

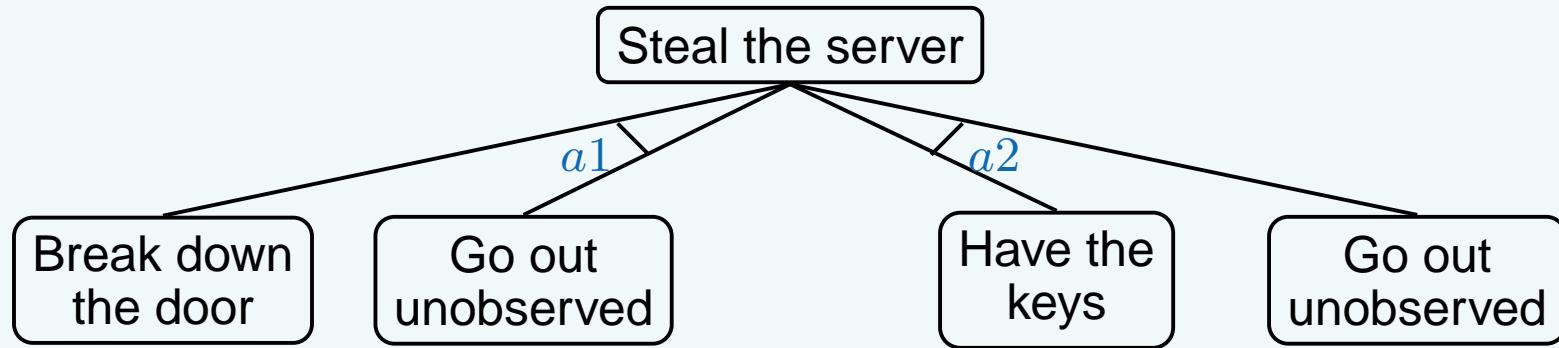
# Strategic games on defense trees

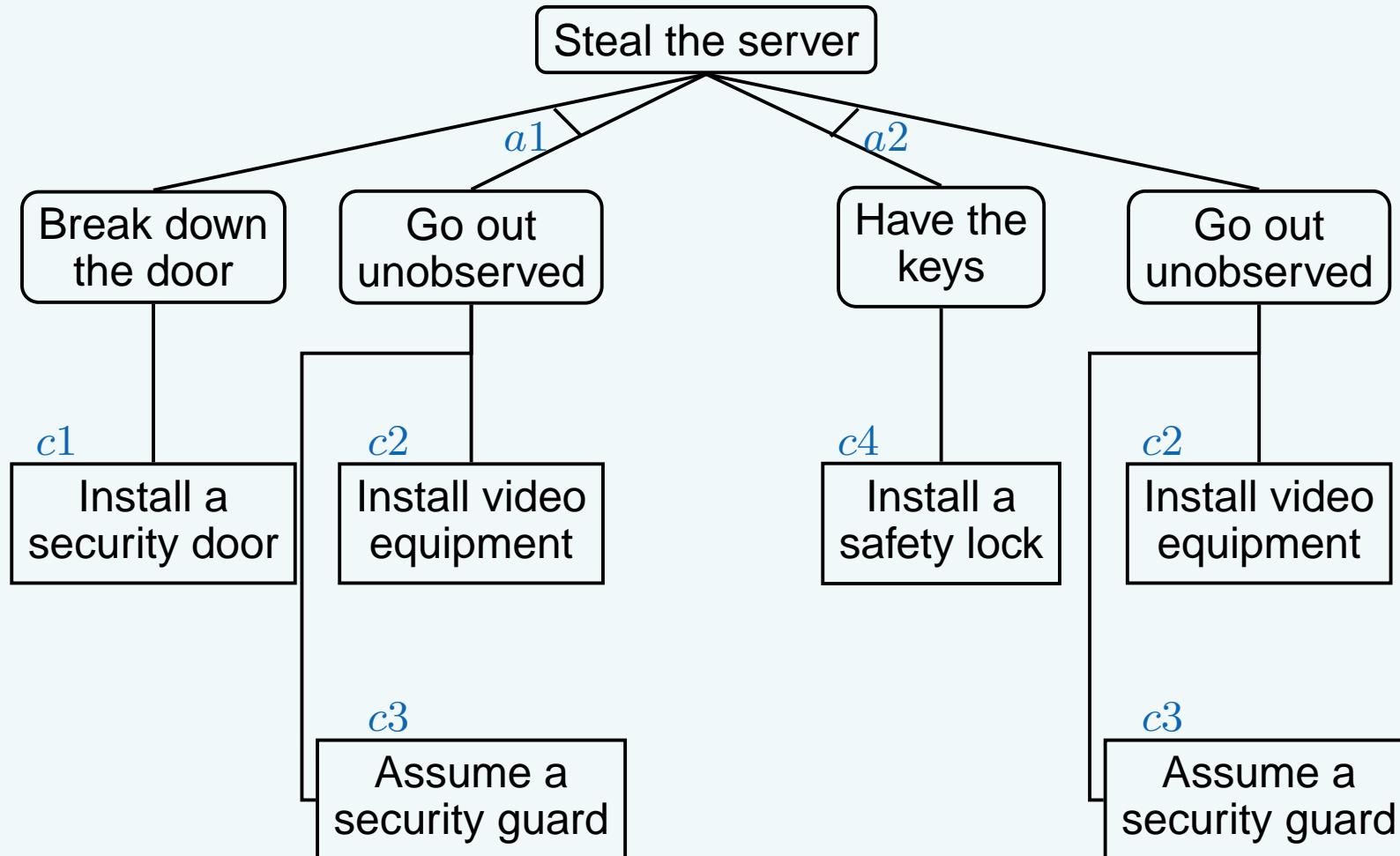
(Bistarelli/Dall'Aglio/Peretti) FAST'06

Game theory seminar

Presented by Sjouke Mauw

# A simple attack tree







# Attributes

- Return On Investment (*ROI*) = measure of the efficacy of a specific security investment w.r.t. a specific attack.
- Return On Attack (*ROA*) = measure the convenience of an attack by considering the impact of a security solution on the attacker's behaviour.

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# Return On Investment

$$ROI = \frac{ALE \times RM - CSI}{CSI}$$

Where

- Annualized loss Expectancy

$$ALE = AV \times EF \times ARO, \text{ where}$$

- Asset Value ( $AV$ ).
- Exposure Factor ( $EF$ ) is fraction of Asset Value measuring the loss due to a threat.
- Annualized Rate of Occurrence ( $ARO$ ) is the estimated number of annual occurrences of a threat.

- Risk Mitigated by a countermeasure ( $RM$ ) is the effectiveness of the countermeasure (a fraction).

- Cost of Security Investment ( $CSI$ ) is cost of implementing the countermeasure.

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# ROI computation

Attack	EF	ARO	Countermeasures	RM	CSI	ROI
$a_1$ Break down the door and go out unobserved	90%	0.1	$c_1$ Install a security door $c_2$ Install video surveillance $c_3$ Employ security guard $c_3$ Install security lock	0.7 0.1 0.5 0	1500 3000 12000 300	3.20 -0.70 -0.63 -1
$a_2$ Open door with keys and go out unobserved	93%	0.1	$c_1$ Install a security door $c_2$ Install video surveillance $c_3$ Employ security guard $c_3$ Install security lock	0 0.1 0.5 0.2	1500 3000 12000 300	-1 -0.69 -0.61 5.20

AV = 100000 euro.



# Return On Attack

$$ROA = \frac{GI \times (1 - RM) - (cost_a + cost_{ac})}{cost_a + cost_{ac}}$$

Where

- $GI$  is the expected gain from a successful attack.
- $cost_a$  is the cost sustained by the attacker to succeed.
- $cost_{ac}$  is the additional cost brought by the countermeasure  $c$  adopted by the defender to mitigate the attack  $a$ .

