A Quadratic Lower Bound for Simulation

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Where innovation starts

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Process Algebra as a Tool for the Specification and Verification of CIM-architectures

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Now the complete definition of the Workcell (W) is

$$W = \tau_I \partial_H (WC \| T^\lambda \| WA \| WB)$$

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Process equivalences.

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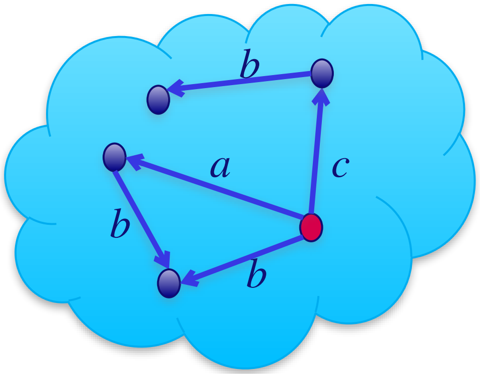

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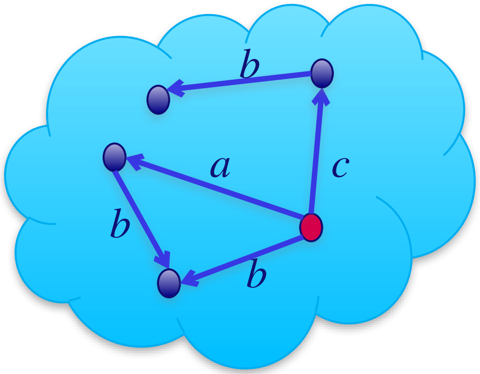

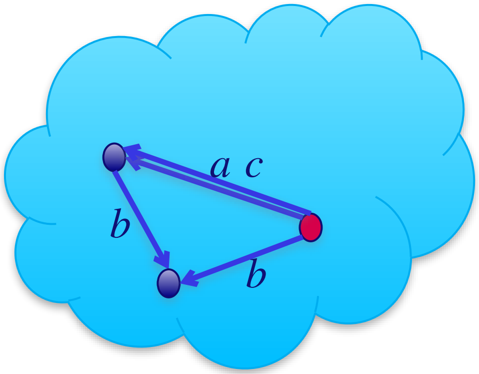

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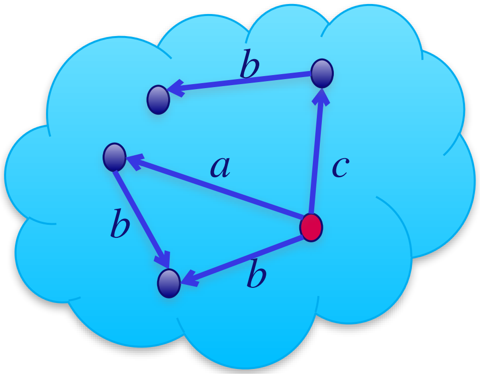

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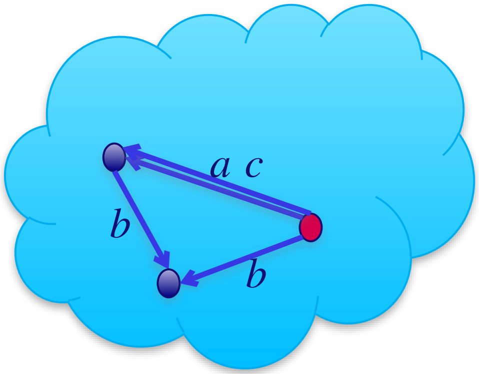

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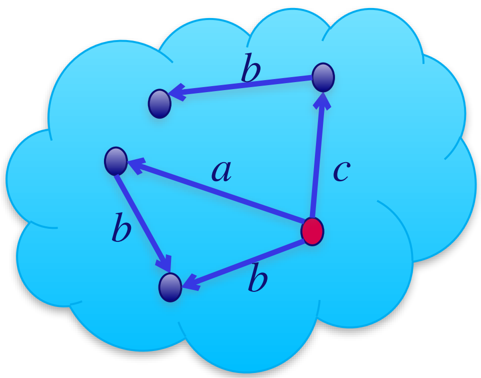

