \ell C_{\ell}[C_{P_w}[\nu \Omega.(\nu\eta.(C[P_f] \mid P_\gamma) \mid R_D^{(\Theta,\Delta,\Pi)})] \mid R_T^{(\Psi,\Phi,c_{\text{out}},c_{\text{in}})}]
\]

2) By applying the evaluation context \( C_{\theta}[,]:=\nu c_{\text{out}},\nu c_{\text{in}},(-)Q \) (x is a fresh variable), on both sides of (eqciee5), we have \( \text{eqciee6:} \)

\[
C_{\ell}[C_{P_w}[C[R_i^{(\Psi,\Phi,c_{\text{out}},c_{\text{in}})}\{\text{id}\}/\text{id}_i, t/\tau]] \mid R_D \mid R_T^{(\Psi,\Phi,c_{\text{out}},c_{\text{in}})}]
\approx \ell C_{\ell}[C_{P_w}[\nu \Omega.(\nu\eta.(C[P_f] \mid P_\gamma) \mid R_D^{(\Theta,\Delta,\Pi)})] \mid R_T^{(\Psi,\Phi,c_{\text{out}},c_{\text{in}})}]
\]

3) By Lemma \( \text{I} \) and Lemma \( \text{5} \), we move the position of context \( C_{\theta}[,]:=\nu c_{\text{out}},\nu c_{\text{in}},(-)Q \) and \( C_{\ell}[\] in (eqciee6), and have \( \text{eqciee7:} \)

\[
C_{P_w}[C[R_i^{(\Psi,\Phi,c_{\text{out}},c_{\text{in}})}\{\text{id}\}/\text{id}_i, t/\tau]] \mid R_D \mid C_{\ell}[C_{P_w}[R_T^{(\Psi,\Phi,c_{\text{out}},c_{\text{in}})}]]
\approx \ell C_{P_w}[\nu \Omega.(\nu\eta.(C[P_f] \mid P_\gamma) \mid R_D^{(\Theta,\Delta,\Pi)})] \mid C_{\ell}[C_{P_w}[R_T^{(\Psi,\Phi,c_{\text{out}},c_{\text{in}})}]]
\]
4) By applying context $C'_{P_w}[C[R_i(\psi,\phi,\text{out},c_i,\text{cm})\{\text{id}/\text{id}_i, t/\tau\}] | R_D | \_\_]$ on both sides of (eqciee4), we have

\[ C'_h[C[R_i(\psi,\phi,\text{out},c_i,\text{cm})\{\text{id}/\text{id}_i, t/\tau\}] | R_D | C_i[R_T(\psi,\phi,\text{out},c_i,\text{cm})]] \]

\[ \approx \_\_\_ C'_h[C[R_i(\psi,\phi,\text{out},c_i,\text{cm})\{\text{id}/\text{id}_i, t/\tau\}] | R_D | C_i[R_T(\psi,\phi,\text{out},c_i,\text{cm})]]. \]

5) By Lemma 5, we move the position of context $C'_{\_\_\_}$ in (eqciee8) and have

\[ C_P[C[R_i(\psi,\phi,\text{out},c_i,\text{cm})\{\text{id}/\text{id}_i, t/\tau\}] | R_D | C_i[R_T(\psi,\phi,\text{out},c_i,\text{cm})]] \]

\[ \approx \_\_\_ C'_P[C[R_i(\psi,\phi,\text{out},c_i,\text{cm})\{\text{id}/\text{id}_i, t/\tau\}] | R_D | C_i[R_T(\psi,\phi,\text{out},c_i,\text{cm})]]. \]

6) By Lemma 6 combining (eqciee7) and (eqciee9), we have

\[ C_P[\nu\Omega.(\nu\eta.(C[P]) | P_\gamma) | R_D(\phi,\Delta,H)] | C_i[R_T(\psi,\phi,\text{out},c_i,\text{cm})]] \]

\[ \approx \_\_\_ C'_P[C[R_i(\psi,\phi,\text{out},c_i,\text{cm})\{\text{id}/\text{id}_i, t/\tau\}] | R_D | C_i[R_T(\psi,\phi,\text{out},c_i,\text{cm})]]. \]