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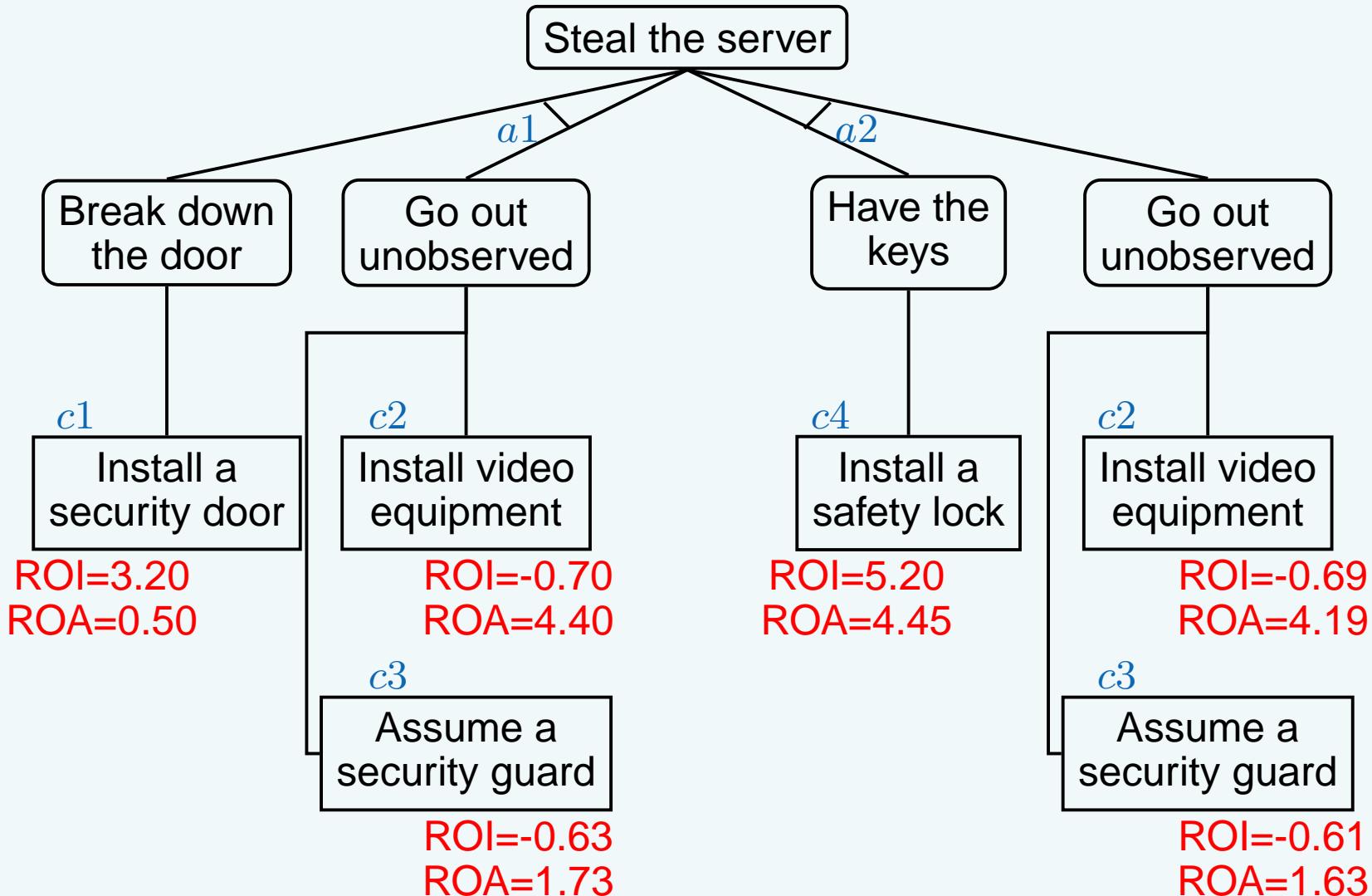
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## ROA computation

Attack	$\text{COST}_a$	Countermeasures	$\text{Cost}_{ac}$	ROA
$a1$ Break down the door and go out unobserved	4000	$c1$ Install a security door $c2$ Install video surveillance $c3$ Employ security guard $c3$ Install security lock	2000 1000 1500 0	0.50 4.40 1.73 6.50
$a2$ Open door with keys and go out unobserved	4200	$c1$ Install a security door $c2$ Install video surveillance $c3$ Employ security guard $c3$ Install security lock	0 1000 1500 200	5.14 4.19 1.63 4.45

$Gi = 30000$  euro.