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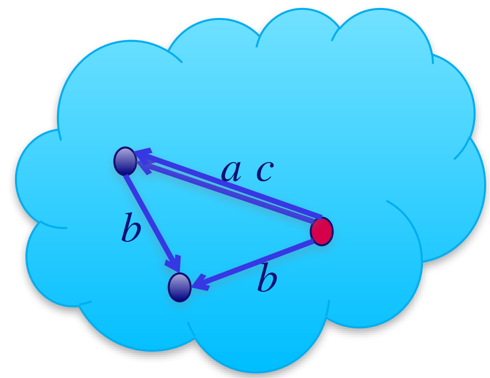

Equivalences:
Strong bisimulation
Branching bisimulation
Simulation equivalence
Simulation preorder

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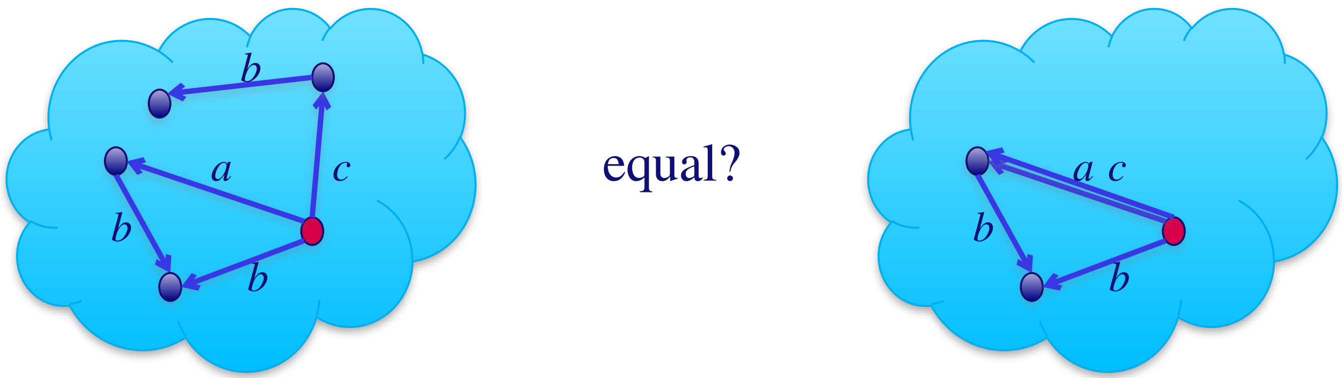

n: number of states
m: number or transitions

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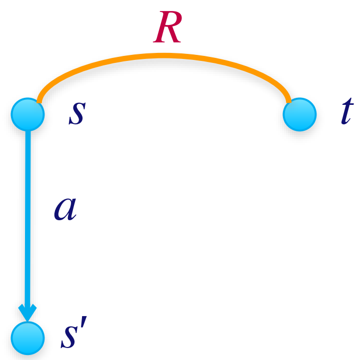
Simulation preorder and equivalence



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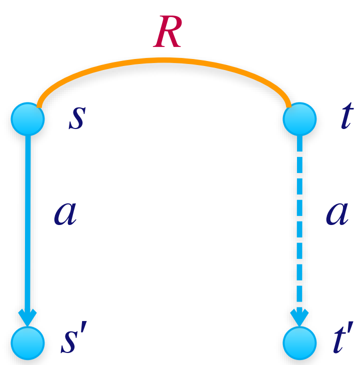

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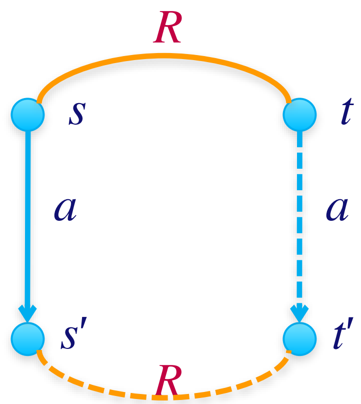

