Quantitative Questions and Attack–Defense Trees

Patrick Schweitzer*

Joint work with Barbara Kordy and Sjouke Mauw

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Outline

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Attack–Defense Trees: Introduction

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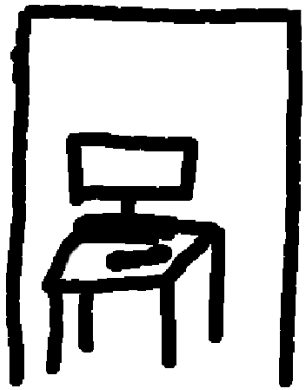

Attack Trees

Methodology to describe security weaknesses of a system

Definition

Attack Trees (ATrees): Tree-like representation of an attacker's goal recursively refined into conjunctive and disjunctive subgoals.

- Successor of fault trees (Safety modeling)
- Security adaptation as threat trees (Weiss, Amoroso)
- Name coined in 1998 (Schneier)
- Formalization in 2005 (Mauw, Oostdijk)

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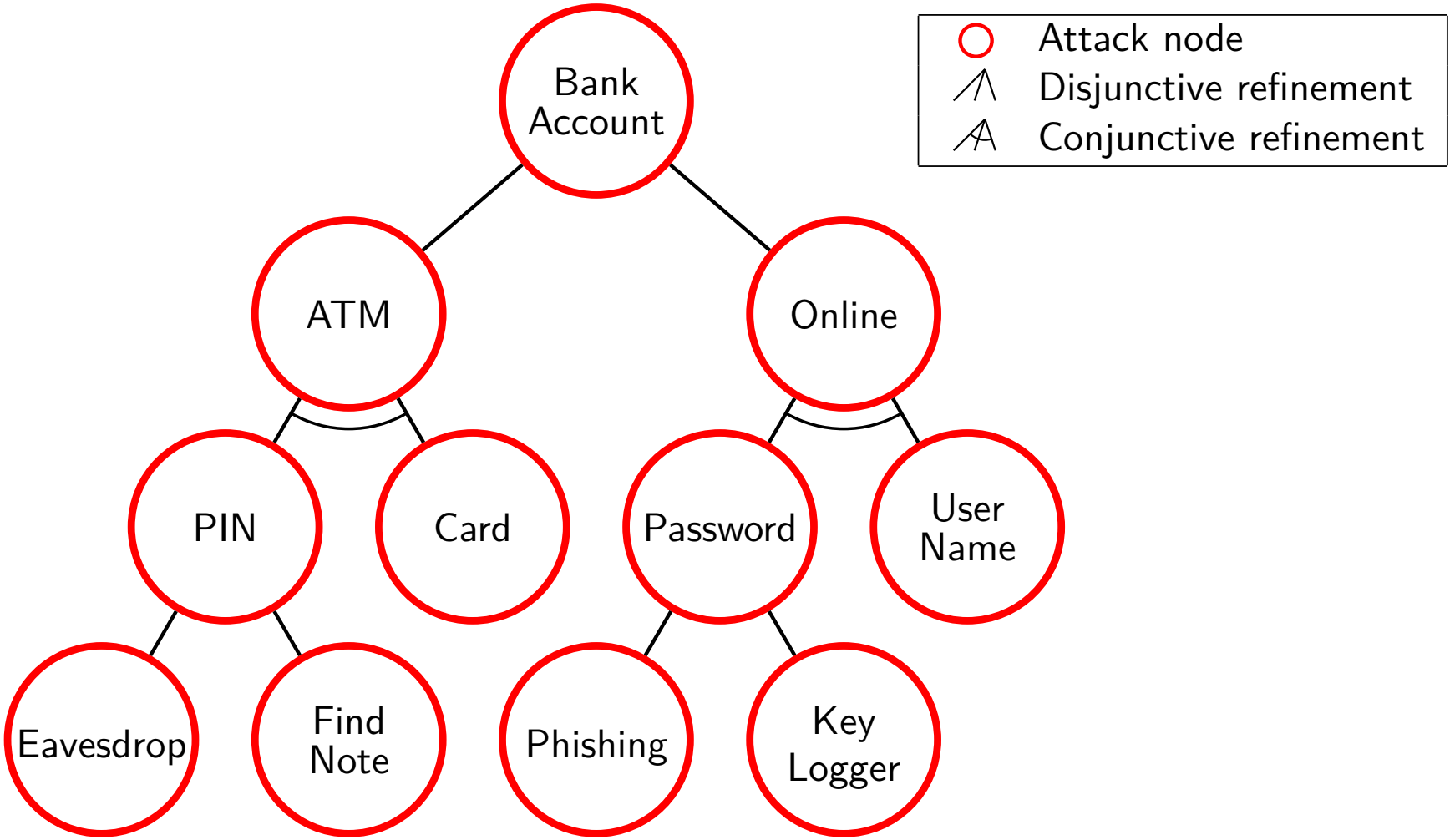
Example: Attacking a Bank Account



Limitations of Attack Trees

- Only attacker's point of view
- No defensive measures
- No attacker/defender interactions
- No evolutionary aspects

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Attack–Defense Trees

Definition

Attack–Defense Trees (ADTrees): ATrees extended with possibly refined or countered defensive actions.

- Introduced by Kordy, Mauw, Radomirović and Schweitzer in *Foundations of Attack–Defense Trees* [FAST'10]
- Related approaches by
  – Attack-Countermeasure trees: Roy et al. ('10)
  – Augmented attack trees: Wang et al. ('10)

