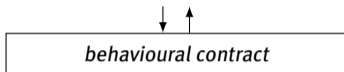


Deciding Impossible Futures

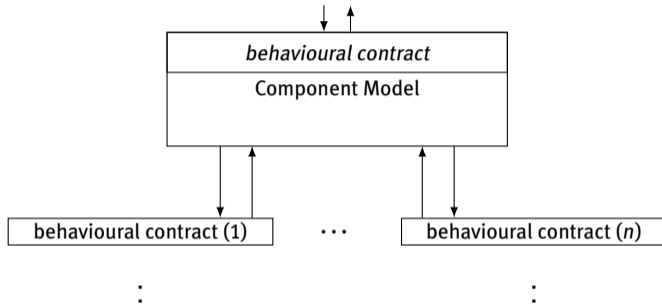
Maurice Laveaux and Tim Willemse
{m.laveaux,t.a.c.willemse}@tue.nl



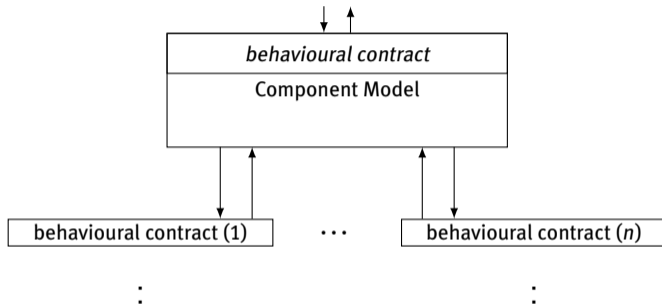




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- ▶ Contract is **refined** into component and other contracts and **proven correct**
- ▶ Generate executable code that is behaviourally **equivalent** to component model



Ideation



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Prototyping





Ideation



Prototyping



Consolidation



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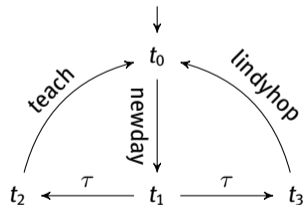
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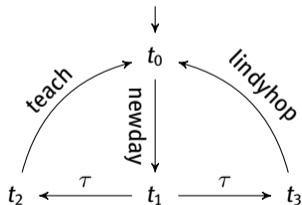
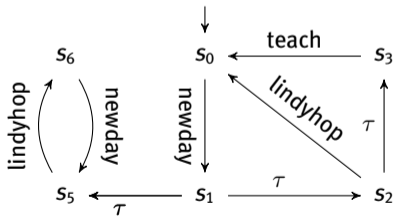
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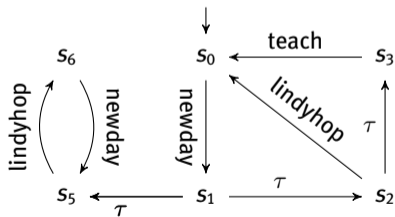
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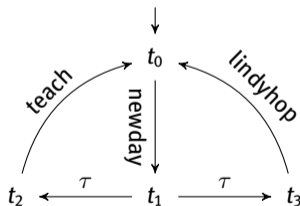
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- ▶ **Impossible Future inclusion:** $s \sqsubseteq_{if} t$ iff $IF(s) \subseteq IF(t)$

(Voorhoeve&Mauw)

$$IF(s) = \{(w, X) \in \text{Act}^* \times 2^{\text{Act}^*} \mid \exists s' : s \xRightarrow{w} s' \text{ and } X \cap WT(s') = \emptyset\}$$

HOW TO EFFECTIVELY DECIDE $s \sqsubseteq_{if} t$?