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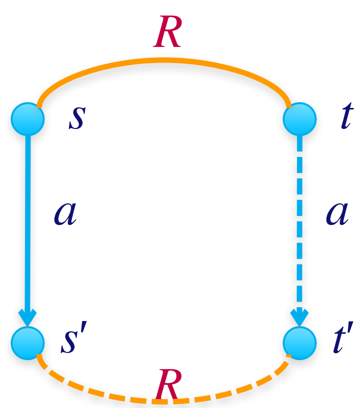
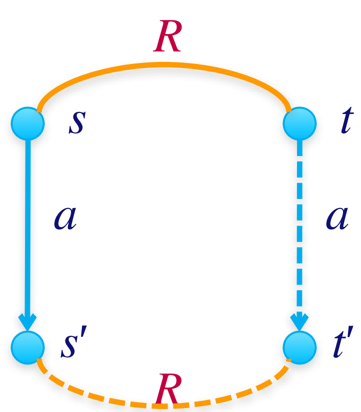


A state $t \in S$ *simulates* a state $s \in S$, notation $s \subsetneqq t$, iff there is a simulation relation R such that sRt.

Simulation preorder and equivalence



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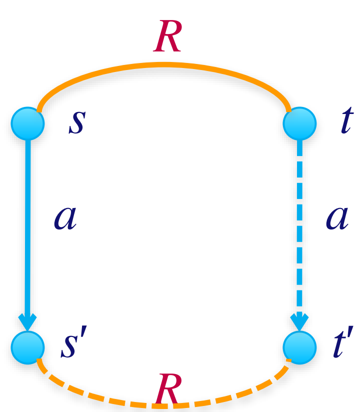

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An FSA B *simulates* an FSA A, notation $A \subsetneq B$, iff the initial state of B simulates the initial state of A.

Complexities to calculate equivalences on regular transition systems.

Strong bisimulation	$O(m \log n)$	Paige/Tarjan, 1987
- in parallel	$O(n)$	Martens/Groote/Haak/Hijma/Wijs, 2022
Branching bisimulation	$O(m \log n)$	Groote/Jansen/Keiren/Wijs, 2020
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(Strong) Exponential Time Hypothesis
Impagliazza & Paturi, 1999

ETH: Satisfiability for 3-CNF requires time $O(2^{\varepsilon n})$, for some $\varepsilon>0$; n the number of proposition letters

SETH: Satisfiability for CNF cannot be solved in time $O(2^{\delta n})$, for any $\delta<1$

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$$\text{ETH} \Rightarrow \text{SETH} \Rightarrow \text{P} \neq \text{NP}$$

Intersection of regular automata.

(Non-)emptiness of language intersection for FSAs.

$$L(A_1) \cap L(A_2) = \varnothing \quad ????$$

Algorithm for NEI: $O(n^2)$.

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SETH $\Rightarrow$

NEI for k automata is not solvable in $O(n^{k-\varepsilon})$ for any $\varepsilon>0$

Proof sketch that SETH implies NEI not solvable in $O(n^{2-\varepsilon})$

$$(x_1 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2)$$

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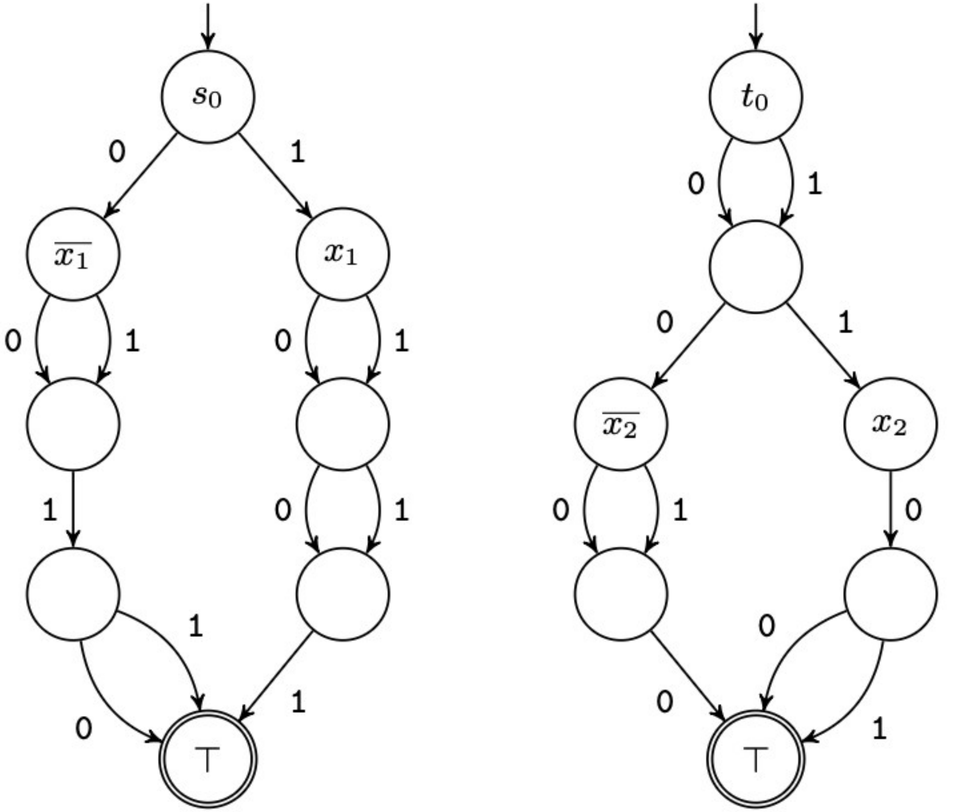