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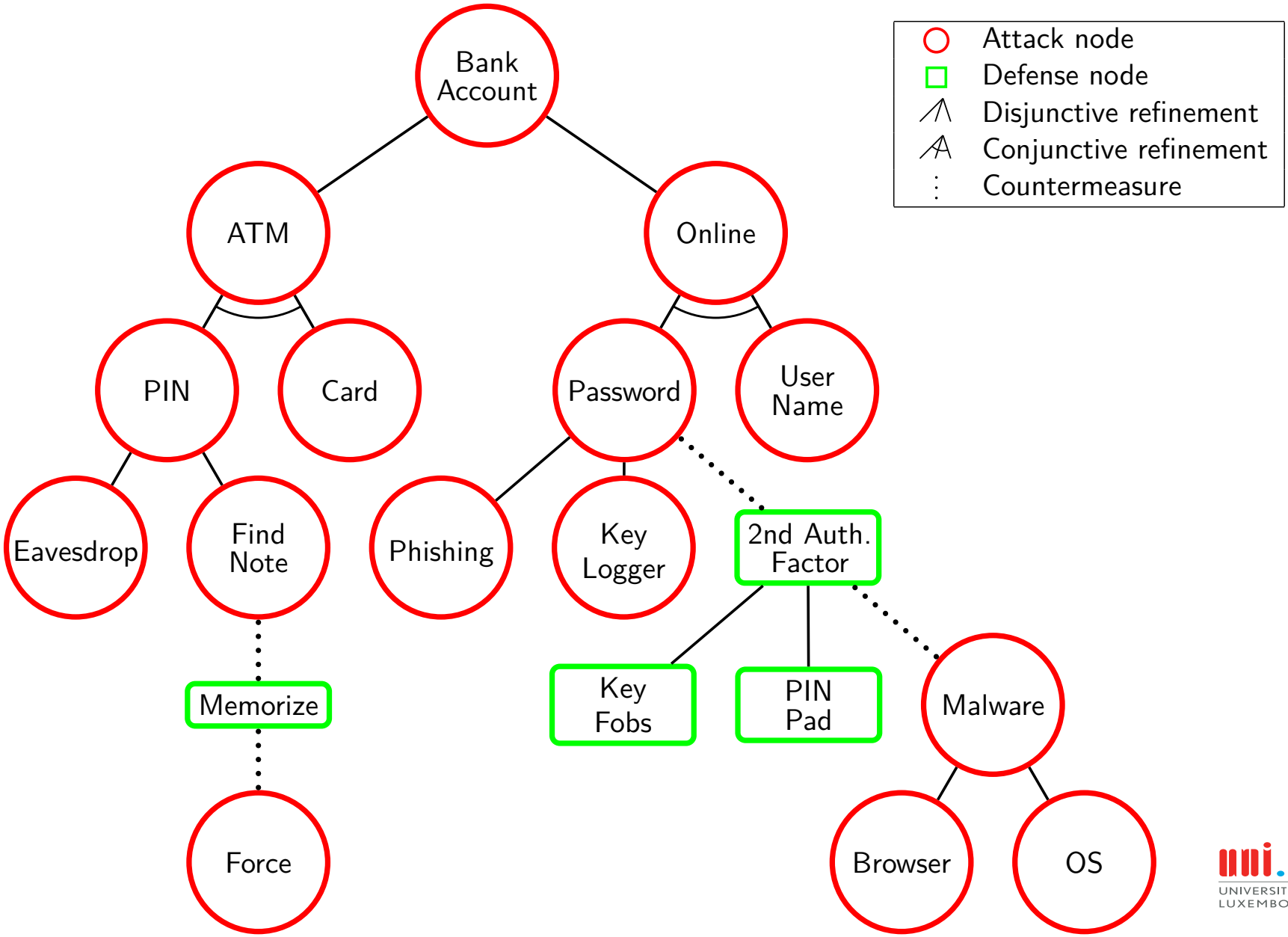
Example: Attacking and Defending a Bank Account



Interesting Questions

- Equivalent representations of the same scenario (semantics)
  PS in 2010
- Practical applications (case studies)
  PS in 2011
- Computational complexity of ATrees and ADTrees (querying)
  BK in 2011
- Quantitative analysis (attributes)
  Today

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Quantitative Analysis: Introduction

- Standard questions (ATrees)
  - What are the costs of attacking the system?
  - How many specialists are needed to attack a system?
  - Can the attack succeed?

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  - What are the minimal costs of an attack when the defender tries to maximally protect his system?
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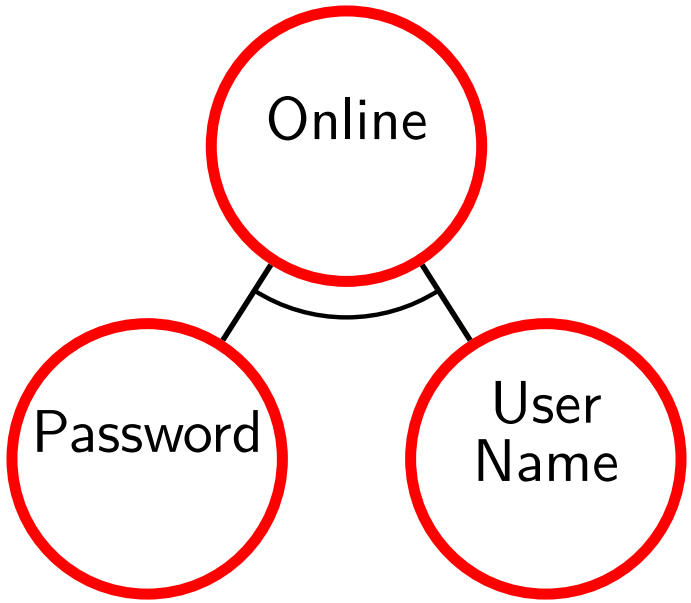
Underspecified Questions

How many specialists are needed to attack the system?

Intuitive answers welcome



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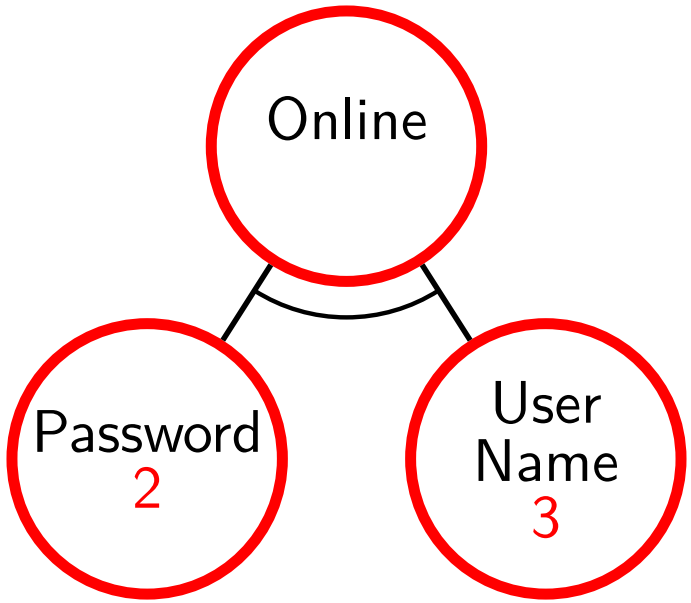

Underspecified Questions

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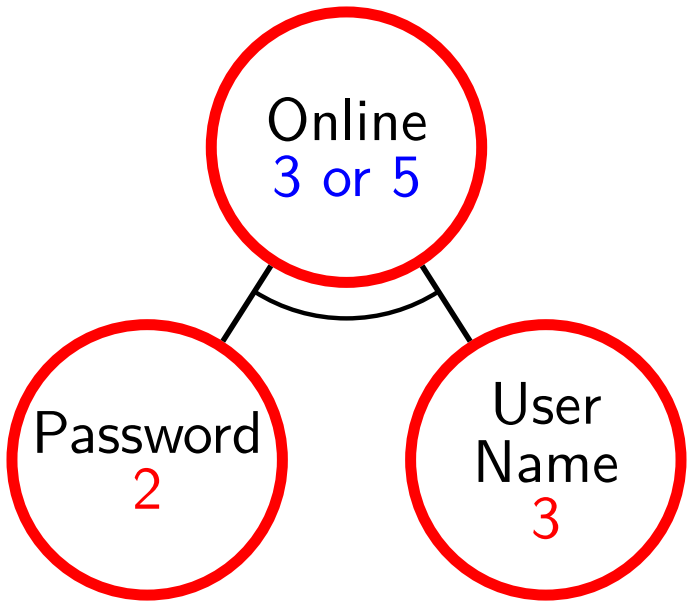

$\longrightarrow$ The question is underspecified

Calculation of Attributes

Bottom-up algorithm

- Attribute domain: operators specifying how to compute values for refined nodes
- Basic assignment: values assigned to non-refined nodes

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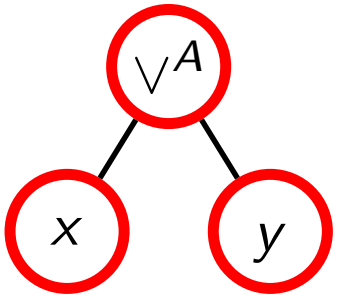
Minimal Number of Specialists Needed to Attack

Question:

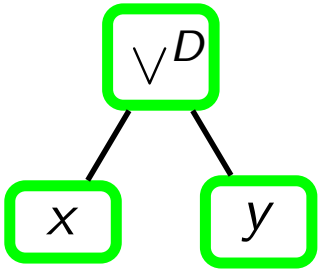
What are the minimal number of specialists needed to attack?