7) According to Lemma 8 we have

\[ C'_h[R_T(\psi,\phi,\text{out},c_i,\text{cm})] \approx \_\_\_ R_T. \]

8) By Lemma 10 combining the above equivalence and (eqciee4), we have

\[ C'_h[C_i[R_T(\psi,\phi,\text{out},c_i,\text{cm})]] \approx \_\_\_ R_T. \]

9) Thus, the left side of (eqciee10) is bisimilar to

\[ C_P[\nu\Omega.(\nu\eta.(C[P]) | P_\gamma) | R_D(\phi,\Delta,H)] | R_T] \]

and the right side of (eqciee10) is bisimilar to

\[ C_P[C[R_i(\psi,\phi,\text{out},c_i,\text{cm})\{\text{id}/\text{id}_i, t/\tau\}] | R_D | R_T]. \]

Thus, (eqciee11):

\[ C_P[\nu\Omega.(\nu\eta.(C[P]) | P_\gamma) | R_D(\phi,\Delta,H)] | R_T] \]

\[ \approx \_\_\_ C'_P[C[R_i(\psi,\phi,\text{out},c_i,\text{cm})\{\text{id}/\text{id}_i, t/\tau\}] | R_D | R_T]. \]

10) Because of rule

\[ !P \equiv P \parallel P, \]

$R_T$ can be absorbed by the context. $C_P[\_ | R_T]$ is a type of context where there requires $R_T$ to be present. We define $C'_{P_w}[\_ | R_T] := C_{P_w}[\_ | R_T]$, where $R_T$ has to be present in the context. Thus, we have

(eqciee12):

\[ C'_{P_w}[\nu\Omega.(\nu\eta.(C[P]) | P_\gamma) | R_D(\phi,\Delta,H)] \]

\[ \approx \_\_\_ C'_{P_w}[C[R_i(\psi,\phi,\text{out},c_i,\text{cm})\{\text{id}/\text{id}_i, t/\tau\}] | R_D]. \]

From (eqciee11), we can obtain

(eqciee13):

\[ C'_{P_w}[C[R_i(\psi,\phi,\text{out},c_i,\text{cm})\{\text{id}/\text{id}_i, t/\tau\}] | R_D] \approx \_\_\_ C'_{P_w}[R_i(\psi,\phi,\text{out},c_i,\text{cm})\{\text{id}/\text{id}_i, t_1/\tau\}] | R_D], \]

Therefore, for any context $C[\_]$ satisfying (eqciee13), (eqciee2) and (eqciee12) hold. Thus, the protocol satisfies $cepriv_\rho$. \qed
2. There exists \( \theta, \rho, \delta \) such that \( c\text{epriv}_{\rho,\delta} \not\Rightarrow c\text{epriv}_{\rho,\theta,\delta} \).

We prove the statement by showing an example in which a protocol satisfies \( c\text{epriv}_{\rho,\delta} \) but not \( c\text{iepriv}_{\rho,\theta,\delta} \) for some \( \rho, \theta, \delta \). For instance, Dreier et al. proved that the voting protocol by Lee et al. \cite{LBD+03} satisfies receipt-freeness – an instance of \( c\text{epriv} \) where the coalition is the counter-balancing voter votes differently from the target voter and the collaboration is forwarding private information to the adversary, but not vote-independence with passive collaboration – an instance of \( c\text{iepriv} \) where the coalition and collaboration are the same as in \( c\text{epriv} \) and the defending third party is the third voter \cite{DLL11}.

B.4 Theorem. 6.22

\[ \text{(4)} \quad \text{priv} \implies \exists \delta, \text{cpriv}_\delta \]

We prove the statement in the following two directions: 1. \( \text{priv} \implies \exists \delta, \text{cpriv}_\delta \) 2. \( \exists \delta, \text{cpriv}_\delta \not\Rightarrow \text{priv} \)

1. When a protocol satisfies \( \text{priv} \), then there exists a coalition \( \delta \) such that the protocol satisfies \( \text{cpriv}_\delta \).