A relation $R \subseteq S_1 \times S_2$ is a **weak simulation** iff whenever $s R t$ and $\alpha \in \text{Act} \cup \{\tau\}$:

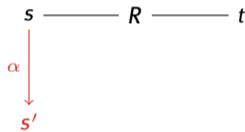
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*If $s \xrightarrow{\alpha} s'$ for some s' , then there is some t'
such that $t \xRightarrow{\bar{\alpha}} t'$ and $s' R t'$*

$$s \text{ ————— } R \text{ ————— } t$$

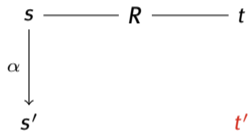
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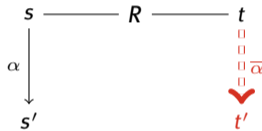
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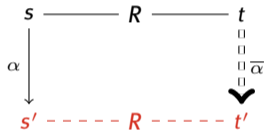
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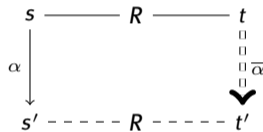
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State s is **weakly simulated by** t , written $s \leq t$ iff there is a weak simulation R such that $s R t$

A **weak simulation game** is a game played by Attacker and Defender on a graph $(V_A \cup V_D, E)$, where:

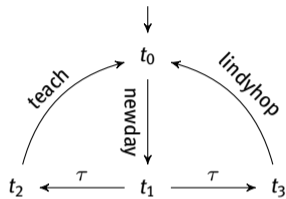
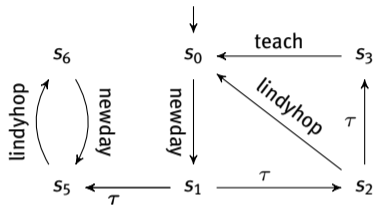
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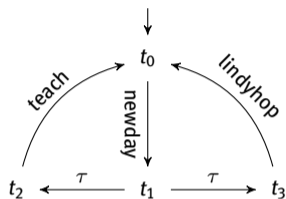
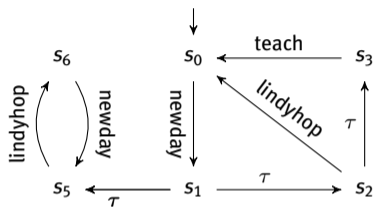
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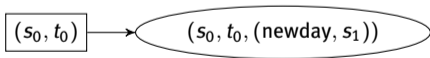
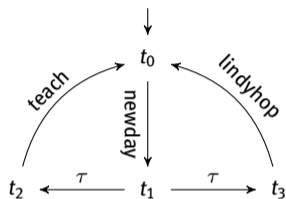
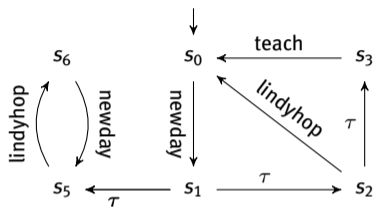
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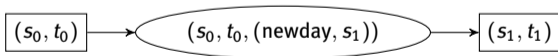
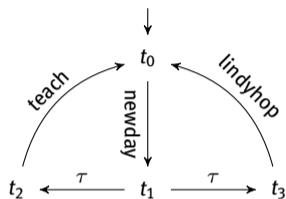
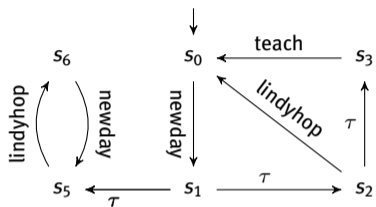
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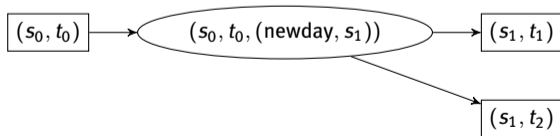
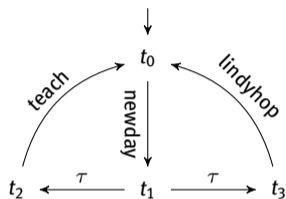
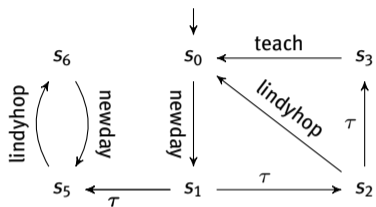