- Values from $\mathbb{N} \cup \{\infty\}$
- $\infty =$ action not under control of the attacker
- $(\vee^A, \wedge^A, \vee^D, \wedge^D, \mathsf{c}^A, \mathsf{c}^D) \mapsto (\min, +, +, \min, +, \min)$
- Attribute domain A consists of a value domain plus mapping of the six functional operators.

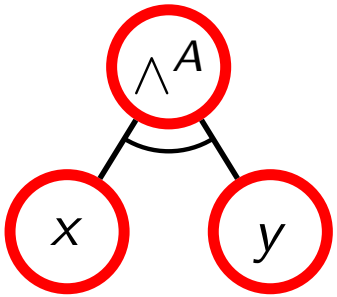
Attribute Domain for Minimal Number of Specialists (Ind.)



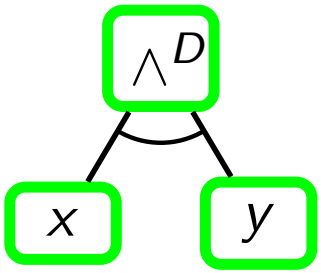
$\vee^A$: $\min\{x, y\}$



$\vee^D$: $x + y$



$\wedge^A$: $x + y$



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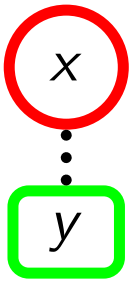
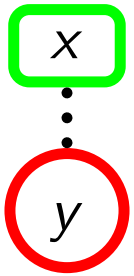


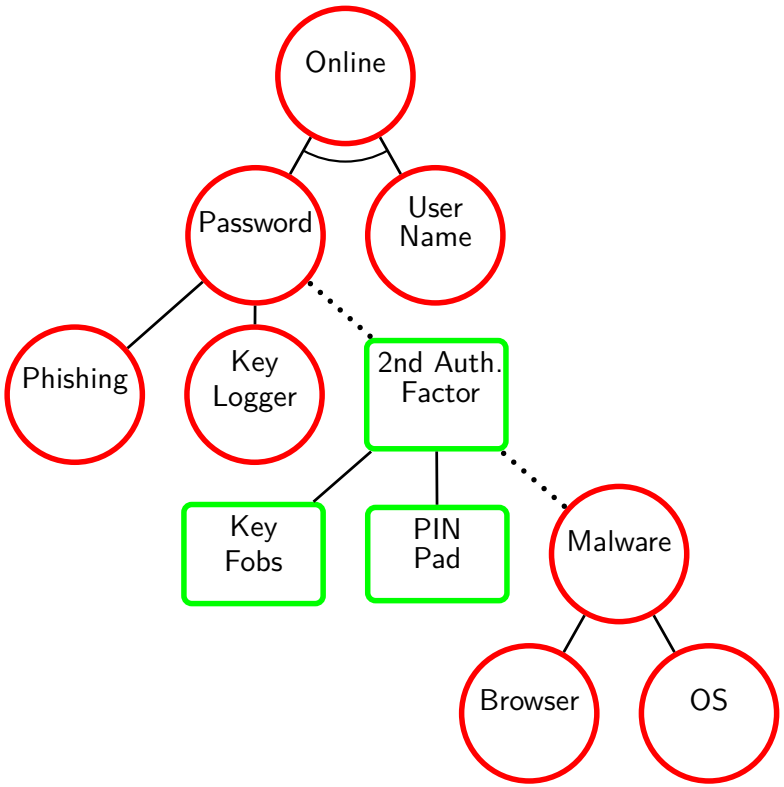
c^A: $x + y$



c^D: $\min\{x, y\}$

Computation of the Minimal Number of Specialists (Ind.)

$$A = (D, \vee^A, \wedge^A, \vee^D, \wedge^D, \mathsf{c}^A, \mathsf{c}^D) \mapsto (\mathbb{N} \cup \{\infty\}, \min, +, +, \min, +, \min)$$



- Attribute Domain

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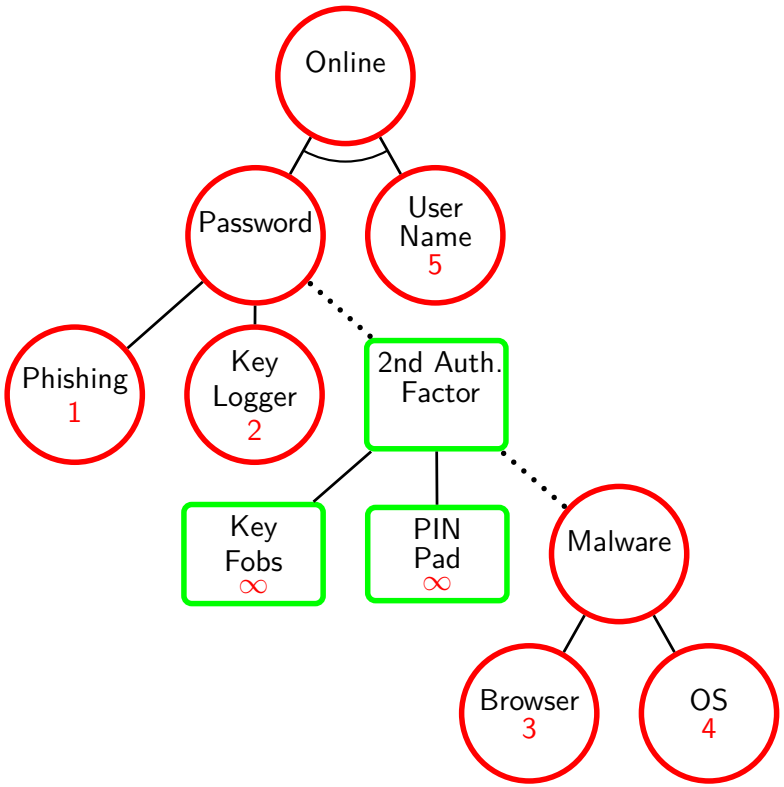