When a well-formed protocol \( P_w \) satisfies \( \text{priv} \) w.r.t. \( \tau \) we have

\[ \text{eqcc1: } C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_1/\tau\}] \approx \ell C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_2/\tau\}] \]

The context \( C_{P_w}[\_] \) has the following form

\[ C_{P_w}[\_] := \nu\tilde{c}.(\text{genkey} \ || R_1 \ | \ldots \ || R_p \ | \_) \]

Because of rule

\[ !P \equiv P \ || P, \]

we have (for a set of defending third parties \( R_D \))

\[ \text{eqcc2: } C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_1/\tau\} \ | \ R_D] \approx \ell C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_2/\tau\} \ | \ R_D] \]

Let \( \delta = (R_D, \emptyset, \emptyset, \emptyset) \) be a coalition,

\[ \text{eqcc3: } \nu\Omega.(\hat{R}_i\{\text{id}/id_i, t_2/\tau\} \ | \ R_D)^{\theta,\Delta,\Pi} = \hat{R}_i\{\text{id}/id_i, t_2/\tau\} \ | \ R_D \]

Thus,

\[ \text{eqcc4: } C_{P_w}[\nu\Omega.(\hat{R}_i\{\text{id}/id_i, t_2/\tau\} \ | \ R_D)^{\theta,\Delta,\Pi}] \approx \ell C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_2/\tau\} \ | \ R_D] \]

Because of (eqcc2), we have

\[ \text{eqcc5: } C_{P_w}[\hat{R}_i\{\text{id}/id_i, t_1/\tau\} \ | \ R_D] \approx \ell C_{P_w}[\nu\Omega.(\hat{R}_i\{\text{id}/id_i, t_2/\tau\} \ | \ R_D)^{\theta,\Delta,\Pi}] \]

Thus, the protocol satisfies \( \text{cpriv}_\delta \). \qed
2. There exists $\delta$ such that $\text{cepriv}_{\theta,\delta} \iff \text{priv}$.

We prove the statement by showing an example in which a protocol satisfies $\text{cepriv}_{\theta,\delta}$ but not $\text{priv}$. For instance, FOO92 [FOO92] is shown that it does not satisfy $\text{priv}$ w.r.t. vote $\text{vote}$, but satisfies vote-privacy – an instance of $\text{cepriv}$ where the coalition is the counter-balancing votes differently from the target voter [KR05].

(3) $\text{ipriv}_\theta \implies \exists \delta, \text{cepriv}_{\theta,\delta}$

We prove the statement in the following two directions: 1. $\text{ipriv}_\theta \implies \exists \delta, \text{cepriv}_{\theta,\delta}$

2. $\exists \theta, \delta, \text{cepriv}_{\theta,\delta} \iff \text{ipriv}_\theta$

1. When a protocol satisfies $\text{ipriv}_\theta$ for some $\theta$, then there exists a coalition $\delta$ such that the protocol satisfies $\text{cepriv}_{\theta,\delta}$.

For a collaboration of third parties $\theta = (R_T, \langle \psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t \rangle)$, when a well-formed protocol $P_w$ satisfies $\text{priv}$ w.r.t. $\tau$ and $(R_T, \langle \psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t \rangle)$, the following equivalence holds.

\[ \text{eqcci1:} \]
\[ \forall \ell C_{P_w}[\hat{R}_t\{id/\mathcal{id}_i, t_1/\tau\} | R_T^{\langle \psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t \rangle}] \approx_\ell C_{P_w}[\hat{R}_t\{id/\mathcal{id}_i, t_2/\tau\} | R_T^{\langle \psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t \rangle}] \]

Thus, we have (for a set of defending third parties $R_D$)

\[ \text{eqcci2:} \]
\[ \forall \ell C_{P_w}[\hat{R}_t\{id/\mathcal{id}_i, t_1/\tau\} | R_T^{\langle \psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t \rangle}] \approx_\ell C_{P_w}[\hat{R}_t\{id/\mathcal{id}_i, t_2/\tau\} | R_T^{\langle \psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t \rangle}] \]