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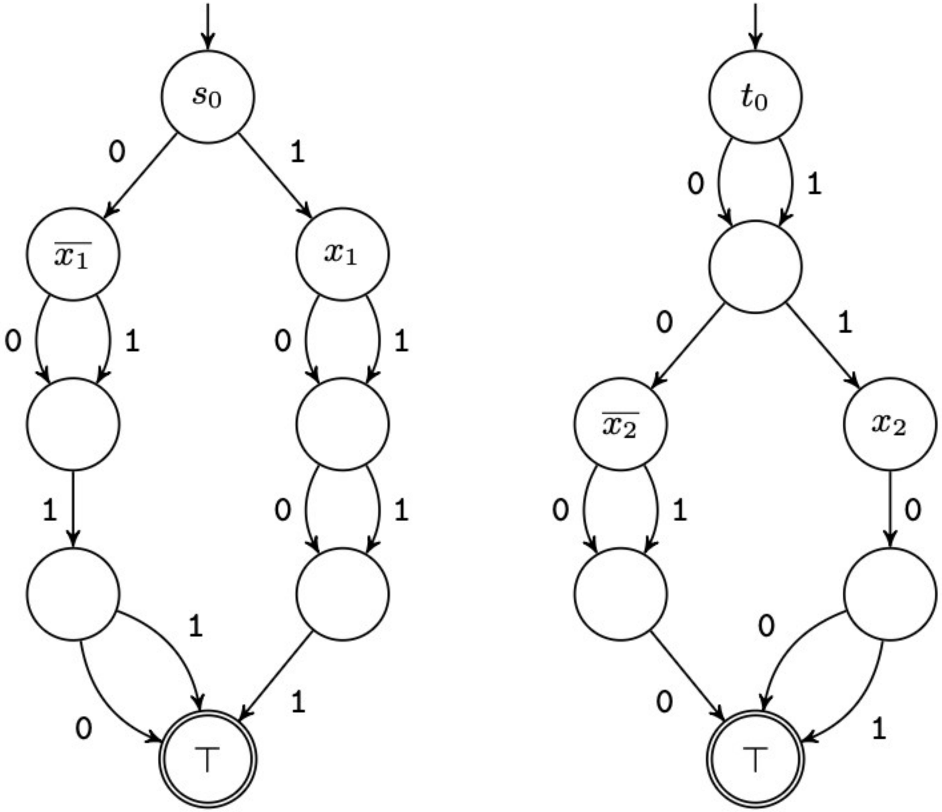