- Attribute Domain
- Basic assignment

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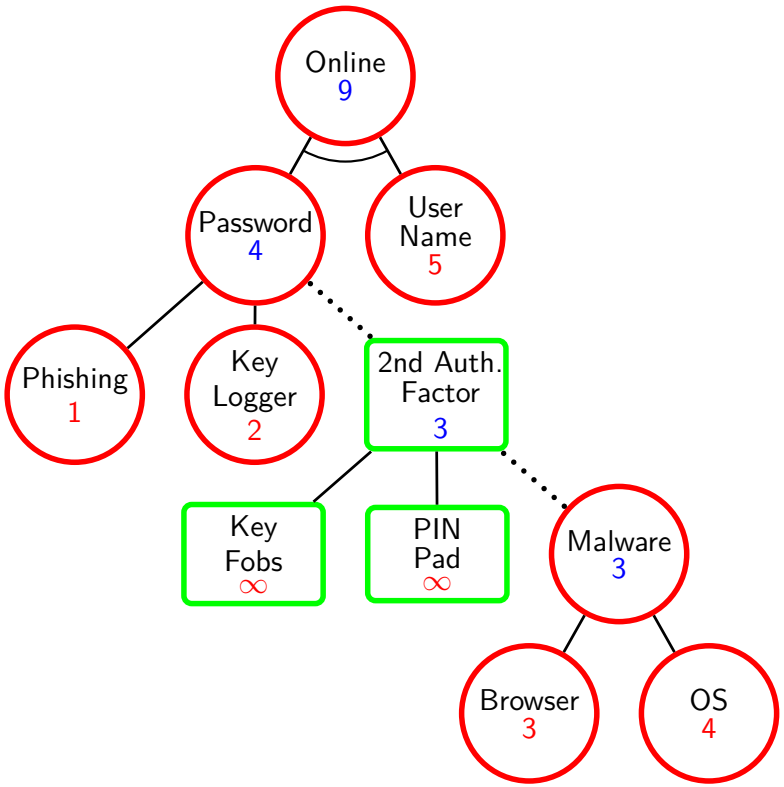

- Attribute Domain
- Basic assignment
- Bottom-up computation

Attribute Domain Summary

Take home message

Given an ADTree, an attribute domain, a basic assignment we can use the bottom up algorithm to provide a value for the root node of the ADTree.

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Components for an Analysis

Component	Functional Example	ADTree Example
Question	What is the functional value?	What is the minimal number of specialists...?
Model	$f(x, y) = x \circ y$	ADTree and Bottom-up algorithm
Interpretation	$\circ = +$	Attribute domain
Input values	$x = 2, y = 3$	Basic assignment

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Outline

Questions vs. Attribute Domains: Introduction

- A question is an informal way to describe a quantity.
- An attribute domain is a formal way to describe a quantity.
- Necessary to link questions and attribute domains.
- Specify format of questions to
  - avoid underspecification of attribute domain.
  - automate selection of attribute domain.
  - classify empirically occurring questions.

Exemplary Questions

Class 1:
– What is the minimal number of specialists needed to attack the system assuming that the specialists work independently?
– What is the minimal time of the attack assuming that all actions are executed one after another?

Class 2:
– Who is the winner of the scenario?
– How likely is the system to be breached?

Class 3:
– How much data traffic will occur?
– What are the environmental costs?

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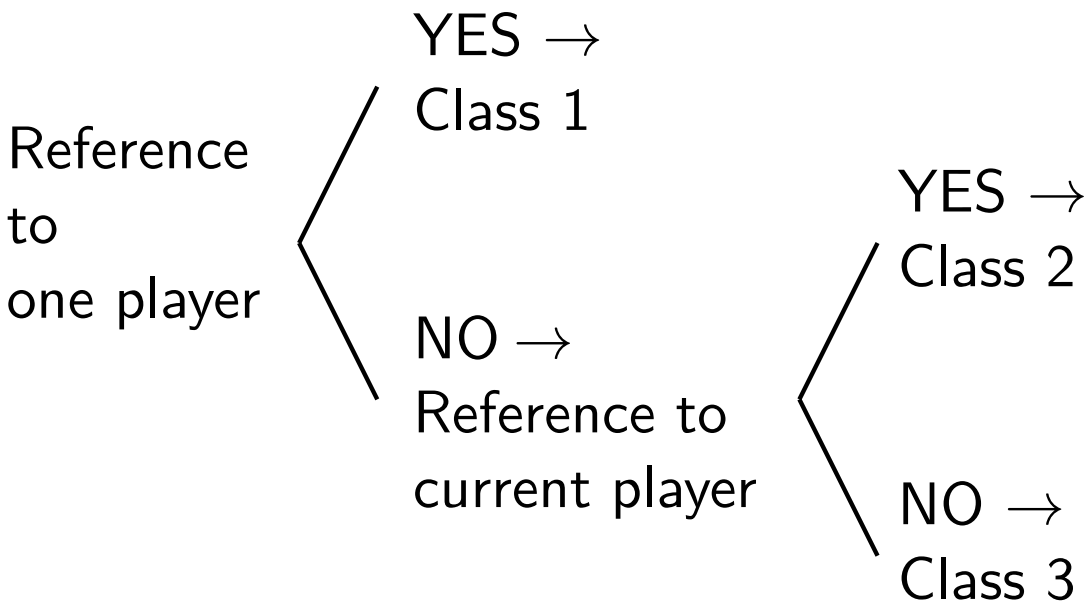
A Classification of Questions into Three Classes



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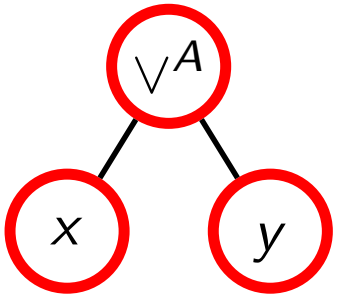
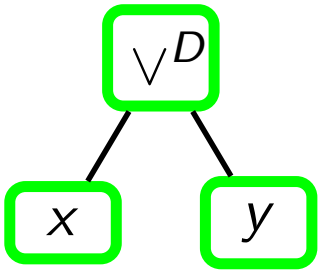
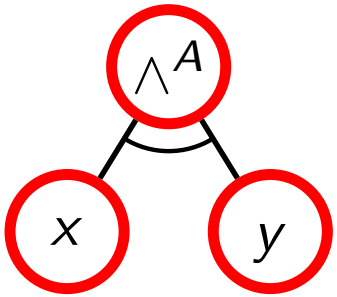
Minimal Number of Specialists (Independent)



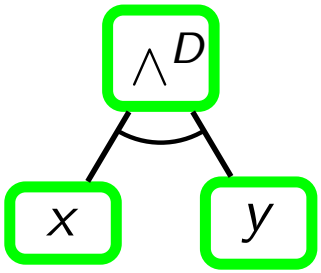
$\vee^A$: minimum



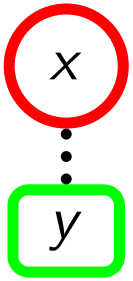
$\vee^D$: addition



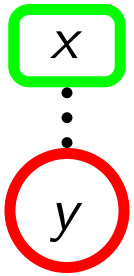
$\wedge^A$: addition



$\wedge^D$: minimum

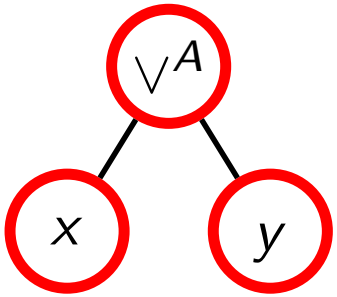


c^A: addition

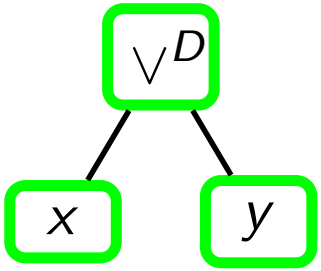


c^D: minimum

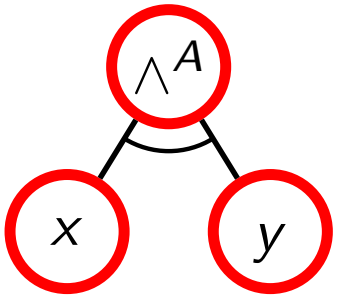
Minimal Number of Specialists (Parallel)