Let $\delta = (R_D, \langle \emptyset, \emptyset, \emptyset \rangle)$ be a coalition, then

\[ \text{eqcci3:} \]
\[ \nu \Omega.(\langle \hat{R}_t\{id/\mathcal{id}_i, t_2/\tau\} | R_D \rangle^{\langle \theta, \Delta, \Pi \rangle}) \approx_\ell \hat{R}_t\{id/\mathcal{id}_i, t_2/\tau\} | R_D \]

Thus, we have

\[ C_{P_w}[\nu \Omega.(\langle \hat{R}_t\{id/\mathcal{id}_i, t_2/\tau\} | R_D \rangle^{\langle \theta, \Delta, \Pi \rangle}) | R_T^{\langle \psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t \rangle}] \approx_\ell C_{P_w}[\hat{R}_t\{id/\mathcal{id}_i, t_1/\tau\} | R_D | R_T^{\langle \psi^t, \Phi^t, c_{\text{out}}^t, c_{\text{in}}^t \rangle}] \]

Therefore, the protocol satisfies $\text{cepriv}_{\theta,\delta}$. \hfill \square

2. There exists $\theta, \rho$ such that $\text{cepriv}_{\theta,\delta} \iff \text{ipriv}_\theta$.

We prove the statement by showing an example in which a protocol satisfies $\text{cepriv}_{\theta,\delta}$ but not $\text{ipriv}_\theta$. For instance, voting protocols FOO92 are shown does not satisfies $\text{priv}$ w.r.t. vote $\text{vote}$ [KR05], thus does not satisfy $\text{ipriv}$, but satisfies vote-independence – an instance of $\text{ipriv}$ where the coalition is the counter-balancing voter votes differently from the target voter and the attacking third party is the third voter [DLL11].

(2) $\text{epriv}_\rho \implies \exists \delta, \text{cepriv}_{\rho,\delta}$

We prove the statement in the following two directions: 1. $\text{epriv}_\rho \implies \exists \delta, \text{cepriv}_{\rho,\delta}$

2. $\exists \rho, \delta, \text{cepriv}_{\rho,\delta} \iff \text{epriv}_\rho$
1. When a protocol satisfies $\text{epriv}_\rho$ for some $\rho$, then there exists a coalition $\delta$ such that the protocol satisfies $\text{cepriv}_{\rho, \delta}$.

When a well-formed protocol $P_w$ satisfies $\text{epriv}$ w.r.t. $\tau$ and $\rho$ where $\rho = \langle \psi, \phi, c_{\text{out}}, c_m \rangle$, there exists a closed plain process $P_f$, such that for any context $C[] := \nu c_{\text{out}}.\nu c_{\text{in}}.(-) | Q$ satisfying $\text{bn}(P_w) \cap \text{fn}(C[]) = \emptyset$ and

**eqcce1:**

$$C_{P_w}[\hat{R}_i^{\langle \psi, \phi, c_{\text{out}}, c_m \rangle} \{\text{id}/\text{id}_i, t/\tau\}] \approx_{\ell} C_{P_w}[\hat{R}_i^{\langle \psi, \phi, c_{\text{out}}, c_m \rangle} \{\text{id}/\text{id}_i, t_1/\tau\}],$$

we have

**eqcce2:**

$$C[P_f]^{\text{out}(c_{\text{out}}')} \approx_{\ell} \hat{R}_i \{\text{id}/\text{id}_i, t_2/\tau\},$$

**eqcce3:**

$$C_{P_w}[C[\hat{R}_i^{\langle \psi, \phi, c_{\text{out}}, c_m \rangle} \{\text{id}/\text{id}_i, t/\tau\}] \approx_{\ell} C_{P_w}[C[P_f]].$$

From (eqcce3), we have

**eqcce4:**

$$C_{P_w}[C[\hat{R}_i^{\langle \psi, \phi, c_{\text{out}}, c_m \rangle} \{\text{id}/\text{id}_i, t/\tau\}] | R_D] \approx_{\ell} C_{P_w}[C[P_f] | R_D].$$