x_1

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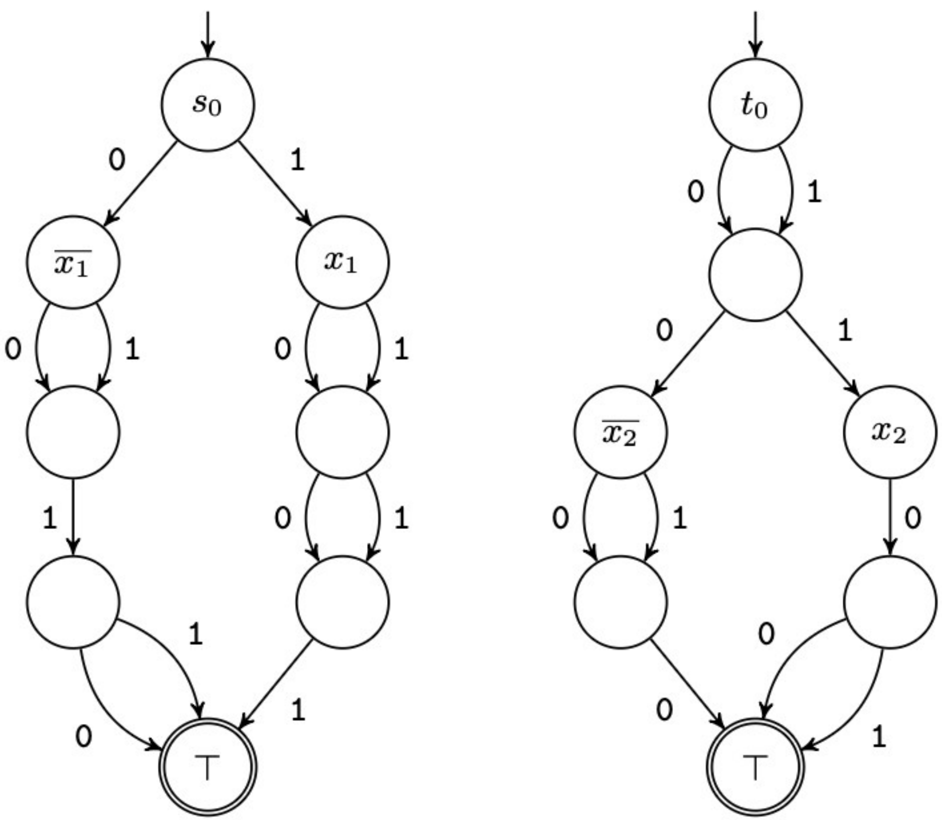

x_1

x_2

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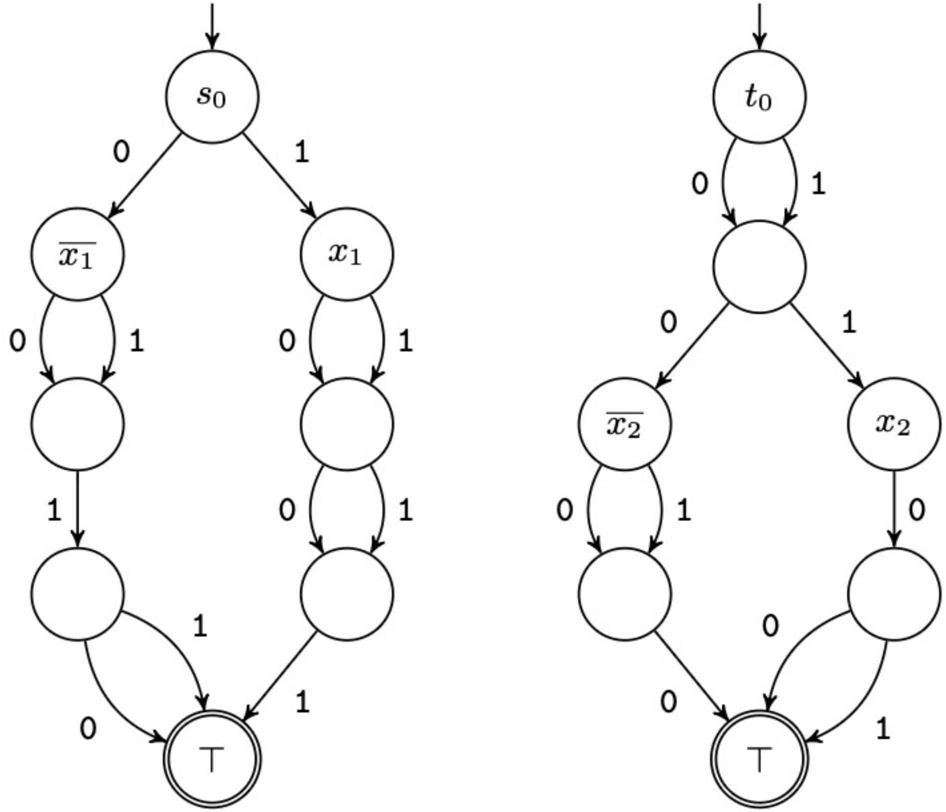