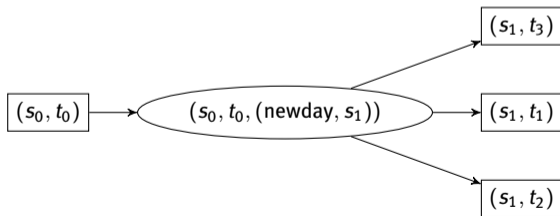
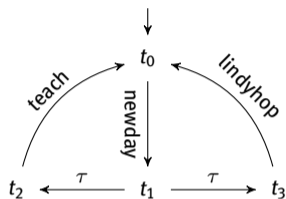
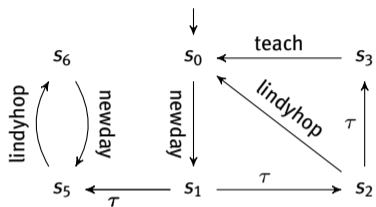


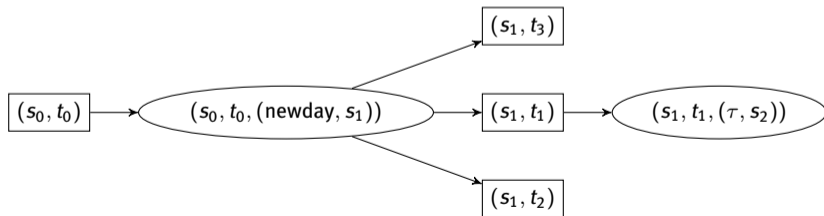
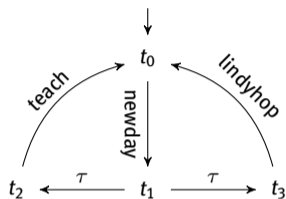
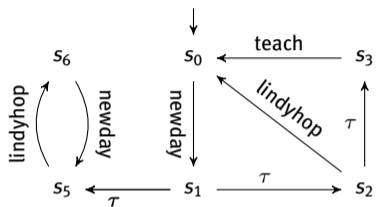
(s_0, t_0)

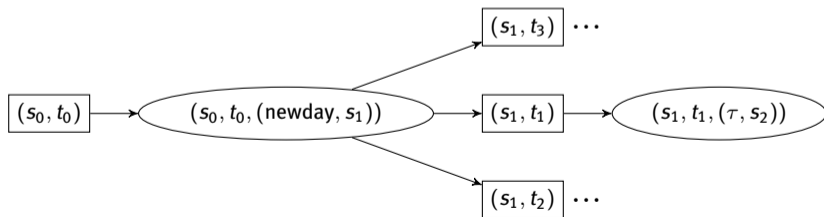
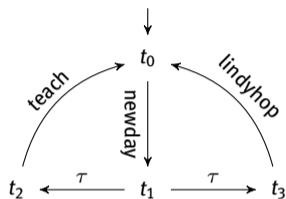
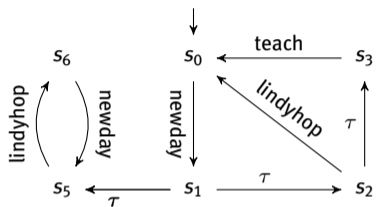


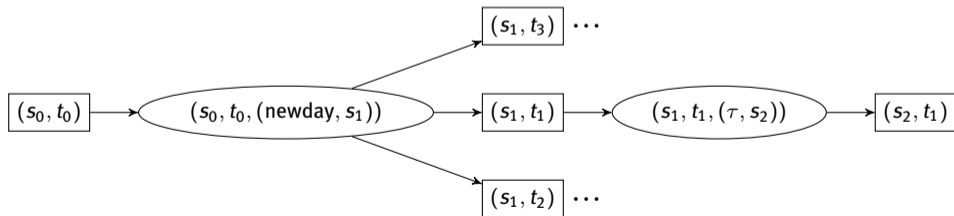
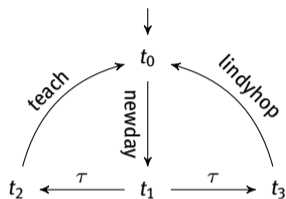
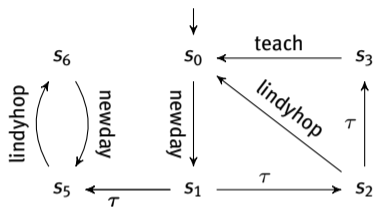