# Defence trees as strategic games

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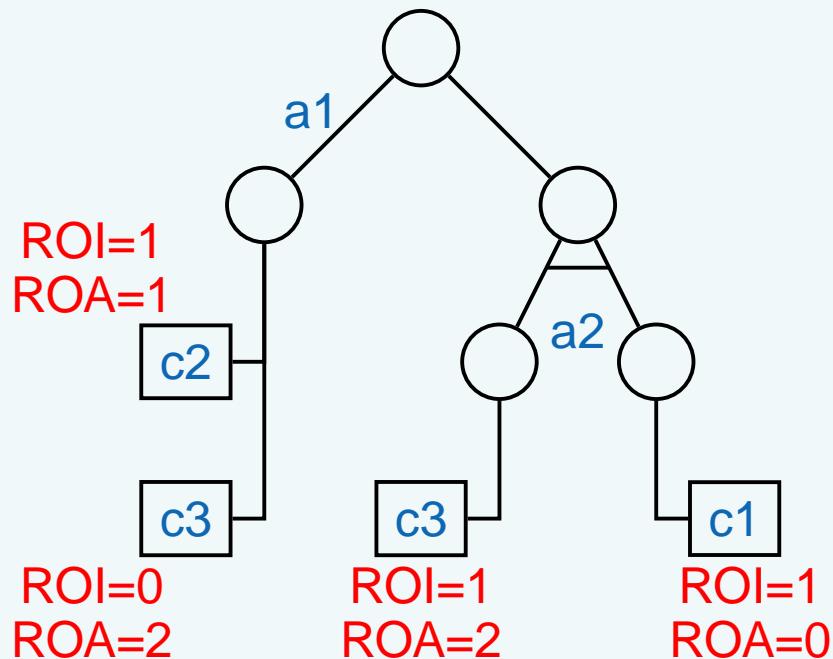
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- Two players: attacker D and defender A.
- Defender's strategies: possible countermeasures  $\{c1, c2, c3, c4\}$ .
- Attacker's strategies: possible attacks  $\{a1, a2\}$ .
- Both players want to maximize their payoff functions ROI and ROA.

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	a1	a2
c1	1, 1	1, 0
c2	1, 1	0, 2
c3	0, 2	1, 2

Nash equilibria:  $(c1, a1)$ ,  $(c3, a2)$ .



# Some quotes

- “The Nash equilibrium represents the best strategies for both the attacker and the defender (with the hypothesis that neither the attacker nor the defender have any knowledge of the other).”
- “The defender will select, if possible, both countermeasure  $c_1$  and  $c_3$ . However if the financial resources available to the system administrator are limited, only countermeasure  $c_3$  will be selected (because it will cover both strategy of the attacks).”