x_1

x_2

0 if x_1 satisfies first clause

1 if x_2 satisfies first clause

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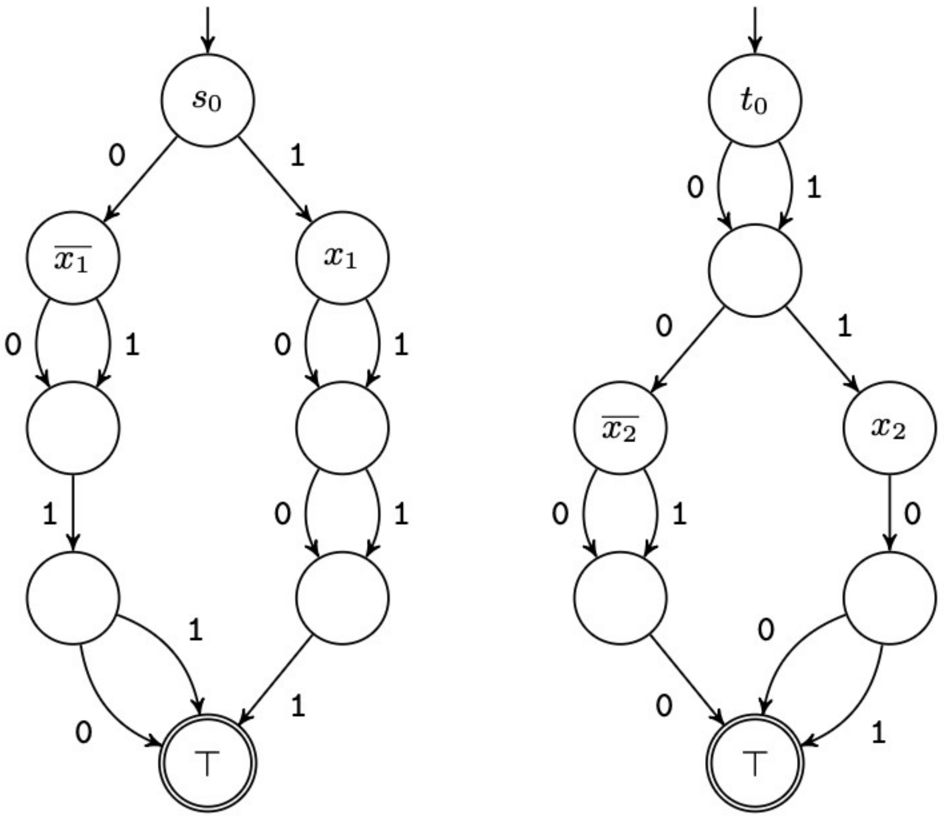

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idem for second clause.