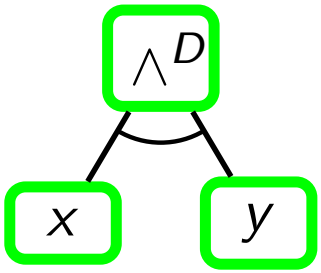
$\vee^A$: minimum



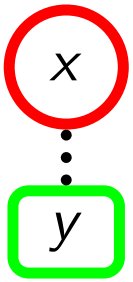
$\vee^D$: maximum



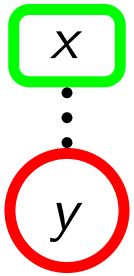
$\wedge^A$: maximum



$\wedge^D$: minimum

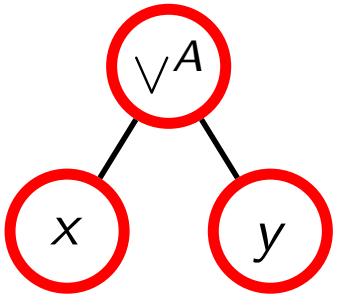


c^A: maximum

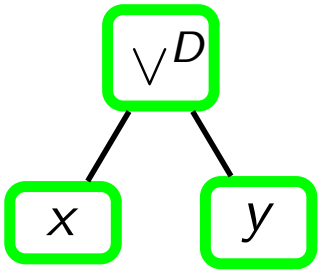


c^D: minimum

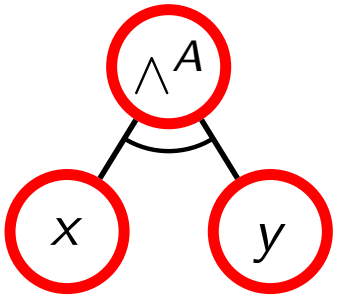
Class 1 Generalization



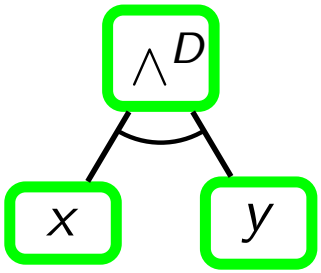
$\vee^A$: at least one



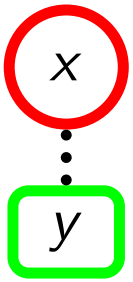
$\vee^D$: all



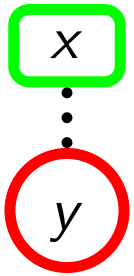
$\wedge^A$: all



$\wedge^D$: at least one



c^A: all



c^D: at least one

$\longrightarrow$ Two different operators suffice

Deducing the Structure of a Question in Class 1

We specify

- the value domain,
- “at least one”,
- “all”,
- whether “at least one” holds for attacker or defender.

$\longrightarrow$ If the question has exactly four parts, the attribute domain is completely determined.

Example of a Question in Class 1

1. What is the minimal
2. number of specialists needed
3. to attack the system
4. assuming that the specialists work independently?

Part 2 - Notion: determines the value domain
Part 1 - Modality: determines the operator for “at least one”
Part 4 - Execution style: determines the operator for “all”
Part 3 - Owner of the question: determines assignment of “at least one” and “all” to attacker and defender.

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Part 2 of a Question in Class 1

- attack potential,
- attack time,
- consequence,
- costs,
- detectability,
- difficulty level,
- elapsed time,
- expertise,
- impact,
- insider required,
- mitigation success,
- outcome,
- penalty,
- profit,
- response time,
- resources,
- severity,
- skill level,
- special equipment needed,
- special skill needed,
- survivability.

Parts 1,3 and 4 of a Question in Class 1

Notion (Part 2)	Modality (Part 1)	Owner (Part 3)	Exec.style (Part 4)	Structure $(D, \mathrm{op}_1, \mathrm{op}_2)$
duration	min	A	sequential	$(\mathbb{R}, \min, +)$
duration	max	D	sequential	$(\mathbb{R}, +, \max)$
duration	comb	A	parallel	$(\mathbb{R}, \max, \max)$
...	...	...	...	...

$$(D, \mathrm{op}_1, \mathrm{op}_2) \longrightarrow (D, \mathrm{op}_1, \mathrm{op}_2, \mathrm{op}_2, \mathrm{op}_1, \mathrm{op}_2, \mathrm{op}_1)$$

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Questions for ADTrees in Class 1 Summary

Message to take home

A question for ADTrees in Class 1 needs to contain four components to construct the attribute domain: notion, modality, owner and execution style.

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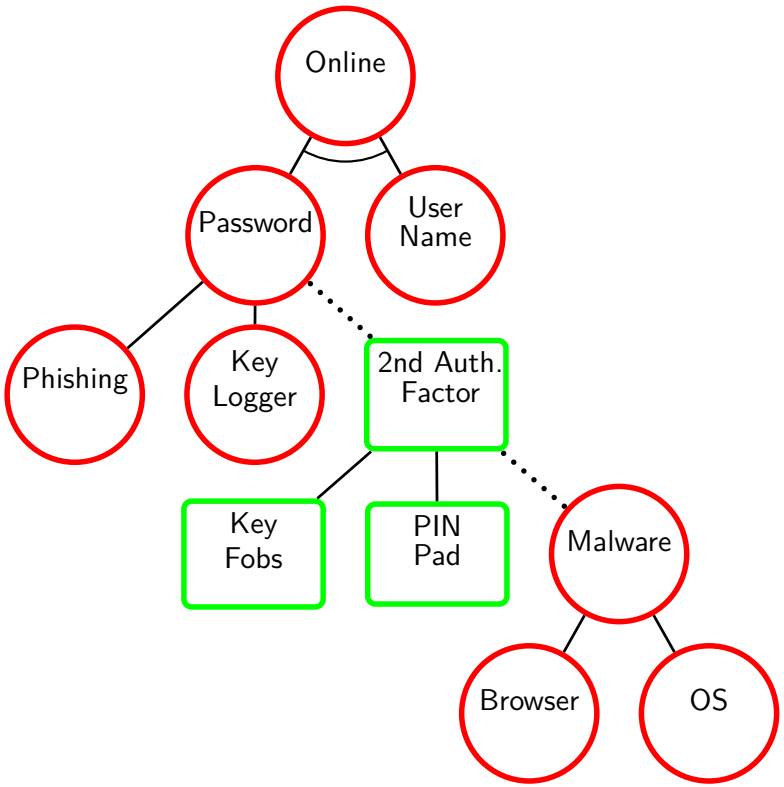
Example: Computation of the Winning Player

$$A = (D, \vee^A, \wedge^A, \vee^D, \wedge^D, \mathrm{c}^A, \mathrm{c}^D) \mapsto (\{0,1\}, \vee, \wedge, \vee, \wedge, \star, \star)$$