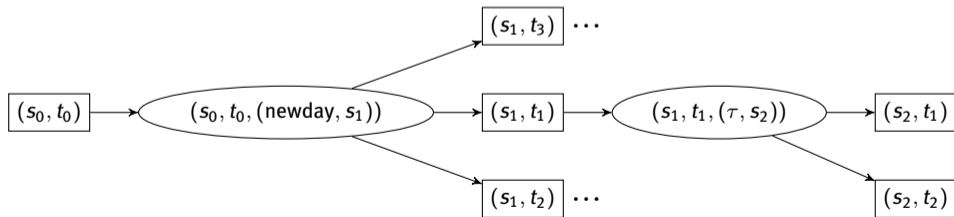
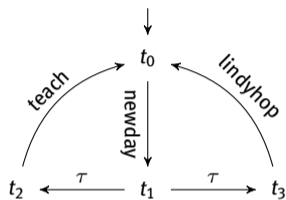
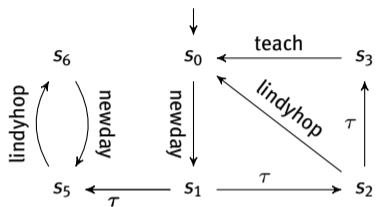


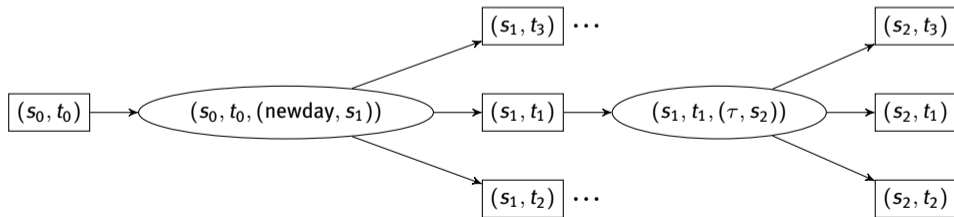
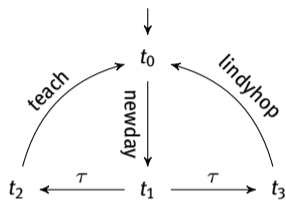
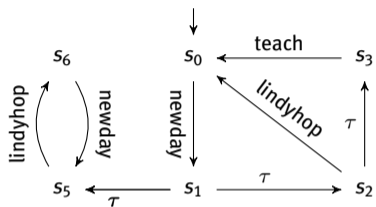


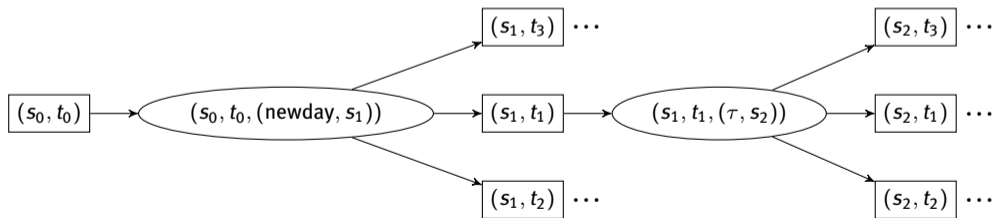
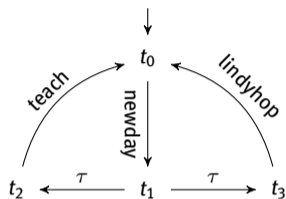
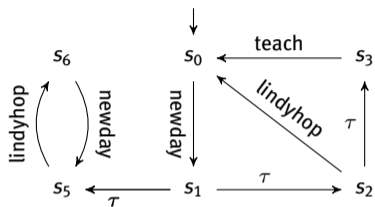












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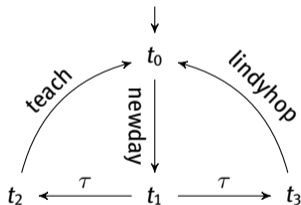
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- ▶ So in this case $s \sqsubseteq_{wt} t$ is a reachability problem solvable using a breadth-first or depth-first search.