Proof sketch that SETH implies NEI not solvable in $O(n^{2-\varepsilon})$

$$(x_1 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2)$$

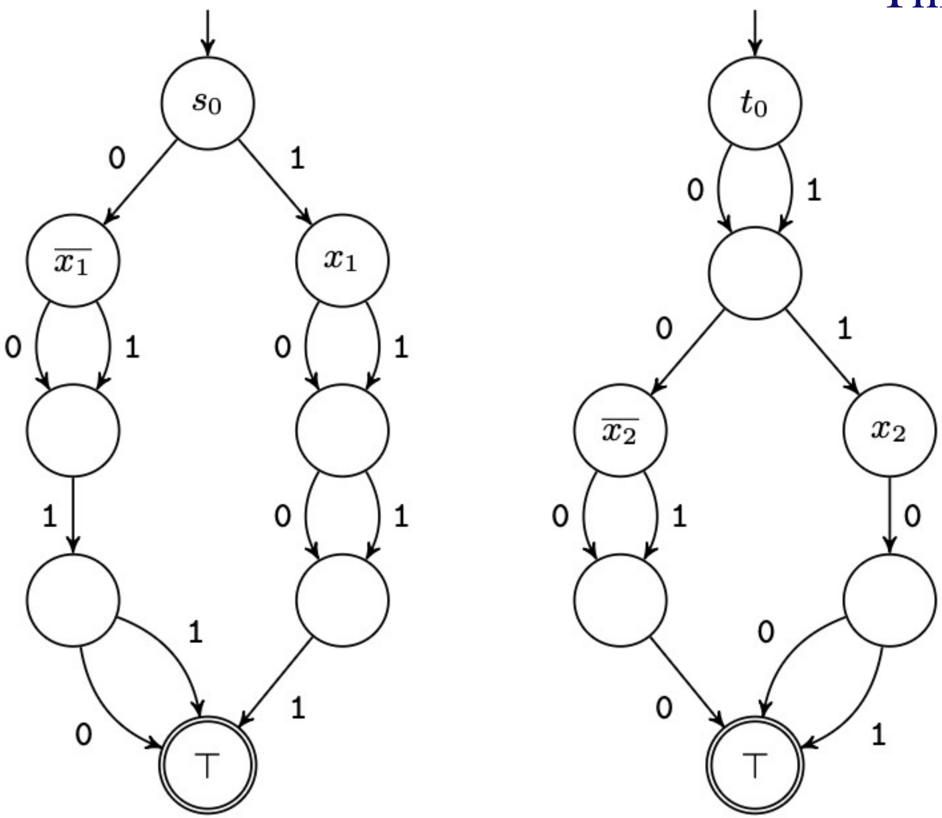
Suppose NEI is solvable in $O(N^{2-\varepsilon})$

$N = mn2^{\frac{1}{2}n}$ is the # of states of each DFA.

Then SAT is solvable in:

$O((mn2^{\frac{1}{2}n})^{2-\varepsilon}) = O(m^{2-\varepsilon}n^{2-\varepsilon}2^{n(1-\frac{1}{2}\varepsilon)})$

This contradicts SETH.



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Back to simulation equivalence.

Lemma. Consider deterministic regular automata (with explicit termination transitions) A and B:

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Theorem. SETH implies similarity ($\subsetneqq$) cannot be decided in $O(n^{2-\varepsilon})$ for any $\varepsilon>0$:

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Lemma. Consider deterministic regular automata (with explicit termination transitions) A and B:

$$L(A) \subseteq L(B) \quad \text{iff} \quad A \precsim B$$

Theorem. SETH implies similarity ($\precsim$) cannot be decided in $O(n^{2-\varepsilon})$ for any $\varepsilon>0$:

Proof. Deciding

$$L(A) \cap L(B) = \varnothing$$

is equivalent to deciding:

$$L(A) \subseteq \Sigma^* \backslash L(B)$$

Simulation equivalence.

Theorem. SETH implies that simulation equivalence cannot be decided in $O(n^{2-\varepsilon})$ for any $\varepsilon>0$. FSAs need to be nondeterministic, but can have finite behaviour.

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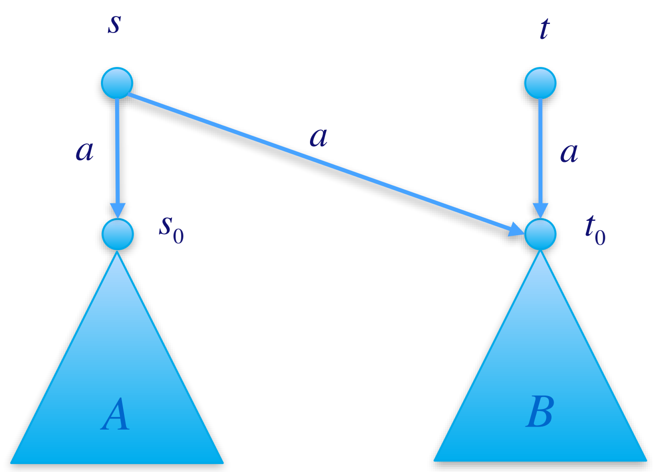
Proof. Consider DFAs A and B. Then

$$s_0 \subsetneq t_0 \quad \text{iff} \quad s \sim t.$$



Conclusion

Assuming SETH (which seems very reasonable)

- Calculating simulation preorder is essentially quadratic in the number of states for DFAs.
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