- Attribute Domain, where $a \star b = a \wedge \neg b$

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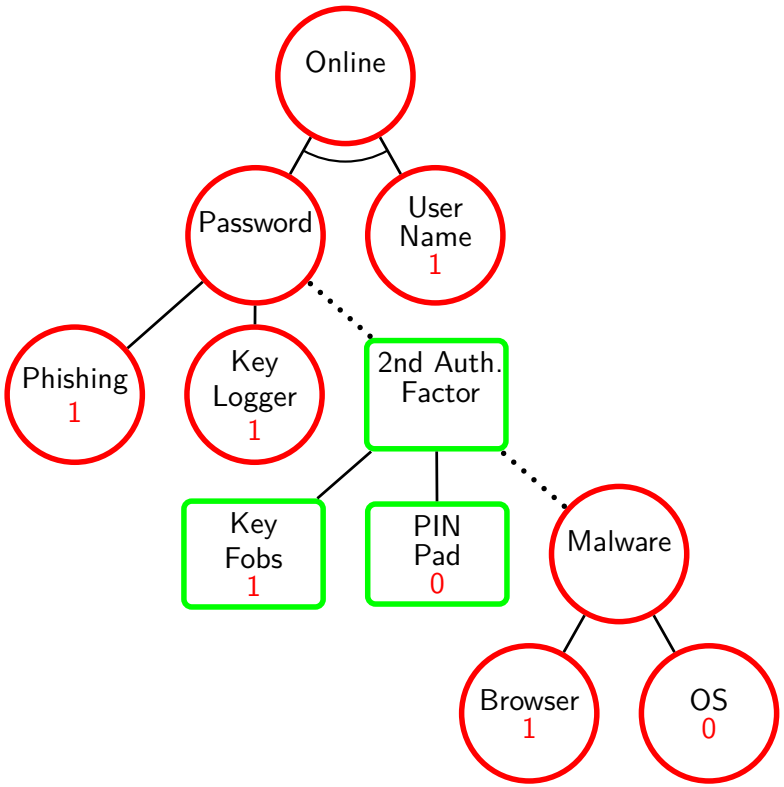
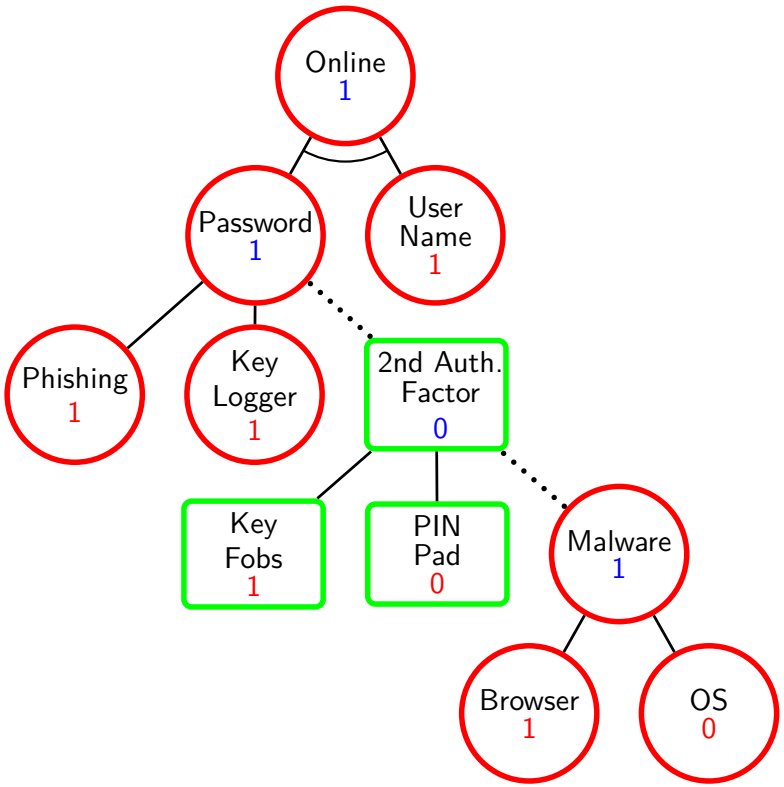


- Attribute Domain, where $a \star b = a \wedge \neg b$
- Basic assignment: 1's and 0's

Example: Computation of the Winning Player

$$A = (D, \vee^A, \wedge^A, \vee^D, \wedge^D, \mathrm{c}^A, \mathrm{c}^D) \mapsto (\{0,1\}, \vee, \wedge, \vee, \wedge, \star, \star)$$



- Attribute Domain, where $a \star b = a \wedge \neg b$
- Basic assignment: 1's and 0's
- Bottom-up computation

Class 2 Generalization

- Reference to the current player.
- The value for a subtree answers the question for the subtree.
- We need three operators.
- Values of a player can be transformed to values of the other player, using a negation function.
- Given the negation function the third operator can be expressed in terms for the second operator.
- Deduction of the parts of the question is similar to deduction in Class 1.

Exemplary Questions

Class 1:
– What is the minimal number of specialists needed to attack the system assuming that the specialists work independently?
– What is the minimal time of the attack assuming that all actions are executed one after another?

Class 2:
– Who is the winner of the scenario?
– How likely is the system to be breached?

Class 3:
– How much data traffic will occur?
– What are the environmental costs?

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Attributes in Class 3

- Questions neither have a reference to the current player nor the owner of the question.
- Both the attacker and the defender contribute equally to the result.
- Deduction of the parts of the question is similar to Class 1.

Class Summary

	Attribute Domain $(D, \vee^A, \wedge^A, \vee^D, \wedge^D, \mathsf{c}^A, \mathsf{c}^D)$
Class 1	$(D, \mathrm{op}_1, \mathrm{op}_2, \mathrm{op}_2, \mathrm{op}_1, \mathrm{op}_2, \mathrm{op}_1)$
Class 2	$(D, \mathrm{op}_1, \mathrm{op}_2, \mathrm{op}_1, \mathrm{op}_2, \mathrm{op}_3, \mathrm{op}_3)$
Class 3	$(D, \mathrm{op}_1, \mathrm{op}_2, \mathrm{op}_1, \mathrm{op}_2, \mathrm{op}_2, \mathrm{op}_2)$

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How to Apply our Work to Attack Trees

- Attribute domains equally relevant for attack trees.
- An attribute domain for attack trees is the three tuple $(D, \vee^A, \wedge^A)$.
- Simpler case, because the owner of a question is always the attacker.
- Properly specified questions for attack tree contain three of the four parts.
- Classes 1, 2 and 3 coincide for attack trees.

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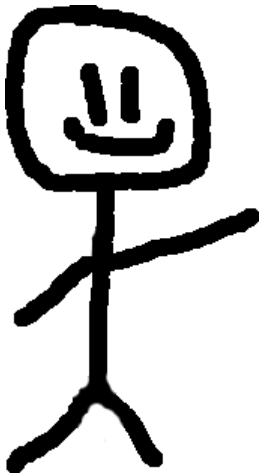
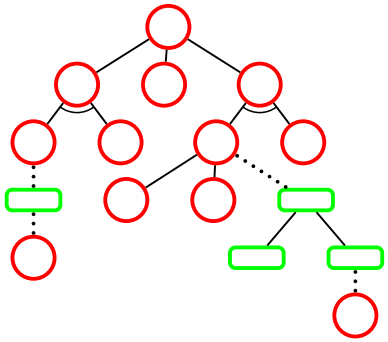
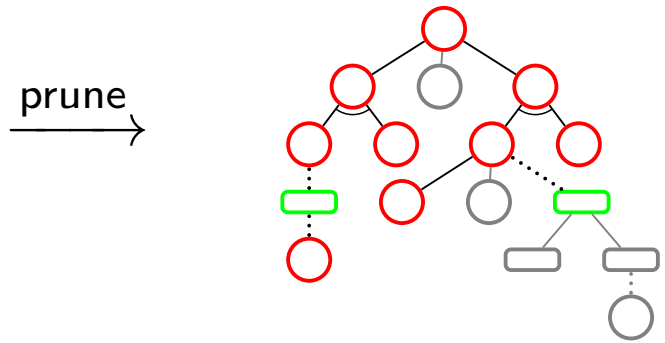
Outline