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# Mixed strategy

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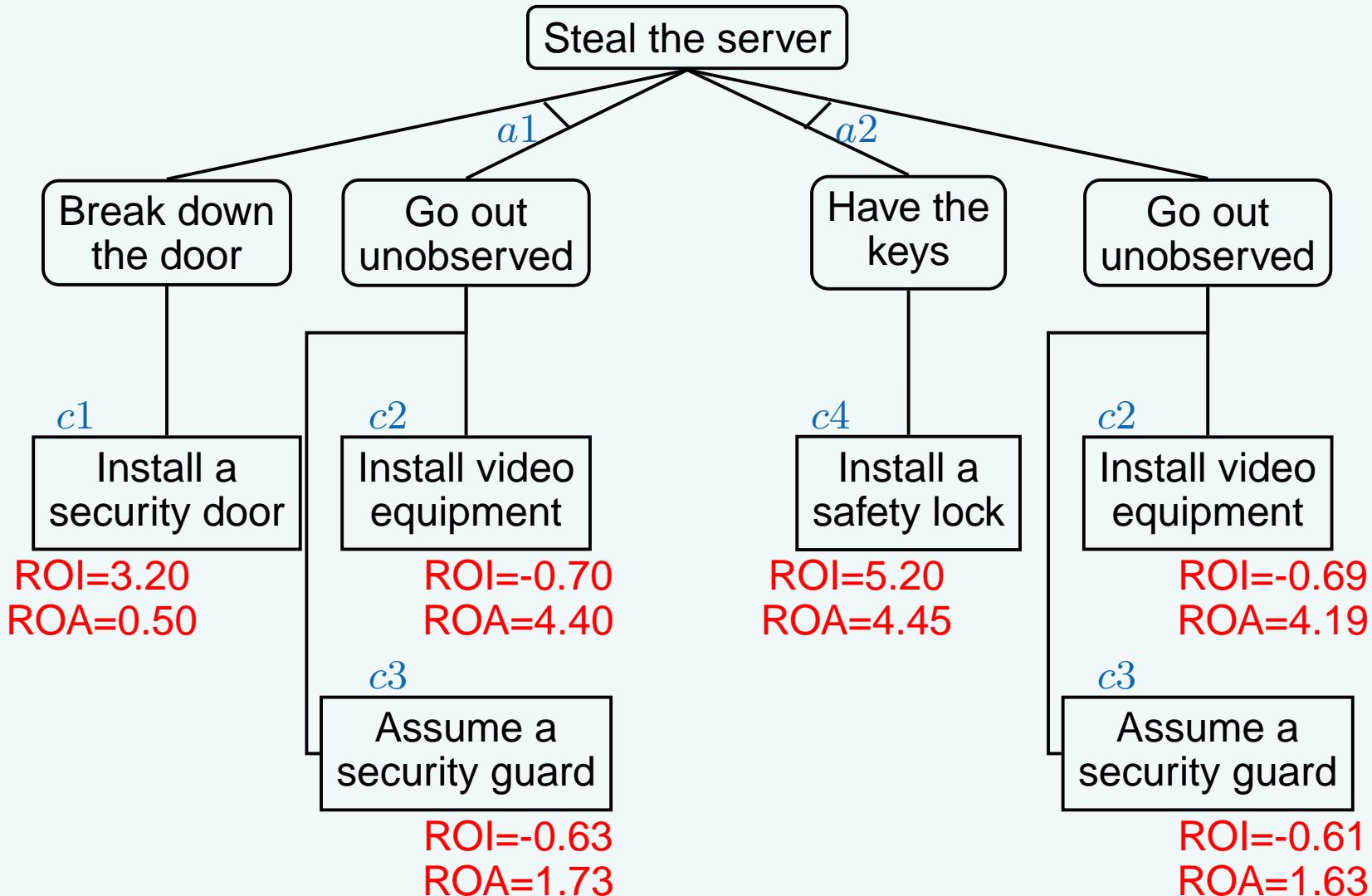
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- “Player (especially defender) deals with single attacker drawn from a population of attackers whose actions can be estimated as frequencies from previous attacks.”
- Therefore, consider a mixed strategy, consisting of a probability distribution over attacks/defences.





# In strategic form

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	a1	a2
c1	3.20,0.50	-1.00,6.14
c2	-0.70,4.40	-0.69,4.19
c3	-0.63,1.73	-0.61,1.63
c4	-1.00,6.50	5.20,4.45

No Nash equilibrium.



# With mixed strategies

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- Use Gambit to compute equilibria.
- Defender plays:
  - $c_1$  with probability  $\frac{205}{769}$
  - $c_4$  with probability  $\frac{564}{769}$
- Attacker plays:
  - $a_1$  with probability  $\frac{31}{52}$
  - $a_2$  with probability  $\frac{21}{52}$
- “the best that a system administrator can do is to invest in  $c_1$  to avoid the first attack and in  $c_4$  to avoid the second attack.”



# Consider multiple attacks/countermeasures

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- Still no Nash equilibrium with pure strategy.
- Mixed equilibrium:
  - Defender plays:
    - $c_4$  with probability  $\frac{39}{55}$
    - $\{c_1, c_4\}$  with probability  $\frac{16}{55}$
  - Attacker plays:
    - $a_1$  with probability  $\frac{5}{21}$
    - $a_2$  with probability  $\frac{16}{21}$
  - Note: strategies  $\emptyset$  and  $\{a_1, a_2\}$  are uniformly dominated by simple strategies  $a_1$  and  $a_2$ . So the attacker has no interest in combining the actions together.



# Future

- Extend to 1 defender and n attackers.

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<http://www.sci.unich.it/~bista/papers/papers-download/DG4.pdf>