Let $\delta = (R_D, \langle \emptyset, \emptyset, \emptyset \rangle)$ be a coalition, then

**eqcce5:**

$$\nu \Omega. (\nu \eta. (C[P_f] | P_\gamma) \approx_{\ell} C[P_f] | R_D)$$

By applying context $C_{P_w}[]$ on both sides of (eqcce5) we have

**eqcce6:**

$$C_{P_w}[\nu \Omega. (\nu \eta. (C[P_f] | P_\gamma) \approx_{\ell} C[P_f] | R_D)] \approx_{\ell} C_{P_w}[C[P_f] | R_D]$$

Combining (eqcce4) and (eqcce6), by Lemma [1] we have

**eqcce7:**

$$C_{P_w}[\nu \Omega. (\nu \eta. (C[P_f] | P_\gamma) \approx_{\ell} C[P_f] | R_D)] \approx_{\ell} C_{P_w}[C[\hat{R}_i^{\langle \psi, \phi, c_{\text{out}}, c_m \rangle} \{\text{id}/\text{id}_i, t/\tau\}] | R_D]$$

Since $\delta = (R_D, \langle \emptyset, \emptyset, \emptyset \rangle)$, we have

$$\nu \Omega. (\nu \eta. (C[P_f]^{\text{out}(c_{\text{out}}')} | P_\gamma) \approx_{\ell} C[P_f]^{\text{out}(c_{\text{out}}')} | R_D)$$

Because of (eqcce2), we have

**eqcce8:**

$$C[P_f]^{\text{out}(c_{\text{out}}')} \approx_{\ell} \hat{R}_i \{\text{id}/\text{id}_i, t_2/\tau\} | R_D.$$ 

Since $\delta = (R_D, \langle \emptyset, \emptyset, \emptyset \rangle)$, we have

$$\Omega. (\hat{R}_i \{\text{id}/\text{id}_i, t_2/\tau\} | R_D)^{\langle \theta, \Delta, \Pi \rangle} \approx_{\ell} \hat{R}_i \{\text{id}/\text{id}_i, t_2/\tau\} | R_D$$

Thus,

**eqcce9:**

$$\nu \Omega. (\nu \eta. (C[P_f]^{\text{out}(c_{\text{out}}')} | P_\gamma) \approx_{\ell} C[P_f]^{\text{out}(c_{\text{out}}')} | R_D) \approx_{\ell} \Omega. (\hat{R}_i \{\text{id}/\text{id}_i, t_2/\tau\} | R_D)^{\langle \theta, \Delta, \Pi \rangle}$$

Because of (eqcce1), we have

**eqcce10:**

$$C_{P_w}[C[\hat{R}_i^{\langle \psi, \phi, c_{\text{out}}, c_m \rangle} \{\text{id}/\text{id}_i, t/\tau\}] | R_D] \approx_{\ell} C_{P_w}[\hat{R}_i^{\langle \psi, \phi, c_{\text{out}}, c_m \rangle} \{\text{id}/\text{id}_i, t_1/\tau\}] | R_D],$$

Therefore, for any context $C[]$ satisfying (eqcce10), the protocol satisfies (eqcce7) and (eqcce9), thus, the protocol satisfies $\text{cepriv}_{\rho, \delta}$.  \qed
2. There exists $\delta, \rho$ such that $\text{cepriv}_{\rho, \delta} \iff \text{epriv}_\rho$.

We prove the statement by showing an example in which a protocol satisfies $\text{cepriv}_{\rho, \delta}$ but not $\text{epriv}_\rho$. For instance, voting protocol by Okamoto [Oka96] does not satisfy $\text{priv}$ w.r.t. vote [KR05] in the case of unanimous result, thus does not satisfy $\text{epriv}$ where $\rho$ is forwarding private information to the adversary, but satisfies receipt-freeness – an instance of $\text{cepriv}$ where the coalition is the counter-balancing votes differently from the target voter and the collaboration is forwarding private information to the adversary [DKR09].