Pruning: Introduction





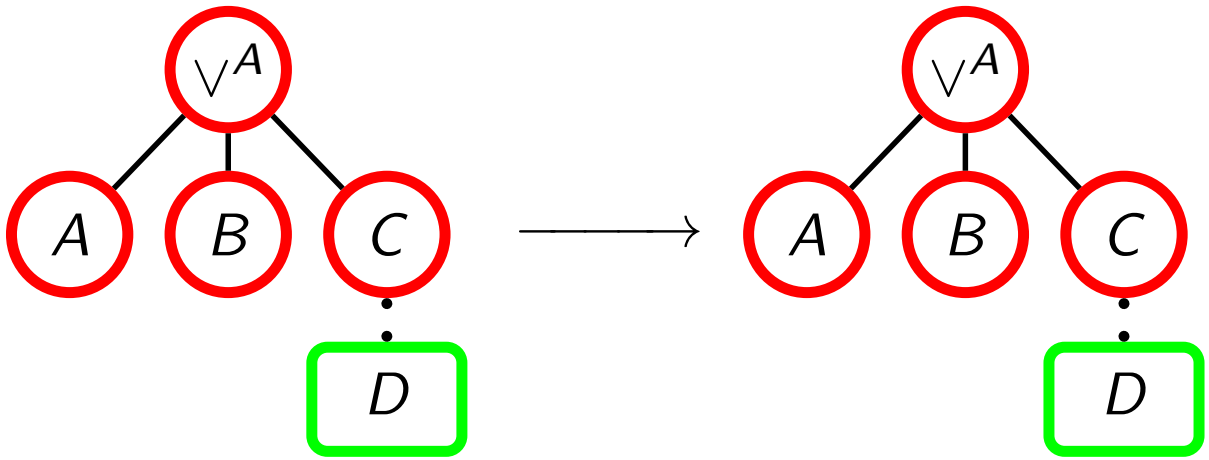


The Issue of the Assignment for the Non-Owner

Average cost of the attacker with serial execution, when all defenses are in place.

Basic assignment: $A = 1$; $B = 2$; $C = 3$; $D = \infty$

$\vee^A = avg$; $c^A = +$



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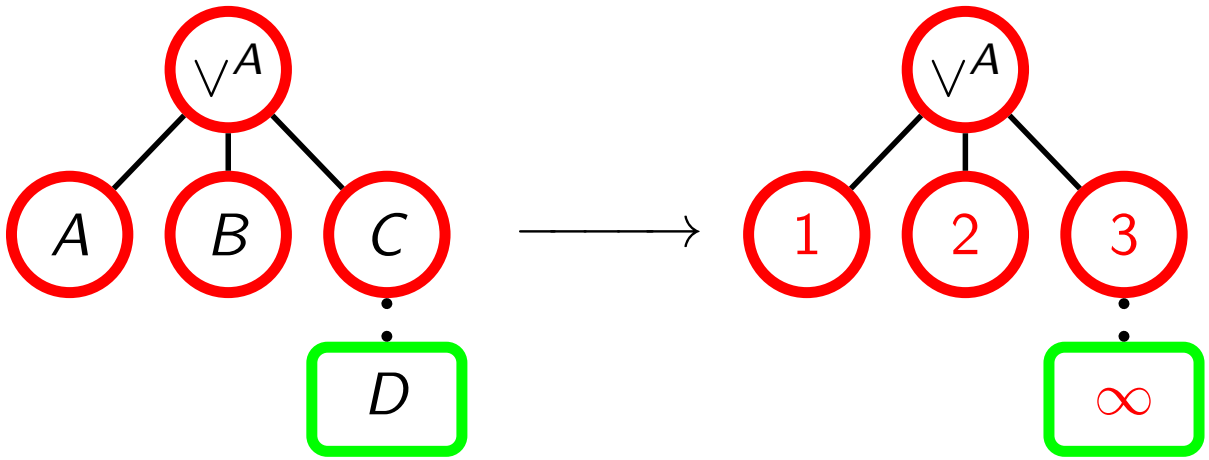

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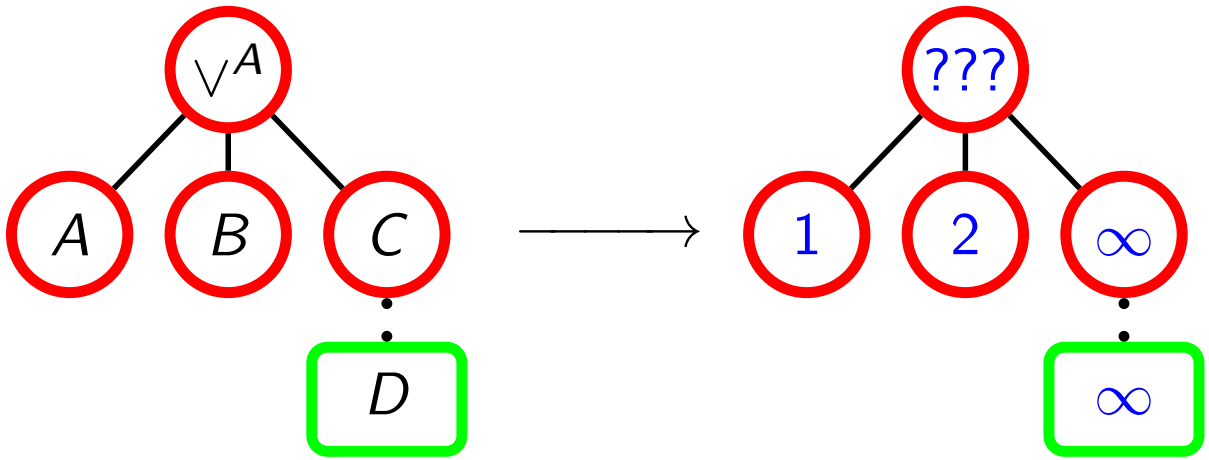

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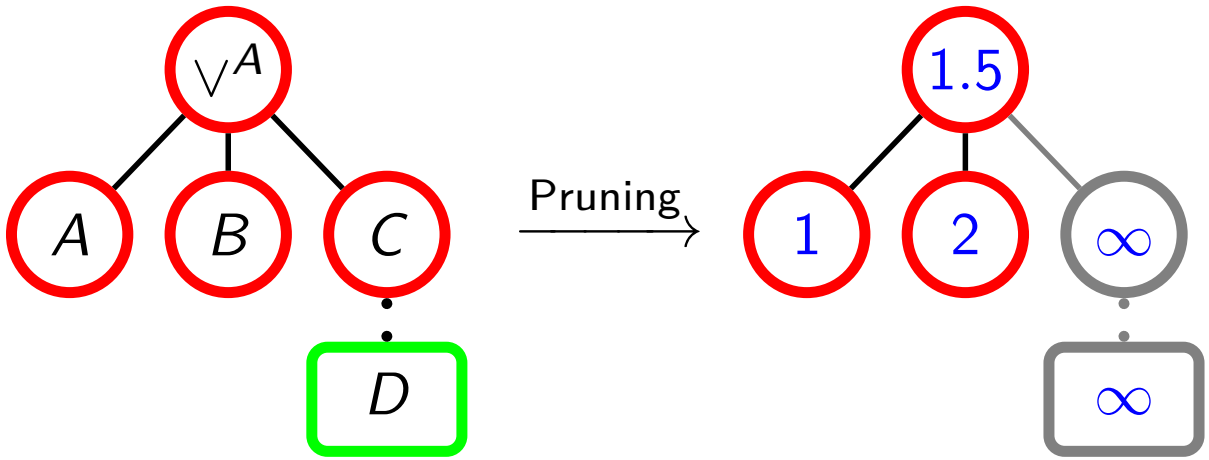

Average / Expected costs?