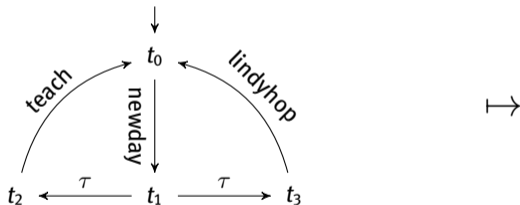
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Example



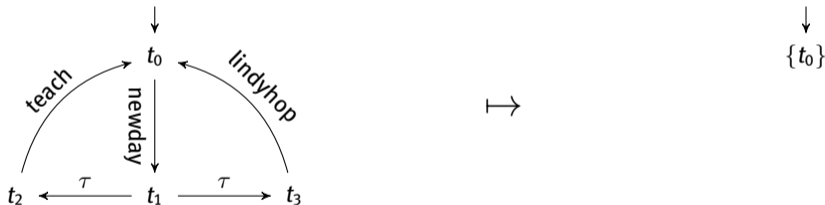
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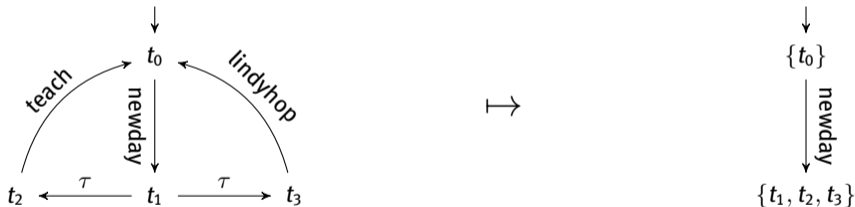
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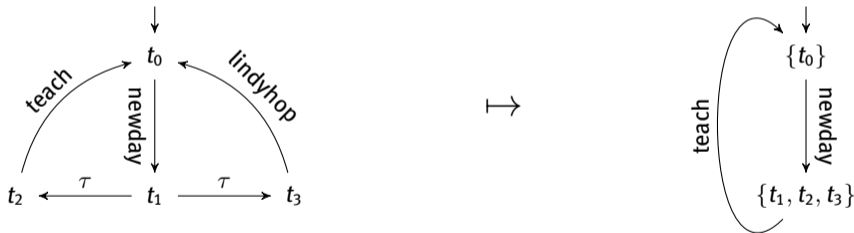
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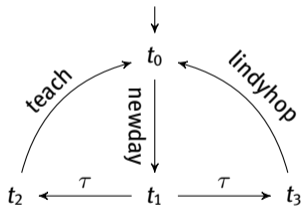
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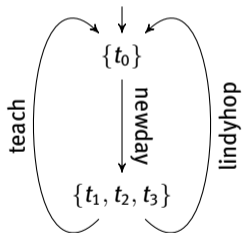


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Example



\mapsto



So:

$$s \sqsubseteq_{wt} t$$

if and only if

$$s \sqsubseteq_{wt} \text{norm}(t)$$

if and only if

$$s \leq \text{norm}(t)$$

if and only if

No Defender-sink is reachable from $(s, \text{norm}(t))$

Modifying the simulation game to decide Impossible Futures:

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But: implemented naively this approach is sloooooooooooooow and does not even terminate in many cases

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Optimising the search for Defender-sinks in the simulation game:

- ▶ Store visited Attacker vertices in an **antichain** $A \subseteq S_1 \times 2^{S_2} \setminus \{\emptyset\}$
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But: implemented naively this approach is also **slow** though it typically terminates

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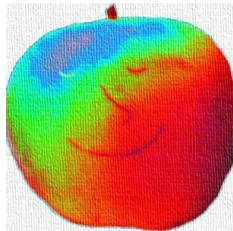
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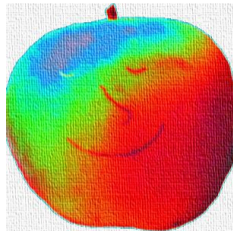
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Towards consolidation: Verum is currently investigating how to integrate Impossible Futures in their Dezyne tool

Portrait of an Artist 1



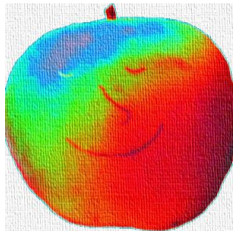
Portrait of an Artist 1



Portrait of an Artist 2



Portrait of an Artist 1



Portrait of an Artist 2



*Artist informally known as Sjouke
who created both self-portraits*