(1) $\text{iepriv}_{\rho, \theta} \implies \exists \delta, \text{ciepriv}_{\rho, \theta, \delta}$

We prove the statement in the following two directions:

1. $\text{iepriv}_{\rho, \theta} \implies \exists \delta, \text{ciepriv}_{\rho, \theta, \delta}$

2. $\exists \rho, \theta, \delta, \text{ciepriv}_{\rho, \theta, \delta} \iff \text{iepriv}_{\rho, \theta}$

1. When a protocol satisfies $\text{iepriv}_{\rho, \theta}$ for some $\rho, \theta$, then there exists a coalition $\delta$ such that the protocol satisfies $\text{ciepriv}_{\rho, \theta, \delta}$.

For a collaboration of the target user $\rho = (\Psi, \Phi, c_{out}, c_{in})$ and a collaboration of third parties $\theta = (R_T, (\Psi^{f_i}, \Phi^{f_i}, c^{f_i}_{out}, c^{f_i}_{in}))$, when a well-formed protocol $P_w$ satisfies $\text{iepriv}$ w.r.t. $\tau$, $(\Psi, \Phi, c_{out}, c_{in})$ and $(R_T, (\Psi^{f_i}, \Phi^{f_i}, c^{f_i}_{out}, c^{f_i}_{in}))$, there exists a closed plain process $P_f$, such that for any context $C[..] := \nu c_{out}.\nu c_{in}.(\nu Q)$ satisfying $\text{bn}(P_w) \cap \text{fn}(C[..]) = \emptyset$ and

\begin{align*}
\text{eqccee1:} & \quad C_{P_w}[C[R_i^{(\Psi, \Phi, c_{out}, c_{in})}|id/\text{id}_i, t/\tau]|R_T] \approx \ell C_{P_w}[\hat{R}_i^{(\Psi, \Phi, c^{f_i}_{out}, c^{f_i}_{in})}|id/\text{id}_i, t_1/\tau]|R_T],
\end{align*}

\begin{align*}
\text{eqccee2:} & \quad C[P_f]|^{out(c^{f_i}_{out})} \approx \ell \hat{R}_i|\text{id}/\text{id}_i, t_2/\tau],
\end{align*}

\begin{align*}
\text{eqccee3:} & \quad C_{P_w}[C[R_i^{(\Psi, \Phi, c_{out}, c_{in})}|id/\text{id}_i, t/\tau]|R_T^{(\Psi^{f_i}, \Phi^{f_i}, c^{f_i}_{out}, c^{f_i}_{in})}] \\
& \approx \ell C_{P_w}[C[P_f]|R_T^{(\Psi^{f_i}, \Phi^{f_i}, c^{f_i}_{out}, c^{f_i}_{in})}|R_D].
\end{align*}

Because of (eqccee3), we have

\begin{align*}
\text{eqccee4:} & \quad C_{P_w}[C[R_i^{(\Psi, \Phi, c_{out}, c_{in})}|id/\text{id}_i, t/\tau]|R_T^{(\Psi^{f_i}, \Phi^{f_i}, c^{f_i}_{out}, c^{f_i}_{in})}|R_D] \\
& \approx \ell C_{P_w}[C[P_f]|R_T^{(\Psi^{f_i}, \Phi^{f_i}, c^{f_i}_{out}, c^{f_i}_{in})}|R_D].
\end{align*}

Let $\delta = (R_D, (\emptyset, \emptyset, \emptyset))$ be a coalition, then

\begin{align*}
\text{eqccee5:} & \quad \nu \Omega.(\nu \eta.(C[P_f]|P_\gamma)|R_D^{(\theta, \Delta, \Pi)}) \approx C[P_f]|R_D \\
& \text{By applying context } C_{P_w}[..]|R_T^{(\Psi^{f_i}, \Phi^{f_i}, c^{f_i}_{out}, c^{f_i}_{in})}] \text{ on both sides of (eqccee5), we have}
\end{align*}

\begin{align*}
\text{eqccee6:} & \quad C_{P_w}[\nu \Omega.(\nu \eta.(C[P_f]|P_\gamma)|R_D^{(\theta, \Delta, \Pi)})|R_T^{(\Psi^{f_i}, \Phi^{f_i}, c^{f_i}_{out}, c^{f_i}_{in})}] \\
& \approx \ell C_{P_w}[C[P_f]|R_D|R_T^{(\Psi^{f_i}, \Phi^{f_i}, c^{f_i}_{out}, c^{f_i}_{in})}]
\end{align*}
By Lemma 1 combining (eqccee4) and (eqccee6), we have