Pruning

To overcome this inconsistency, we can

1. prune graphically,
2. prune mathematically, or
3. ensure that assigned element of non-owner is neutral element w.r.t. c^A and absorbing w.r.t. c^D.

Pruning Algorithm

Pruning

Starting from a leaf of the non-owner, we traverse the tree towards the root until we reach a node v such that

- v is a node of the owner and part of a proper[a] disjunctive refinement;
- v is a node of the non-owner and part of a proper conjunctive refinement;
- v is a node of the owner that counteracts a refined node of the non-owner.

The subtree rooted in node v is removed from the ADTree. The procedure is repeated, starting from all leaves of the non-owner.

[a] A refinement is called proper if it contains at least two refining nodes.

Pruning Theorems

Theorem (Semiring)

When $(D, \mathrm{op}_1, \mathrm{op}_2)$ *forms a semiring, pruning is unnecessary; The pruned and the unpruned tree yield the same results.*

Theorem (Pruning equivalence)

Graphical and mathematical pruning are equivalent.

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Pruning Summary

Message to take home

Pruning is removing unsuccessful subtrees of the owner of a question.

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Summary

1. Attack–Defense Trees
2. Quantitative Analysis
3. Questions vs. Attribute Domains
   - Attributes with reference to one player (Class 1)
   - Attributes with reference to current player (Class 2)
   - Attributes with reference to third party (Class 3)
4. Pruning

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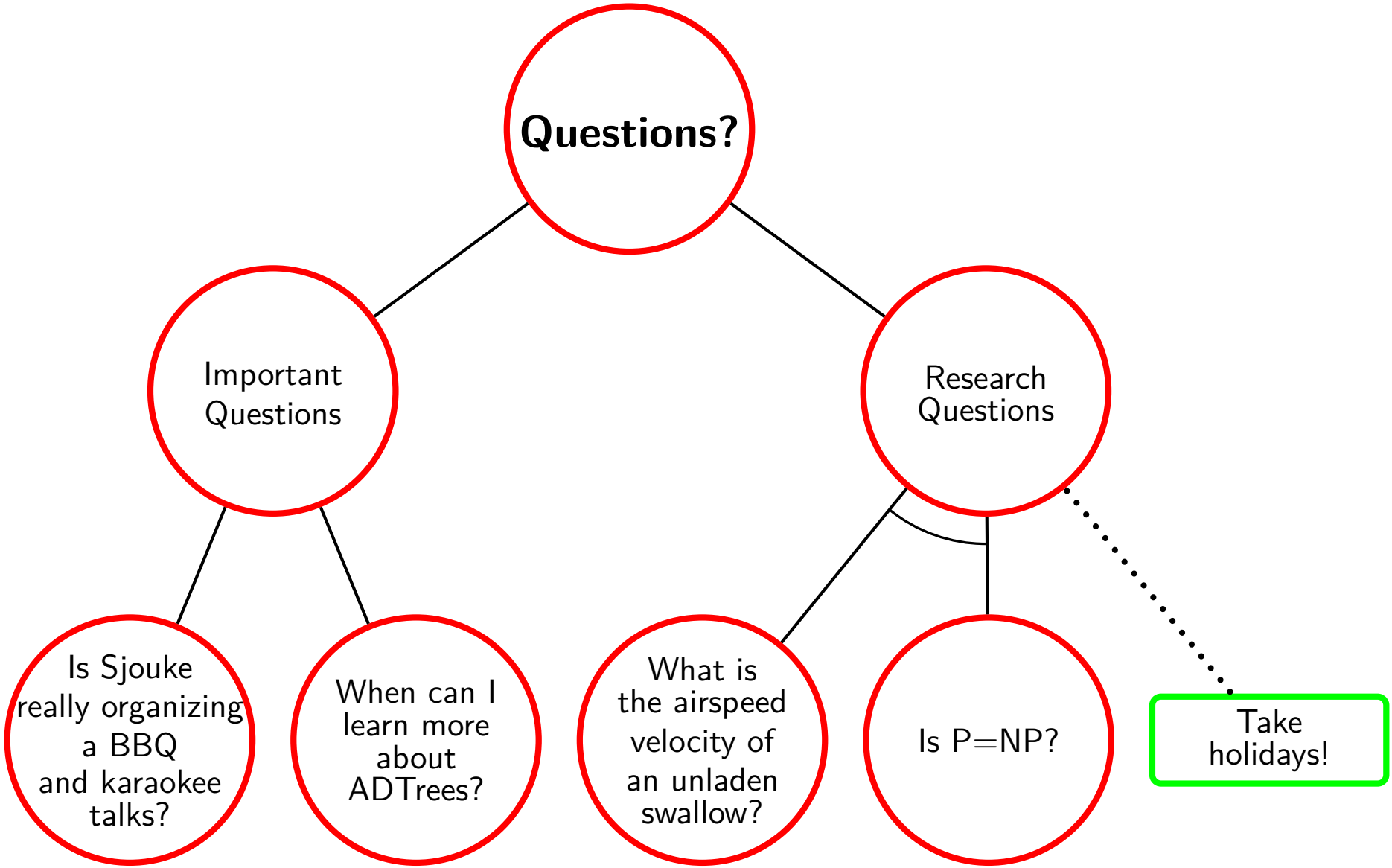
The End


Questions?
Important Questions
Research Questions
Is Sjouke really organizing a BBQ and karaokee talks?
When can I learn more about ADTrees?
What is the airspeed velocity of an unladen swallow?
Is P=NP?
Take holidays!

References


[FAST'10] Kordy, Mauw, Radomirović and Schweitzer
Foundations of Attack–Defense Trees. In Proceedings of FAST 2010, volume 6561 of LNCS. Springer 2011.


[GameSec'10] Kordy, Mauw, Melissen and Schweitzer
Attack-defense trees and two-player binary zero-sum extensive form games are equivalent. In Proceedings of GameSec 2010, volume 6442 of LNCS. Springer, 2010.


[SIIS'11] Kordy, Pouly and Schweitzer
Computational Aspects of Attack–Defense Trees. In Proceedings of SIIS 2011, volume 7053 of LNCS. Springer, 2011.


[JLC'12] Kordy, Mauw, Radomirović and Schweitzer
Attack–Defense Trees. Journal of Logic and Computation, 2012.


[IJSSE'12] Bagnato, Kordy, Meland and Schweitzer
Attribute Decoration of Attack–Defense Trees. International Journal of Secure Software Engineering, Special Issue on Security Modeling, 2012.