\[ C_{P_w}[\hat{R}_i^{(\Psi, \phi, c_{\text{out}}, c_{\text{in}})} \{ \text{id}/\text{id}_i, t/\tau \}] \mid R_T^{(\Psi, \phi, c_{\text{out}}, c_{\text{in}})} \mid R_D \]
\[ \approx_{\ell} C_{P_w}[\nu \Omega. (\nu \eta. (C[P_f] \mid P_\gamma) \mid R_D^{(\theta, \Delta, \Pi)}) \mid R_T^{(\Psi, \phi, c_{\text{out}}, c_{\text{in}})}] \]

Since \( \delta = (R_D, (0, 0, 0)) \), we have

\[ \nu \Omega. (\nu \eta. (C[P_f])^{\text{out}(c_{\text{out}}')} \mid P_\gamma) \mid R_D^{(\theta, \Delta, \Pi)} \]
\[ \approx_{\ell} \hat{R}_i \{ \text{id}/\text{id}_i, t_2/\tau \} \mid R_D \]

Because of (eqccee2), we have

\[ \nu \Omega. (\nu \eta. (C[P_f])^{\text{out}(c_{\text{out}}')} \mid P_\gamma) \mid R_D^{(\theta, \Delta, \Pi)} \]
\[ \approx_{\ell} \hat{R}_i \{ \text{id}/\text{id}_i, t_2/\tau \} \mid R_D \]

Thus, we have

\[ \nu \Omega. (\nu \eta. (C[P_f])^{\text{out}(c_{\text{out}}')} \mid P_\gamma) \mid R_D^{(\theta, \Delta, \Pi)} \]
\[ \approx_{\ell} \nu \Omega. (\hat{R}_i \{ \text{id}/\text{id}_i, t_2/\tau \} \mid R_D)^{(\theta, \Delta, \Pi)} \]

From (eqccee1), we have

\[ C_{P_w}[\hat{R}_i^{(\Psi, \phi, c_{\text{out}}, c_{\text{in}})} \{ \text{id}/\text{id}_i, t/\tau \}] \mid R_T \mid R_D \]
\[ \approx_{\ell} C_{P_w}[\hat{R}_i^{(\Psi, \phi, c_{\text{out}}', c_{\text{in}}')} \{ \text{id}/\text{id}_i, t_1/\tau \} \mid R_T \mid R_D] \]

Therefore, for any context \( C[\cdot] \) satisfying (eqccee10), (eqccee7) and (eqccee9) are satisfies. Thus, the protocol satisfies \( \text{ciepriv}_{\rho, \theta, \delta} \).

\[ \square \]

2. There exists \( \theta, \rho, \delta \) such that \( \text{ciepriv}_{\rho, \theta, \delta} \implies \text{iepriv}_{\rho, \theta} \).

We prove the statement by showing an example in which a protocol satisfies \( \text{ciepriv}_{\rho, \theta, \delta} \) but not \( \text{iepriv}_{\rho, \theta} \). For instance, voting protocol by Okamoto [Oka96] does not satisfy \( \text{priv} \) w.r.t. vote \( \text{vote} \) when all votes are unanimous. Thus, the protocol does not satisfy \( \text{iepriv} \) w.r.t. vote \( \text{vote} \), \( \rho \) and \( \theta \), where \( \rho \) is the target voter forwarding information to the adversary, \( \theta \) is the collaborating third voter communicating with the adversary. However, the protocol satisfies vote-independence with passive collaboration – an instance of \( \text{ciepriv} \) w.r.t. vote \( \text{vote} \), \( \rho \), \( \theta \) and \( \delta \) where \( \rho \) and \( \theta \) are the same as in \( \text{iepriv} \) and \( \delta \) is the counter-balancing voter voting differently from the target voter [DLL11].
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